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ON BIPOLAR SOFT POINTS

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Abstract. The importance of point concept in all sub-branches of mathematics cannot be ignored. The point concept is needed in the establishment and continuation of some theories. Therefore, it is essential to define the point concept in bipolar soft theory. In this paper, the concept of bipolar soft points is defined and the relationship between bipolar soft points and bipolar soft sets is considered. Moreover, we investigate some bipolar soft point sets, define neighborhoods of bipolar soft points and reveal the structure of bipolar soft topological spaces such as interior, closure, boundary and bases by using neighborhoods of bipolar soft points.

Keywords: Bipolar soft set, bipolar soft point, bipolar soft interior point, bipolar soft adherent point, bipolar soft boundary point, bipolar soft limiting point.

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1. Introduction

When we talk about the modeling of real world problems which are ranging from engineering to medical and medical to social fields, we come across with the presence of uncertainty in data. Recently, new set theories such as fuzzy sets [17], rough sets [11], intuitionistic sets [4], soft sets [10] and neutrosophic sets [14] have been obtained for solving the problems that classical methods in mathematics was insufficient to deal with. Besides these novel set theories, their different combinations have been presented in the studies [1, 3, 6, 9, 16]. Not only they have been applied in various fields of mathematics, but also in other fields of science such as social science, natural science, medical science and engineering, there have been applications of these set theories. [5]

In 2013, Shabir and Naz [13] introduced bipolar soft set structure which may provide more general and clear results than soft set structure. There have been different studies on that concept [7,12,15] but as it is known that there is a need for researches on the point concept to get more progress and developments in all directions of mathematics. Bipolar soft set theory considers the care features of data granules. Bipolarity is important to distinguish between positive information which is quaranteed to be possible and negative information which is forbidden or surely false.

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In this study, by defining the point concept on bipolar soft set structure, fundamental properties for this point concept are presented. Furthermore, by constructing system of neighborhoods for bipolar soft point concept, relations and connections with bipolar soft topologic spaces are obtained with examples supporting our results.

2. Preliminary

In this section, we will give some preliminary information about bipolar soft sets and bipolar soft topological spaces. Throughout this paper X, E, $P(X)$ denote initial universe, set of parameters and power set of X, respectively. Let A, B, C be subsets of E.

Definition 2.1. *[8] Let $E = \{e_1,\ e_2,\ e_3, ..., e_n\}$ be a set of parameters. The not set of E, denoted by $\neg E$, is defined by $\neg E = \{\neg e_1,\ \neg e_2,\ \neg e_3, ..., \neg e_n\}$ where for all i, $\neg e_i = not\ e_i$.*

Definition 2.2. *[10] A pair (F, E) is called a soft set over X, where F is a mapping given by $F : E \to P(X)$. That is,*

$$(F, E) = \{(e, F(e)) : e \in E,\ F : E \to P(X)\}.$$

Definition 2.3. *[2] Let (F, E) be a soft set over X. (F, E) is called a soft point, if for the parameter $e \in E$, $F(e) = \{x\}$ and $F(e^c) = \varnothing$ for all $e^c \in E - \{e\}$ (briefly denoted by x_e).*

Definition 2.4. *[13] A triplet (F, G, A) is called a bipolar soft set over X, where F and G are mappings, $F : A \to P(X)$ and $G : \neg A \to P(X)$ such that $F(e) \cap G(\neg e) = \emptyset$ for all $e \in A$ and $\neg e \in \neg A$.*

Definition 2.5. *[13] For two bipolar soft sets (F_1, G_1, A) and (F_2, G_2, B) over X, (F_1, G_1, A) is called a bipolar soft subset of (F_2, G_2, B) if*

(1) *$A \subseteq B$ and*
(2) *$F_1(e) \subseteq F_2(e)$ and $G_2(\neg e) \subseteq G_1(\neg e)$ for all $e \in A$.*
This relationship is denoted by $(F_1, G_1, A) \widetilde{\subseteq} (F_2, G_2, B)$. (F_1, G_1, A) and (F_2, G_2, B) are said to be equal if (F_1, G_1, A) is a bipolar soft subset of (F_2, G_2, B) and (F_2, G_2, A) is a bipolar soft subset of (F_1, G_1, B).

