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Research Article

Fitting Various Growth Equations to the Daily Milk Yield Data of Nili-Ravi Buffaloes and Cholistani Cows at Intake at Maintenance Levels

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ABSTRACT

This study described the daily MY in buffalo and cow under restricted feeding conditions using the five growth models (Brody, Von Bertalanffy, Logistic, Gompertz and Wood). In addition, the species-wise differences in lactation parameters were also tested. These models were fitted to the lactation data of four whole lactations (two buffaloes and two cows) using 1200 unadjusted MY records. Fitting of the model was evaluated through appropriate fitness indicators such as the adjusted R^2, Akaike's Information Criterion (AIC), Bayesian Information Criterion (BIC) and Root Means Squared Error (RMSE). The Wood's model provided the best fit of the lactation curves with logical values of parameter estimates owing to higher R^2 and lower AIC, BIC and RMSE than other equations. The Wood's model had a better fit of lactation data of cows than of buffaloes. The average estimated values for the initial MY (a), ascending phase before peak MY (b) and descending phase after peak MY (c) were 4.75, 0.238 and 0.004, and 2.56, 0.321 and 0.006 kg/day for buffaloes and cows, respectively. The magnitude of lactation parameters remained higher (P<0.05) in buffaloes for a, peak MY, persistency and lactation yield than in cows. The Woods' model fairly accurately described the lactation data than other equations under restricted feeding conditions, with poor fitting in buffaloes to moderate fitting in cows.

Keywords: Intake at maintenance, Growth models, Lactation curves, Species differences

Introduction

Alterations in daily milk yield (MY) are evident such that it increases from calving to the peak production and thereafter decreases smoothly until the end of lactation [1]. Knowledge of lactation curves in dairy cattle is important for decision making on herd management involving feeding and selection strategies, and it is also a key element in determining optimum strategies for insemination and replacement of dairy cows [2,3]. The data on MY has been well described in cows [1,4,5] and buffaloes [6,7] using various lactation functions [8]. Variations in the shape and form of the lactation curve arise from the factors such as genetic make-up, parity, diet, and other environmental influences [3,6,9]; the diet being one of the important factors [10,11]. The MY data described by various researchers presented data of lactating animals fed at intake at production levels [6, 11] or if there were feed restrictions, they were carried for a certain period of lactation cycle [12,13] but not for the whole length of lactation. We therefore, expected to observe different shape and form of lactation curve of lactating animals under intake at maintenance levels than intake at production levels.

To describe the milk yield, two types of models can be used i.e., the mechanistic ones, to describe the causative mechanism under the biology of lactation [14,15]; and the empirical ones, mainly for the quantification [6]. The selection of the model is the basic tool between the fitting biological properties of a model and its biological interpretations [16]. For instance, the mechanistic model [15] have parametric advantages for biological interpretations, but difficult to fit in when parameters showing greater standard errors and multi-collinearity. Conversely, the empirical ones can provide the acceptable solutions and adequate fits to data, but not on the biological basis. The mechanistic model proposed by Dijkstra et al.[14], may be the solution for such a gap providing fitting properties and biological interpretations [17].

The purpose of the current study was to investigate the suitability of lactation curve model in describing the daily MY data of buffaloes and cows at intake at maintenance levels. The second objective was to compare buffaloes with cows regarding differences in the magnitude of lactation parameters.

Material and Methods

Ethical Statement

All experiments were conducted according to the criteria of the University's Animal Care and Management Committee (The IUB, 2015).