Definition 2.6. *[13] Bipolar soft complement of a bipolar soft set (F, G, A) over X is denoted by $(F, G, A)^c$ and is defined by $(F, G, A)^c = (F^c, G^c, A)$ where $F^c : A \to P(X)$ and $G^c : \neg A \to P(X)$ are given by $F^c(e) = G(\neg e)$ and $G^c(\neg e) = F(e)$ for all $e \in A$ and $\neg e \in \neg A$.*

Definition 2.7. *[13] Bipolar soft union of two bipolar soft sets (F_1, G_1, A) and (F_2, G_2, B) over X is the bipolar soft set (H, I, C) over X where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F_1(e), & \text{if } e \in A - B, \\ F_2(e), & \text{if } e \in B - A, \\ F_1(e) \cup F_2(e), & \text{if } e \in A \cap B. \end{cases}$$

$$I(\neg e) = \begin{cases} G_1(\neg e), & \text{if } \neg e \in (\neg A) - (\neg B), \\ G_2(\neg e), & \text{if } \neg e \in (\neg B) - (\neg A), \\ G_1(\neg e) \cap G_2(\neg e), & \text{if } \neg e \in (\neg A) \cap (\neg B). \end{cases}$$

It is denoted by $(F_1, G_1, A) \widetilde{\cup} (F_2, G_2, B) = (H, I, C)$.

Definition 2.8. *[13] Bipolar soft intersection of two bipolar soft sets (F_1, G_1, A) and (F_2, G_2, B) over X is the bipolar soft set (H, I, C) over X where $C = A \cup B$ is non-empty and for all $e \in C$,*

$$H(e) = F_1(e) \cap F_2(e) \text{ and } I(\neg e) = G_1(\neg e) \cup G_2(\neg e).$$

It is denoted by $(F_1, G_1, A) \widetilde{\cap} (F_2, G_2, B) = (H, I, C)$.

Definition 2.9. *[13] Let* (F_1, G_1, A) *and* (F_2, G_2, B) *be two bipolar soft sets over* X. *Then,*

(1) $\left((F_1, G_1, A) \tilde{\cup} (F_2, G_2, B)\right)^c = (F_1, G_1, A)^c \tilde{\cap} (F_2, G_2, B)^c$,

(2) $\left((F_1, G_1, A) \tilde{\cap} (F_2, G_2, B)\right)^c = (F_1, G_1, A)^c \tilde{\cup} (F_2, G_2, B)^c$.

Definition 2.10. *[13] A bipolar soft set* (F, G, A) *over* X *is said to be relative null bipolar soft set, denoted by* $\left(\Phi, \widetilde{X}, A\right)$, *if for all* $e \in A$, $F(e) = \emptyset$ *and for all* $\neg e \in \neg A$, $G(\neg e) = X$.

The relative null bipolar soft set with respect to the universe set of parameters E *is called a NULL bipolar soft set over* X *and is denoted by* $\left(\Phi, \widetilde{X}, E\right)$.

Definition 2.11. *[13] A bipolar soft set* (F, G, A) *over* X *is said to be relative absolute bipolar soft set, denoted by* $\left(\widetilde{X}, \Phi, A\right)$, *if for all* $e \in A$, $F(e) = X$ *and for all* $\neg e \in \neg A$, $G(\neg e) = \emptyset$.

The relative absolute bipolar soft set with respect to the universe set of parameters E *is called a ABSOLUTE bipolar soft set over* X *and is denoted by* $\left(\widetilde{X}, \Phi, E\right)$.

Definition 2.12. *[12] Let* $\tilde{\tilde{\tau}}$ *be the collection of bipolar soft sets over* X *with* E *as the set of parameters. If*

(1) $\left(\Phi, \widetilde{X}, E\right)$ *and* $\left(\widetilde{X}, \Phi, E\right)$ *belong to* $\tilde{\tilde{\tau}}$,

(2) *the bipolar soft union of any number of bipolar soft sets in* $\tilde{\tilde{\tau}}$ *belongs to* $\tilde{\tilde{\tau}}$,

(3) *the bipolar soft intersection of finite number of bipolar soft sets in* $\tilde{\tilde{\tau}}$ *belongs to* $\tilde{\tilde{\tau}}$,

Then $\tilde{\tilde{\tau}}$ *is said to be a bipolar soft topology over* X *and* $\left(X, \tilde{\tilde{\tau}}, E, \neg E\right)$ *is called a bipolar soft topological space over* X .

Definition 2.13. *[12] Let* $\left(X, \tilde{\tilde{\tau}}, E, \neg E\right)$ *be a bipolar soft topological space over* X. *Then the members of* $\tilde{\tilde{\tau}}$ *are said to be bipolar soft open sets in* X.

Definition 2.14. *[12] Let* $\left(X, \tilde{\tilde{\tau}}, E, \neg E\right)$ *be a bipolar soft topological space over* X. *A bipolar soft set* (F, G, E) *over* X *is said to be a bipolar soft closed set in* X, *if its bipolar soft complemet* $(F, G, E)^c$ *belongs to* $\tilde{\tilde{\tau}}$.

Definition 2.15. *[15] Let* (F, G, A) *be a bipolar soft set over* X. *The presentation of*

$$(F, G, A) = \left\{ \begin{array}{c} (e,\ F(e),\ G(\neg e)) : e \in A \subseteq E,\ \neg e \in \neg A \subseteq \neg E \\ \text{and } F(e),\ G(\neg e) \in P(X) \end{array} \right\}$$

is said to be a short expansion of bipolar soft set (F, G, A).

From now on, $BSS\,(X)_{E,\neg E}$ denotes the family of all bipolar soft sets over X with E as the set of parameters and $BSTS$ denotes a bipolar soft topolological space.

Proposition 2.1. *[15] Let* $\left(X, \tilde{\tilde{\tau}}, E, \neg E\right)$ *be a* $BSTS$ *over* X. *Then the collection* $\tilde{\tau} = \left\{(F, E) : (F, G, E) \in \tilde{\tilde{\tau}}\right\}$ *defines a soft topology and* $(X, \tilde{\tau}, E)$ *is a soft topological space over* X.

Remark 2.1. *[15] Let* $\left(X, \tilde{\tilde{\tau}}, E, \neg E\right)$ *be a* $BSTS$ *over* X. *It can be easily shown that, if the collection* $\tilde{\tilde{\tau}}$ *is finite then* $\widetilde{\neg\tau} = \left\{(G, \neg E) : (F, G, E) \in \tilde{\tilde{\tau}}\right\}$ *defines a soft topology and* $(X, \widetilde{\neg\tau}, \neg E)$ *is a soft topological space over* X.

Similarly, if the collection $\tilde{\tilde{\tau}}$ *is finite then*

$\widetilde{\tau}_{\neg e} = \left\{ G(\neg e) : (F, G, E) \in \widetilde{\widetilde{\tau}}, \text{ for all } \neg e \in \neg E \right\}$
defines a topology and $(X, \widetilde{\tau}_{\neg e})$ *is a topological space over* X.

Definition 2.16. *[15] Let* $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ *be a* $BSTS$ *over* X *and* (F, G, E) *be a bipolar soft set over* X. *Then the bipolar soft closure of* (F, G, E), *denoted by* $\overline{(F, G, E)}$ *is the bipolar soft intersection of all bipolar soft closed super sets of* (F, G, E).

Obviously, $\overline{(F, G, E)}$ *is the smallest bipolar soft closed set over* X *that containing* (F, G, E).

Definition 2.17. *[15] Let* $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ *be a* $BSTS$ *over* X *and* (F, G, E) *be a bipolar soft set over* X. *Then the bipolar soft interior of* (F, G, E), *denoted by* $(F, G, E)^{\circ}$ *is the bipolar soft union of all bipolar soft open subsets of* (F, G, E).

Obviously, $(F, G, E)^{\circ}$ *is the biggest bipolar soft open set over* X *that is contained by* (F, G, E).

Definition 2.18. *[15] Let* $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ *be a* $BSTS$ *over* X *and* $\widetilde{\widetilde{B}} \subseteq \widetilde{\widetilde{\tau}}$. $\widetilde{\widetilde{B}}$ *is said to be a bipolar soft basis for the bipolar soft topology* $\widetilde{\widetilde{\tau}}$ *if every element of* $\widetilde{\widetilde{\tau}}$ *can be written as the bipolar soft union of elements of* $\widetilde{\widetilde{B}}$.

Definition 2.19. *[15] Let* $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ *be a* $BSTS$ *over* X *and* $(F, G, E) \widetilde{\subseteq} \left(\widetilde{X}, \Phi, E\right)$. *Then the collection*

$$\widetilde{\widetilde{\tau}}_{(F,G,E)} = \left\{ (F, G, E) \widetilde{\cap} (F_i, G_i, E) : (F_i, G_i, E) \in \widetilde{\widetilde{\tau}} \text{ for } i \in I \right\}$$

is called a bipolar soft subspace topology on (F, G, E) *and* $\left(X_{(F,G,E)}, \widetilde{\widetilde{\tau}}_{(F,G,E)}, E, \neg E\right)$ *is called a bipolar soft topological subspace of* $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$.