Milk Yield Performance of Lactating Animals under Maintenance Feeding Regime

This study was conducted at The Department of Livestock Management, The Islamia University of Bahawalpur (29.39°N, 71.68°E), Bahawalpur, Pakistan. Four rumen-cannulated (Bar Diamond, Parma, ID, USA) animals including 2 lactating Nili-Ravi buffaloes, mean live weight (LW) = 509±43.4 kg, age = 2225±49.5 days; and 2 lactating Cholistani cows, LW = 289±29.4 kg, age = 1115±21.9 days at the start of the experiment were used for the production trial. The animals were offered a standard diet slightly above the maintenance level for meeting the requirements of milk production but not ad libitum throughout the experiment. The animals were restricted to consume dry matter at 1.80% of LW instead of 3.0% (as recommended by National Research Council, 2001), with a forage to concentrate ratio of 80:20 on dry matter basis. Ingredients and mean chemical composition of the diets are presented elsewhere [18]. The animals were confined to individual stalls, individually fed and given access to fresh clean water as per requirements. Animal were milked twice daily and MY was recorded manually by the milk man. The animals were placed in the trial from the 1st day just after parturition. Data of daily MY of 4 complete lactations with 1200 daily milk records were used which were collected between the years 2015 and 2016. Milk yield was recorded daily from day 5 to 305 after parturition and milk samples for quality analyses were collected weekly. We did not exclude data based on minimum daily MY, fat or protein content. The following formula was used for the calculation of 4% FCM of each cow: FCM = [(0.4 x kg milk) + (0.15 x kg milk x fat %)] [19]. Also, the yield of energy-corrected milk (ECM) was calculated by the following formula: ECM (kg) = kg milk x (383 x fat% + 242 x protein% + 783.2)/3140 [20]. The standard milk fat and protein contents i.e. 5.5 and 4.0%, and 4.0 and 3.73% based on data obtained using wet chemistry analyses for buffaloes and cows, respectively were used to calculate the FCM and ECM.

Model Fitting to the Lactation Data

The lactation data were cut at a standard lactation length of 305 days. No adjustments of the raw data were made i.e. outliers and out-of-range productive records were not deleted from the analyses and the models presented in *Table 1* were fitted to these data.

The models were fitted using R (ver. 4.2.1; The R Foundation for Statistical Computing, 2015), using the statement nlsfit in software package Easyreg® (easyreg:

Table 1. *Growth equations used to describe the lactation curve in buffaloes and cows*

Equation	Functional Form	Time to Peak MY	Peak MY	Persistency	Lactation Yield
Brody [21,22]	$y = a(1 - bexp^{-ct})$	---	---	---	---
Gompertz-Laird [22,23]	$y = abexp\left[\frac{b(1-e^{-ct})}{c-ct}\right]$	---	---	---	---
Logistic-Nelder [24]	$y = \frac{abc(c-b)e^{-at}}{[b+(c-b)e^{-at}]\quad 2}$	---	---	---	---
Wood [5]	$y = at^{b}exp(-ct)$	b/c	$a(\frac{b}{c})^{b}exp^{-b}$	$at^{b-1}\exp(-ct)^{(b-ct)}$	$\left(\frac{a}{c}\right)^{(b+1)}(b+1)$
Von Bertalanffy [25,26]	$y = a(1 - bexp^{-ct})\quad 3$	---	---	---	---

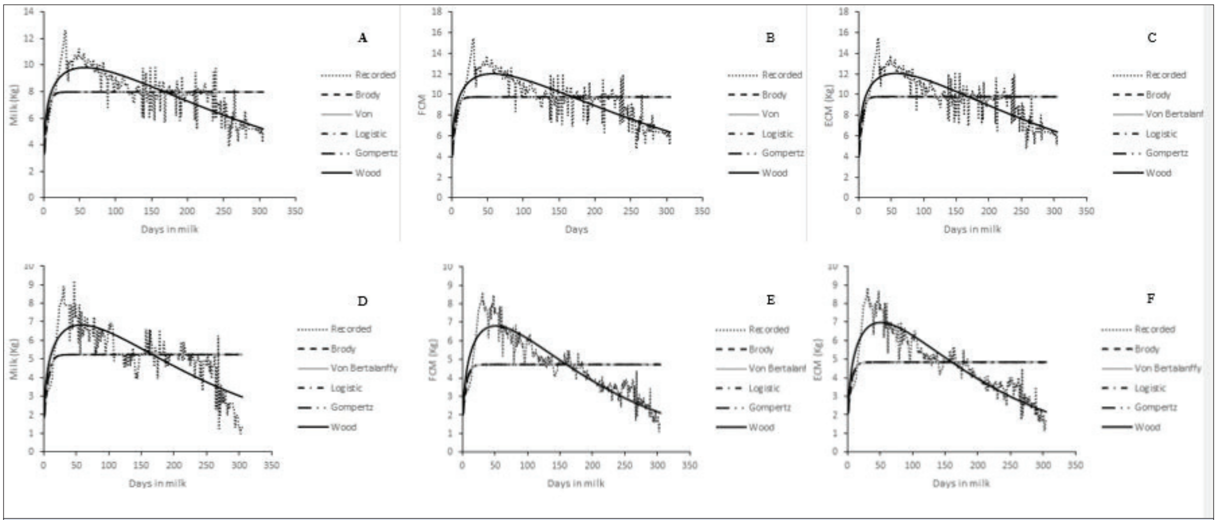

Fig 1. Fitting the growth equations on unadjusted milk, fat corrected milk (FCM) and energy corrected milk (ECM) records of buffaloes (A, B, C) and cows (D, E, F) plotted as days in milk on x-axis vs. milk yield (kg/day) on y-axis