3. Bipolar Soft Points

Let X be an initial universe set, E be a set of parameters and $\neg E$ be not set of parameters set E.

Definition 3.1. *Let* (F, G, E) *be a bipolar soft set over* X. *The bipolar soft set* (F, G, E) *is called a bipolar soft point if there exists* $x, y \in X$, $x \neq y$, $e \in E$ *and* $\neg e \in \neg E$ *such that*

$$F(\alpha) = \begin{cases} \{x\}, & \text{if } \alpha = e \\ \varnothing, & \text{if } \alpha \in E \backslash \{e\} \end{cases}$$

$$G(\alpha') = \begin{cases} X \backslash \{x, y\}, & \text{if } \alpha' = \neg e \\ X, & \text{if } \alpha' \in \neg E \backslash \{\neg e\} \end{cases}$$

We denote the bipolar soft point (F, G, E) *briefly by* x^{e}_{y} *and denote the family of all bipolar soft points over* X *briefly by* $BSP(X)_{E,\neg E}$.

Example 3.1. *Let* $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2\}$ *and* $\neg E = \{\neg e_1, \neg e_2\}$. *Then*

$$BSP(X)_{E,\neg E} = \left\{ \begin{array}{l} x^{e_1}_{1x_2}, x^{e_1}_{1x_3}, x^{e_1}_{1x_4}, x^{e_1}_{2x_1}, x^{e_1}_{2x_3}, x^{e_1}_{2x_4}, x^{e_1}_{3x_1}, x^{e_1}_{3x_2}, x^{e_1}_{3x_4}, x^{e_1}_{4x_1}, x^{e_1}_{4x_2}, x^{e_1}_{4x_3} \\ x^{e_2}_{1x_2}, x^{e_2}_{1x_3}, x^{e_2}_{1x_4}, x^{e_2}_{2x_1}, x^{e_2}_{2x_3}, x^{e_2}_{2x_4}, x^{e_2}_{3x_1}, x^{e_2}_{3x_2}, x^{e_2}_{3x_4}, x^{e_2}_{4x_1}, x^{e_2}_{4x_2}, x^{e_2}_{4x_3} \end{array} \right\}.$$

Definition 3.2. *Let* $BSP(X)_{E,\neg E}$ *be all bipolar soft points over* X *with* E *as the set of parameters,* $|X| = n$ *be the number of elements in the set* X *and* $|E| = j$ *be the number of elements in the set* E. *Then, the number of elements of* $BSP(X)_{E,\neg E}$ *is defined by*

$$\left| BSP(X)_{E,\neg E} \right| = n.(n-1).j.$$

Definition 3.3. *Let $x_y^e, x'^{e'}_{y'} \widetilde{\in} BSP(X)_{E,\neg E}$ be two bipolar soft points. Then x_y^e and $x'^{e'}_{y'}$ are called different bipolar soft points if $x \neq x'$ or $e \neq e'$.*

Definition 3.4. *Let (F, G, E) be a bipolar soft set over X and $x_y^e \widetilde{\in} BSP(X)_{E,\neg E}$ be a bipolar soft point. Then x_y^e is said to be contained in (F, G, E) iff $x \in F(e)$ and $y \in X \backslash G(\neg e)$. It is denoted by $x_y^e \widetilde{\in} (F, G, E)$.*

Example 3.2. *Let us consider the bipolar soft set $(F, G, E) = \{(e_1, \{x_2, x_4\}, \{x_1\}), (e_2, \{x_1, x_4\}, \{x_2\})\}$ is a bipolar soft set over X that are given in Example 1. In this case, the bipolar soft points belonging to (F, G, E) are as follows:*

$$\left\{x_{2x_3}^{e_1}, x_{2x_4}^{e_1}, x_{4x_2}^{e_1}, x_{4x_3}^{e_1}, x_{1x_3}^{e_2}, x_{1x_4}^{e_2}, x_{4x_1}^{e_2}, x_{4x_3}^{e_3}\right\}.$$

Proposition 3.1. *Let (F, G, E) be a bipolar soft set over X. Then (F, G, E) is the bipolar soft union of its bipolar soft points. That is,*

$$(F, G, E) = \widetilde{\cup} \left\{x_y^e : x_y^e \widetilde{\in} (F, G, E)\right\}.$$

Proof. It is sufficient to show the following equalities:

For each $e \in E$,

$$F(e) = \widetilde{\bigcup_{x_y^e \widetilde{\in} (F,G,E)}} \{x\}$$

for each $\neg e \in \neg E$

$$G(\neg e) = \widetilde{\bigcap_{x_y^e \widetilde{\in} (F,G,E)}} (X \backslash \{x, y\})$$

which are easily obvious. □

Proposition 3.2. *Let $\{(F_i, G_i, E) : i \in I\}$ be a family of bipolar soft set over X. Then*

(1) *$x_y^e \widetilde{\in} \widetilde{\bigcap_{i \in I}} (F_i, G_i, E)$ iff $x_y^e \widetilde{\in} (F_i, G_i, E)$ for each $i \in I$. That is, $x \in F_i(e)$ and $y \in X \backslash G_i(\neg e)$ for every $i \in I$, $e \in E$, $\neg e \in \neg E$.*

(2) *$x_y^e \widetilde{\in} \widetilde{\bigcup_{i \in I}} (F_i, G_i, E)$ iff $\exists i \in I$ such that $x_y^e \widetilde{\in} (F_i, G_i, E)$. That is, $\exists i \in I$ such that $x \in F_i(e)$ and $y \in X \backslash G_i(\neg e)$ for every $e \in E$, $\neg e \in \neg E$.*

Proof. Straightforward. □

Proposition 3.3. *Let (F_1, G_1, E) and (F_2, G_2, E) be two bipolar soft sets over X. Then $(F_1, G_1, E) \widetilde{\subseteq} (F_2, G_2, E)$ iff for each $x_y^e \widetilde{\in} (F_1, G_1, E) \Rightarrow x_y^e \widetilde{\in} (F_2, G_2, E)$.*

Proof. Suppose that $x_y^e \widetilde{\in} (F_1, G_1, E)$ is an arbitrary bipolar soft point and $(F_1, G_1, E) \widetilde{\subseteq} (F_2, G_2, E)$. Then $F_1(e) \subseteq F_2(e)$ and $G_1(\neg e) \supseteq G_2(\neg e)$ for all $e \in E$, $\neg e \in \neg E$. Since $x \in F_1(e) \subseteq F_2(e)$ and $y \in X \backslash G_1(\neg e) \subseteq X \backslash G_2(\neg e)$ for all $e \in E$, $\neg e \in \neg E$. Therefore, $x_y^e \widetilde{\in} (F_2, G_2, E)$. □

Proposition 3.4. *Let (F, G, E) be a bipolar soft set over X and $x_y^e \widetilde{\in} BSP(X)_{E,\neg E}$ be a bipolar soft point. If the bipolar soft point x_y^e belongs to bipolar soft set (F, G, E), then the soft point x_e also belongs to the soft set (F, E).*

Proof. Straightforward. □

Definition 3.5. *Let $\left(X, \widetilde{\widetilde{\tau}}, E, \neg E\right)$ be a $BSTS$ over X, $x_y^e \widetilde{\in} BSP(X)_{E,\neg E}$ and $(F, G, E) \widetilde{\subseteq} \left(\widetilde{X}, \Phi, E\right)$. The bipolar soft set (F, G, E) called a bipolar soft neighborhood of x_y^e if there exists a bipolar soft open set $(U, V, E) \in \widetilde{\widetilde{\tau}}$ such that*

$$x_y^e \widetilde{\in} (U, V, E) \widetilde{\subseteq} (F, G, E), \text{ i.e., } x \in U(e) \subseteq F(e) \text{ and}$$

$$y \in X \backslash V(e) \subseteq X \backslash G(e) \text{ for every } e \in E, \neg e \in \neg E.$$

We will denote the set of all bipolar soft neighborhoods of x_y^e by $N(x_y^e)$.