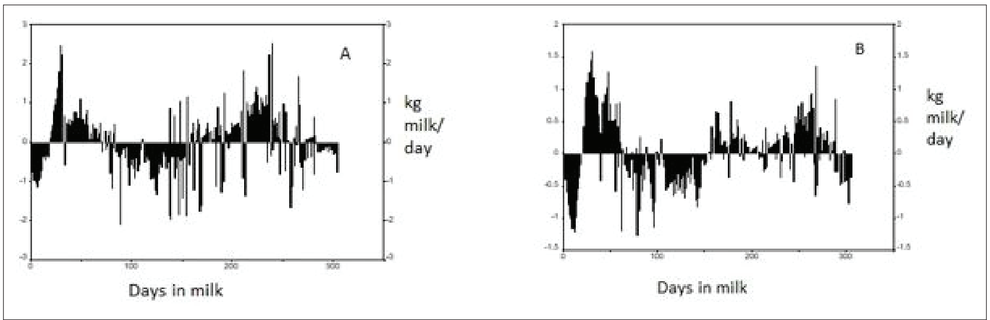

Fig 2. Plot of mean residuals after fitting the Wood's model on unadjusted milk yield records of buffaloes (A) and cows (B) (A; mean residuals = kg/day)

Easy Regression version 4.0: https://CRAN.R-project.org/package=easyreg), whereas Y denotes the MY at a given time t, a is linked to MY at the beginning of the lactation, b the ascending phase before peak MY and, c the descending phase after peak MY, and t the time from parturition. The observed and model predicted curves for all equations *(Fig. 1-A-F)* were constructed using Excel (Office 10, Microsoft Inc.) and standardized residual curves *(Fig. 2-A,B)* based only on Wood's model were constructed using TableCurve® 2D (ver. 5.0, SPSS Inc. NY).

Statistical Analyses

The statistical analyses were performed using the GLM procedure of Minitab® 16.1.1.0. The effects of species on lactation parameters were evaluated according to the model:

$$Y_{ij} = \mu + S_i + \varepsilon_{ij}$$

Where Y_{ij} is the dependent variable, μ is the overall mean, S_i is the effect of ith species and ε_{ij} is the residual error. Results were presented as least square means and were considered statistically significant when the P was ≤0.05 and the trends were considered when P was more than 0.05 but less than 0.10.

Results

The observed and model predicted goodness of fit indicators of unadjusted MY, FCM and ECM determined using various growth equations are shown in *Table 2*, *Table 3*, *Table 4*. The Wood's model provided the best fit with logical values of parameter estimates owing to the highest R^2 and the lowest Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC) and Root Mean Squared Error (RMSE) than other equations. Therefore, the Wood's model was selected to calculate further lactation parameters such as time to peak MY, peak MY, lactation yield, persistency of lactation, etc. The shapes of lactation curves resulting from various growth equations are presented *(Fig. 1-A-F)*.

The observed and model predicted lactation parameter estimates of unadjusted MY, FCM and ECM determined using various growth equations are shown in *Table 2*, *Table 3*, *Table 4*. Observed MY, FCM and ECM increased from day 1 of lactation (1 DIM) to a peak a few weeks later, decreased thereafter until 305 DIM. The average MY, increased from 4.0 and 2.6 kg/day at 1 DIM to the Peak MY of 12.8 and 8.6 kg/day on 30 DIM and subsequently

Table 2. *Least square means of the model predicted and recorded lactation characteristics of unadjusted milk yield records of buffaloes and cows (The values are expressed in kg unless otherwise stated)*