Proposition 3.5. *The bipolar soft neighborhood system $N(x_y^e)$ in the BSTS $\left(X, \tilde{\tilde{\tau}}, E, \neg E\right)$ satisfies the following properties:*

1. *If $(F, G, E) \in N(x_y^e)$, then $x_y^e \widetilde{\in} (F, G, E)$.*
2. *If $(F, G, E) \in N(x_y^e)$ and $(F, G, E) \widetilde{\subseteq} (K, S, E)$, then $(K, S, E) \in N(x_y^e)$.*
3. *If $(F, G, E), (K, S, E) \in N(x_y^e)$, then $(F, G, E) \widetilde{\cap} (K, S, E) \in N(x_y^e)$.*
4. *If $(F, G, E) \in N(x_y^e)$, then there exists $(K, S, E) \in N(x_y^e)$ such that $(F, G, E) \in N(x'^{e'}_{y'})$ for each $x'^{e'}_{y'} \widetilde{\in} (K, S, E)$.*

Proof. 1. Let $(F, G, E) \in N(x_y^e)$. Then there exists a bipolar soft open set $(U, V, E) \in \tilde{\tilde{\tau}}$ such that $x_y^e \widetilde{\in} (U, V, E) \widetilde{\subseteq} (F, G, E)$. Therefore $x_y^e \widetilde{\in} (F, G, E)$.

2. Let $(F, G, E) \in N(x_y^e)$ and $(F, G, E) \widetilde{\subseteq} (K, S, E)$. Since $x_y^e \widetilde{\in} (U, V, E) \widetilde{\subseteq} (F, G, E) \widetilde{\subseteq} (K, S, E)$, then $(K, S, E) \in N(x_y^e)$.

3. Let $(F, G, E), (K, S, E) \in N(x_y^e)$. Then there exists two bipolar soft open sets $(U_1, V_1, E), (U_2, V_2, E) \in \tilde{\tilde{\tau}}$ such that $x_y^e \widetilde{\in} (U_1, V_1, E) \widetilde{\subseteq} (F, G, E)$ and $x_y^e \widetilde{\in} (U_2, V_2, E) \widetilde{\subseteq} (K, S, E)$. Since $(U_1, V_1, E) \widetilde{\cap} (U_2, V_2, E) \in \tilde{\tilde{\tau}}$ and $x_y^e \widetilde{\in} (U_1, V_1, E) \widetilde{\cap} (U_2, V_2, E) \widetilde{\subseteq} (F, G, E) \widetilde{\cap} (K, S, E)$, then $(F, G, E) \widetilde{\cap} (K, S, E) \in N(x_y^e)$.

4. Let $(F, G, E) \in N(x_y^e)$. Then there exists a bipolar soft open set $(U, V, E) \in \tilde{\tilde{\tau}}$ such that $x_y^e \widetilde{\in} (U, V, E) \widetilde{\subseteq} (F, G, E)$. Now, $x'^{e'}_{y'} \widetilde{\in} (U, V, E) \widetilde{\subseteq} (F, G, E)$ is satisfied, for each $x'^{e'}_{y'} \widetilde{\in} (U, V, E)$. If (K, S, E) is taken instead of (U, V, E), the desired result is obtained. □

Theorem 3.1. *A bipolar soft set (F, G, E) over X is bipolar soft open set iff (F, G, E) is a bipolar soft neighborhood of its each bipolar soft points.*

Proof. Let $(F, G, E) \in \tilde{\tilde{\tau}}$ and $x_y^e \widetilde{\in} (F, G, E)$. Then $x_y^e \widetilde{\in} (F, G, E) \widetilde{\subseteq} (F, G, E)$. Therefore, (F, G, E) is a bipolar soft neighborhood of x_y^e.

Conversely, let (F, G, E) be a bipolar soft neighborhood of its each bipolar soft points and $x_y^e \widetilde{\in} (F, G, E)$. Then there exists a bipolar soft open set $(U, V, E) \in \tilde{\tilde{\tau}}$ such that $x_y^e \widetilde{\in} (U, V, E) \widetilde{\subseteq} (F, G, E)$. From here,

$$(F, G, E) = \widetilde{\bigcup_{x_y^e \widetilde{\in} (F,G,E)}} \{x_y^e\} \widetilde{\subseteq} \widetilde{\bigcup_{x_y^e \widetilde{\in} (F,G,E)}} (U, V, E)_{x_y^e} \widetilde{\subseteq} \widetilde{\bigcup_{x_y^e \widetilde{\in} (F,G,E)}} (F, G, E) = (F, G, E).$$

Since (F, G, E) can be written as a bipolar soft union of bipolar soft open sets, then (F, G, E) is bipolar soft open set. □