Parameters	Buffalo (n=2)					Cow (n=2)					Probability	Two Sample t-test
	Milk Records (n=600)					Milk Records (n=600)						
	Brody	Von Bertalanffy	Logistic	Gompertz	Wood	Brody	Von Bertalanffy	Logistic	Gompertz	Wood	Species[a]	
A	7.97	7.97	7.98	7.97	4.75	5.23	5.24	5.24	5.24	2.56	0.035	
B	0.732	0.317	1.643	1.086	0.321	0.775	0.351	2.055	1.239	0.238	0.013	
C	0.214	0.233	0.274	0.243	0.004	0.178	0.195	0.231	0.203	0.006	0.635	
Adj R^2	0.05	0.05	0.05	0.05	0.57	0.04	0.05	0.05	0.05	0.64	0.065	
AIC	2525	2524	2523	2524	2041	2277	2276	2274	2275	1679	0.115	
BIC	2542	2542	2541	2542	2059	2295	2293	2292	2293	1696	0.115	
RMSE	1.91	1.91	1.90	1.90	1.28	1.55	1.56	1.56	1.56	0.95	0.043	
ED											<0.001	3.23
Lactation yield (modeled)					7927					3937	0.004	
Lactation yield (recorded)					2404					1421	0.022	
Peak MY (modeled)					9.81					6.84	0.017	
Peak MY (recorded)					12.80					8.61	0.013	
Time to peak MY (modeled) days					57					51	0.459	
Time to peak MY (recorded) days					30.5					31.0	0.443	
Persistency of lactation					1.67					1.11	0.025	

A = initial milk yield, B = incline in milk yield before peak, C = decline in milk yield after peak, MY = milk yield, AIC = Akiake Information Criteria, BIC = Bayesian Information Criteria, RMSE = Root Mean Squared Error, ED = Estimate of difference was calculated using 2 sample t-test; a = effect of species of the milking animal (buffalo vs. cow)

declined to 5.0 and 1.6 kg/day on 305 DIM for buffaloes and cows, respectively. The Wood's model provided the closely related lactation parameter estimates to the observed values than other equations did. The average estimated values of a, b and c were 4.75, 0.238 and 0.004, and 2.56, 0.321 and 0.006 kg/day for buffaloes and cows, respectively. The observed time to peak MY was 30 DIM for each species with 2946 and 1421 kg total MY during a 305-days lactation period for buffaloes and cows, respectively. The Wood's model under-predicted the peak MY and over-estimated the time to peak MY and lactation yield in both species.

The wood's model had a better fit of lactation data for cows than of buffaloes with higher R^2 (P=0.070), and lower AIC, BIC and RMSE (P<0.05). The magnitude of lactation parameters remained higher (P<0.05) in buffaloes for a, peak MY, persistency and lactation yield than in cows. Model predicted time to peak MY remained unchanged (P>0.05) between buffaloes and cows. The magnitude of b and c were greater in cows than in buffaloes (P<0.05). The fitness of the model to individual species milk data were also compared by standardized residual curves (*Fig. 2-AiB*), respectively. The average residuals distributed uniformly around the zero in case of cows, but distributed widely in case of buffaloes. Whereas the model slightly overestimated MY from 1 to 20 DIM for both species, followed by an under-estimation from 20 to 80 DIM in buffaloes and from 20 to 52 DIM in cows. The model again over-estimated the MY from 80 to 165 DIM in buffaloes and from 52-155 DIM in cows, followed by an under-estimation until the end of lactation. The shape and form of the curve (lactation parameter estimates) in FCM and ECM remained the same as that of unadjusted MY except that average estimated values of a, peak MY and lactation yields were improved (*Table 3*, *Table 4*).

Table 3. *Least square means of the model predicted and recorded lactation characteristics of unadjusted milk yield records of buffaloes and cows (The values are expressed in kg unless otherwise stated)*

Parameters	Buffalo (n=2)					Cow (n=2)					Probability	Two Sample t-test
	Milk Records (n=600)					Milk Records (n=600)						
	Brody	Von Bertalanffy	Logistic	Gompertz	Wood	Brody	Von Bertalanffy	Logistic	Gompertz	Wood	Species[a]	
A	9.77	9.77	9.77	9.77	5.82	4.72	4.72	4.73	4.72	2.32	0.037	
B	0.732	0.317	1.643	1.086	0.367	0.704	0.304	1.581	1.042	0.238	0.019	
C	0.214	0.233	0.274	0.243	0.004	0.202	0.216	0.247	0.224	0.007	0.635	
Adj R^2	0.05	0.05	0.05	0.05	0.57	0.02	0.02	0.02	0.02	0.68	0.070	
AIC	2773	2772	2771	2772	2289	2467	2466	2466	2466	1776	0.445	
BIC	2791	2790	2789	2790	2307	2484	2484	2483	2484	1794	0.445	
RMSE	2.34	2.34	2.34	2.34	1.57	1.82	1.82	1.82	1.82	1.03	0.048	
ED											<0.001	4.91
Lactation yield (modeled)					10194					3811	0.001	
Lactation yield (recorded)					2946					1421	0.022	
Peak MY (modeled)					12.02					6.81	0.017	
Peak MY (recorded)					15.68					8.61	0.013	
Time to peak MY (modeled) days					57					51	0.349	
Time to peak MY (recorded) days					30.5					31.0	0.443	
Persistency of lactation					3.51					1.84	0.025	

A = initial milk yield, B = incline in milk yield before peak, C = decline in milk yield after peak, MY = milk yield, AIC = Akiake Information Criteria, BIC = Bayesian Information Criteria, RMSE = Root Mean Squared Error, ED = Estimate of difference was calculated using 2 sample t-test; a = effect of species of the milking animal (buffalo or cow)

Discussion

Although the number of animals included in the study seems small and there might be larger data sets than ours in studies where shape and form of lactation curves were analyzed. However, a number of 300 MY records per animal per lactation indicate that data are suitable, albeit not optimal, for this type of analysis.

Since the shape and form of lactation curve changes with the genetic make-up of animals, parity number, diet regime and other environmental factors [6,11,27], the choice of growth model to describe the lactation data of example group of lactating animals is very critical. In the present study, all equations other than the Wood's had poor fitting to the lactation data. The common understanding with the use of growth equations to lactation data is to mimic growth of mammary tissues to that of general body taking at intake at ad libitum feeding. Since the animal in the current study were fed at intake at maintenance levels, the growth of mammary glands might have not mimicked the general body growth, thereby, poor fitting of the growth equations. From the evaluation of the different equations used in the current study, it is evident that the non-linear growth equations except that of Wood had poor potential for fitting MY records of buffaloes and cows under intake at maintenance levels. It is therefore suggested that models which can account for the level of feed intake by the animal may be developed to better describe the data under restricted feeding conditions.

Aziz et al.[28] reported higher a value in Egyptian buffaloes, while b and c were consistent with our results. Contrarily, Anwar et al.[8]'s findings of all these parameters are highly

Table 4. *Least square means of the model predicted and recorded lactation characteristics of unadjusted milk yield records of buffaloes and cows (The values are expressed in kg unless otherwise stated)*

Parameters	Buffalo (n=2)					Cow (n=2)					Probability	Two Sample t-test
	Milk Records (n=600)					Milk Records (n=600)						
	Brody	Von Bertalanffy	Logistic	Gompertz	Wood	Brody	Von Bertalanffy	Logistic	Gompertz	Wood	Species[a]	
A	9.79	9.80	9.80	9.80	5.83	4.84	4.84	4.84	4.84	2.38	0.038	
B	0.732	0.317	1.643	1.086	0.367	0.704	0.304	1.581	1.042	0.238	0.020	
C	0.214	0.233	0.274	0.243	0.004	0.202	0.216	0.247	0.224	0.007	0.750	
Adj R^2	0.05	0.05	0.05	0.05	0.57	0.02	0.02	0.02	0.02	0.68	0.069	
AIC	2776	2776	2775	2776	2293	2497	2496	2495	2496	1806	0.337	
BIC	2794	2793	2793	2793	2310	2514	2514	2513	2514	1823	0.337	
RMSE	2.35	2.35	2.35	2.35	1.58	1.87	1.87	1.87	1.87	1.06	0.041	
ED											<0.001	5.0
Lactation yield (modeled)					10216					3947	0.002	
Lactation yield (recorded)					2955					1456	0.022	
Peak MY (modeled)					12.05					6.98	0.017	
Peak MY (recorded)					15.74					8.82	0.013	
Time to peak MY (modeled) days					57					51	0.449	
Time to peak MY ((recorded) days					30.5					31.0	0.443	
Persistency of lactation					6.25					4.39	0.025	