Definition 3.6. *Let $\left(X, \tilde{\tilde{\tau}}, E, \neg E\right)$ be a BSTS over X, $x_y^e \widetilde{\in} BSP(X)_{E, \neg E}$ and $(F, G, E) \widetilde{\subseteq} \left(\widetilde{X}, \Phi, E\right)$.*

1. *x_y^e is called a bipolar soft interior point of (F, G, E), if $(K, S, E) \widetilde{\subseteq} (F, G, E)$ for some $(K, S, E) \in N(x_y^e)$.*
2. *x_y^e is called a bipolar soft adherent point of (F, G, E), if $(K, S, E) \widetilde{\cap} (F, G, E) \neq \left(\Phi, \widetilde{X}, E\right)$ for any $(K, S, E) \in N(x_y^e)$.*
3. *x_y^e is called a bipolar soft boundary point of (F, G, E), if $(K, S, E) \widetilde{\cap} (F, G, E) \neq \left(\Phi, \widetilde{X}, E\right)$ and $(K, S, E) \widetilde{\cap} (F, G, E)^c \neq \left(\Phi, \widetilde{X}, E\right)$ for any $(K, S, E) \in N(x_y^e)$.*

(4) x_y^e *is called a bipolar soft limiting point of* (F,G,E)*, if every bipolar soft neighborhood of* x_y^e *contains at least one bipolar soft point of* (F,G,E) *other than* x_y^e.

Example 3.3. *Let* $X=\{x_1,x_2,x_3,x_4\}$, $E=\{e_1,e_2\}$ *and* $\neg E=\{\neg e_1,\neg e_2\}$. *Suppose that* $\widetilde{\widetilde{\tau}}=\left\{\left(\Phi,\widetilde{X},E\right),\left(\widetilde{X},\Phi,E\right),(U_1,V_1,E),(U_2,V_2,E),(U_3,V_3,E),(U_4,V_4,E)\right\}$ *is a bipolar soft topology defined over* X *where* (U_1,V_1,E), (U_2,V_2,E), (U_3,V_3,E), (U_4,V_4,E) *are bipolar soft sets over* X, *defined as follows:*

$$\begin{aligned}(U_1,V_1,E)&=\{(e_1,\{x_1,x_3\},\{x_2\}),(e_2,\{x_2,x_3\},\{x_1\})\},\\(U_2,V_2,E)&=\{(e_1,\{x_2,x_3\},\{x_4\}),(e_2,\{x_1,x_3\},\{x_2\})\},\\(U_3,V_3,E)&=\{(e_1,\{x_1,x_2,x_3\},\emptyset),(e_2,\{x_1,x_2,x_3\},\emptyset)\},\\(U_4,V_4,E)&=\{(e_1,\{x_3\},\{x_2,x_4\}),(e_2,\{x_3\},\{x_1,x_2\})\}.\end{aligned}$$

Let $(F,G,E)=\{(e_1,\{x_1,x_3\},\{x_2,x_4\}),(e_2,\{x_1,x_3\},\{x_1,x_2\})\}$ *be a bipolar soft set over* X. *Since* $x_{3x_1}^{e_1}$, $x_{3x_4}^{e_2}\widetilde{\in}(U_4,V_4,E)\widetilde{\subseteq}(F,G,E)$, *then the bipolar soft points* $x_{3x_1}^{e_1}$ *and* $x_{3x_4}^{e_2}$ *are bipolar soft interior points of* (F,G,E) *over the bipolar soft topological space* $\left(X,\widetilde{\widetilde{\tau}},E,\neg E\right)$.

Theorem 3.2. *Let* $\left(X,\widetilde{\widetilde{\tau}},E,\neg E\right)$ *be a BSTS over* X *and* $(F,G,E)\widetilde{\subseteq}\left(\widetilde{X},\Phi,E\right)$. *Then*

$$(F,G,E)^\circ=\widetilde{\cup}\left\{x_y^e:x_y^e\text{ is a bipolar soft interior point of }(F,G,E)\right\}.$$

Proof. Since $(F,G,E)^\circ$ is a bipolar soft open set and $x_y^e\widetilde{\in}(F,G,E)^\circ\widetilde{\subseteq}(F,G,E)$ for each $x_y^e\widetilde{\in}(F,G,E)^\circ$, then x_y^e is a bipolar soft interior point of (F,G,E).