A = initial milk yield, B = incline in milk yield before peak, C = decline in milk yield after peak, MY = milk yield, AIC = Akiake Information Criteria, BIC = Bayesian Information Criteria, RMSE = Root Mean Squared Error, ED = Estimate of difference was calculated using 2 sample t-test; [a] = effect of species of the milking animal (buffalo or cow)

valued compared to this study's results. This difference in the results can be due to the parity and production conditions. Tekerli et al.[27] reported lower lactation yield, peak MY and time to peak MY, and higher persistency in lower parity animals, whereas the opposite trends for these parameters were found in higher parity animals. We found a smaller magnitude of b and c in buffaloes compared to cows. These two parameters are an indirect measure of persistency in lactation and therefore our data suggested greater persistency of lactation in buffaloes than local cows, which are similar to the findings of Khan & Chaudhry [29]. The Woods' lactation parameters b and c are directly correlated with lactation length and parity, and greater lactation length and parity are translated into sharper b and slower c in pre and post-peak lactation periods [8].

In the present study, the modeled peak MY was under-estimated than observed one in both species. These findings are consistent with those of Anwar et al.[8] and Dematawewa and Dekkers [17] who found that Wood's model under-predicted MY than the observed at farm. However, Boujenane [30] found no difference among the modeled and observed lactation yields in Holstein-Friesian dairy cows. Moreover, under-estimation was more prominent in buffaloes than in cows (P<0.05). In a similar way, model predicted time to peak MY was over-valued in both species, and numerically, this estimation was poorer in buffaloes than cows. Similar to our findings, other researchers also reported under-estimation of peak MY and over-estimation of the time to peak MY [31,32]. The current study findings of lactation yield in buffaloes are in agreement with the results reported by Sezer et al.[9]. However, lactation yield in buffaloes was lower than the yield determined by Khan [33], and higher than recorded by Anwar et al.[8], Khan and Chaudry [29] in Nili-Ravi buffaloes and by Şahin et al.[34] in Anatolian buffaloes. In Cholistani cows, the complete lactation yields were 1st reported by

Ashfaq et al.[35] who found greater values of lactation yield than ours. This difference in the standardized lactation yields can be attributed to factors related to genetics, feeding and individual animal variability [10]. The model predicted data showed more persistency in lactation for buffaloes (P<0.05) which is evident from the values of b and c.

In this experiment, Woods' model was fitted to the buffaloes and cows' complete lactation length. Preliminary assessment of goodness of fit indicators such as greater adjusted R^2 and lower AIC, BIC and RMSE in case of cows indicated a trend (P<0.10) of better fitting of the model to the lactation data of cows than of buffaloes. These results are similar to the findings of other researchers [8,17,28,34], indicating poor fitting of the Woods' model to the lactation data in buffaloes. On contrary, Soysal et al.[36] reported that the Woods' model had a best for lactation data in Italian origin buffaloes.

Fig. 2-A and B showed larger variations (spread between 0±2.5 kg milk/day) in the spread of standardized residuals in buffaloes than in cows (spread between 0±1.5 kg milk/day). To adequately fit the model to the data, the residuals have to oscillate on both sides showing no trend [30]. It is also apparent that the divergence of residuals was higher around the peak MY and around the end of lactation than around the mid lactation. This deviation was prominently higher in buffaloes and decreased rigorously in cows. These results are similar to Cole et al.[37]'s findings, who all reported a wider range of residual data around the two mentioned stages of lactation using the Woods' model. Inclusion of as many as possible fitness indicators in the regression model improves description of the data and aids in decision making [38,39].

It is concluded that the Woods' model has the potential to fairly accurately describe the lactation data of buffaloes and cows, under restricted feeding conditions, among all growth equations used. Even though, the model under-predicted milk yields in the beginning, around the peak and at the end of lactation than the observed one. The model showed a moderate fitting to the lactation data in cows whereas a poor fitting in buffaloes. Further, the indication of relatively smaller changes in pre- and post-peak milk yields, suggests more persistency in milk yields in buffaloes.

Availability of Data and Materials

The datasets generated and/or analyzed during the current study are available from the corresponding authors on reasonable request.

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Ethical Approval

All experiments were conducted according to the criteria of the University's Animal Care and Management Committee (The IUB, 2015).

Competing Interests

No potential conflict of interest was found by the authors.

Author's Contributions

ZK conducted this experiment under the supervision of SN, TNP, JAB and MNT. NS, RR and MNT conducted the statistical analyses. MÖ and TŞ helped MNT and RR in writing, reviewing and formatting the manuscript.

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