Conversely, if $x_y^e\widetilde{\in}(F,G,E)$ is a bipolar soft interior point, then there exists a bipolar soft open set (U,V,E) such that $x_y^e\widetilde{\in}(U,V,E)\widetilde{\subseteq}(F,G,E)$. From the definition of bipolar soft interior set, $(U,V,E)\widetilde{\subseteq}(F,G,E)^\circ$. This means that, every bipolar soft interior point belongs to $(F,G,E)^\circ$. □

Theorem 3.3. *Let* $\left(X,\widetilde{\widetilde{\tau}},E,\neg E\right)$ *be a BSTS over* X *and* $\widetilde{\widetilde{B}}\subseteq\widetilde{\widetilde{\tau}}$. *Then,*

- **a:** *the family* $\widetilde{\widetilde{B}}$ *is a bipolar soft base of* $\widetilde{\widetilde{\tau}}$ $\Leftrightarrow$ *there exists* $B_{x_y^e}\in\widetilde{\widetilde{B}}$ *such that* $x_y^e\widetilde{\in}B_{x_y^e}\widetilde{\subseteq}(F,G,E)$ *for every* $(F,G,E)\in\widetilde{\widetilde{\tau}}$ *and every* $x_y^e\widetilde{\in}(F,G,E)$.
- **b:** *if the family* $\widetilde{\widetilde{B}}=\{B_i\}_{i\in I}$ *is a bipolar soft base of* $\widetilde{\widetilde{\tau}}$, *then there exists* $B_i\in\widetilde{\widetilde{B}}$ *such that* $x_y^e\widetilde{\in}B_{i_3}\widetilde{\subseteq}B_{i_1}\widetilde{\cap}B_{i_2}$ *for every* B_{i_1}, $B_{i_2}\in\widetilde{\widetilde{B}}$ *and every* $x_y^e\widetilde{\in}B_{i_1}\widetilde{\cap}B_{i_2}$.

Proof. a) Let the family $\widetilde{\widetilde{B}}$ be a bipolar soft base of $\widetilde{\widetilde{\tau}}$, $(F,G,E)\in\widetilde{\widetilde{\tau}}$ and $x_y^e\widetilde{\in}(F,G,E)$. Since $\widetilde{\widetilde{B}}$ is a bipolar soft base of $\widetilde{\widetilde{\tau}}$, then

$$\exists\widetilde{\widetilde{B}}'\subseteq\widetilde{\widetilde{B}}:(F,G,E)=\underset{B\in\widetilde{\widetilde{B}}'}{\widetilde{\cup}}B.$$

From $x_y^e\widetilde{\in}(F,G,E)=\underset{B\in\widetilde{\widetilde{B}}'}{\widetilde{\cup}}B$, $x_y^e\widetilde{\in}B_{x_y^e}\widetilde{\subseteq}(F,G,E)$ is satisfied for at least one $B\in\widetilde{\widetilde{B}}'$.

Coversely, suppose that there exists $B_{x_y^e} \in \widetilde{\widetilde{B}}$ such that $x_y^e \widetilde{\in} B_{x_y^e} \widetilde{\subseteq} (F, G, E)$ for every $(F, G, E) \in \widetilde{\widetilde{\tau}}$ and every $x_y^e \widetilde{\in} (F, G, E)$. In this case,

$$\begin{aligned} (F, G, E) &= \widetilde{\bigcup}_{x_y^e \widetilde{\in} (F,G,E)} \{x_y^e\} \widetilde{\subseteq} \widetilde{\bigcup}_{x_y^e \widetilde{\in} (F,G,E)} B_{x_y^e} \widetilde{\subseteq} (F, G, E) \\ &\Rightarrow \quad (F, G, E) = \widetilde{\bigcup}_{x_y^e \widetilde{\in} (F,G,E)} B_{x_y^e}. \end{aligned}$$

That is, the family $\widetilde{\widetilde{B}}$ is a bipolar soft base of $\widetilde{\widetilde{\tau}}$.
b) It is obtained in a similar way. □

4. Conclusion

In this paper, we introduced the concept of the bipolar soft points, researched some bipolar soft point sets and presented neighbourhoods of bipolar soft points. Development of bipolar soft point structure is thought to contribute to the development of bipolar soft set theory in the topology as well as algebra, geometry and analysis of other sub-branches of mathematics. We hope that, the results of this study may help in the investigation of bipolar soft seperation axioms and in many research.

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