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Analytical and approximate solutions of an epidemic system of HIV/AIDS transmission

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Abstract This paper constructs approximate solutions of the epidemic system of HIV/AIDS transmission model with the help of the conformable fractional differential transform method. The fractional derivatives considered in this paper are described in the sense of the conformable operator. This method provides the approximate solutions speedily in the form of a convergent series. The results show the efficiency of the proposed method.

1. Introduction

HIV infection is an important health problem for many people from all over the world. It is a virus spread through certain body fluids that attack the human body's immune system, specifically the CD4 cells, generally, it is often called as T cells. Moreover, it is estimated that it may destroy so many of these cells which the human body can't fight off infections and disease. HIV/AIDS has had a large impact on society, both as an illness and as a source of discrimination [1]. HIV was first clinically reported on June 5, 1981, with five cases in the United States [2,3]. The initial cases were a cluster of injecting drug users and homosexual men with no known cause of impaired immunity who showed symptoms of Pneumocystis carinii pneumonia (PCP), a rare opportunistic infection that was known to occur in people with much compromised immune systems [4]. Many mathematical models have been proposed for the dynamics of HIV infection [4–7,59]. In the mid-90s, researchers have focused on synaptic transmission and their impact on HIV epidemics [4,8-16]. Mathematical models are also used to explain many real world problems arising in different fields of nonlinear media. In this regard, first epidemic of Hepatitis E infection has been recorded during 1955–56 in New Delhi [26]. Its another version with related to malaria has also been introduced in [27], and also many other models [23,28–57]. On of the most powerful methods to investigate suh models is the differential transform method (DTM) which has been firstly proposed by Zhou for solving linear and nonlinear initial value problems in electric circuit [17]. DTM is based on Taylor series expansion constituting a polynomial form of numerical solution to the governing models [18,19]. Developed version of DTM is named as conformable fractional differential transform method (CFDTM) which is also based a polynomial series solution via an iterative procedure [21]. It is one of the most powerful numerical methods used to solve fractional

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mathematical models. Arikoglu *et al* have investigated some models for obtaining approximate solutions via CFDTM [22].

This paper is composed of the following sections. In section 2, we present some important properties of CFDTM. In section 3, we apply CFDTM to derive approximate numerical solution of the epidemic system of HIV/AIDS transmission model [4] with conformable derivative defined by

$$\mathbf{D}^{\alpha}u(t) = a - b[v(t) + a_2 w(t) + a_1 z(t)]u(t) - mu(t)$$

$$\mathbf{D}^{\alpha}v(t) = b[v(t) + a_2 w(t) + a_1 z(t)]u(t) - (r + p + m)v(t) + cw(t) + yz(t),$$

$$\mathbf{D}^{\alpha}w(t) = pv(t) - (c + m)w(t)$$

$$\mathbf{D}^{\alpha}\mathbf{z}(\mathbf{t}) = \mathbf{rv}(\mathbf{t}) - (\mathbf{y} + \mathbf{m} + \delta)\mathbf{z}(\mathbf{t}), \tag{1}$$

with initial conditions

$$\mathrm{u}(0) = \mathrm{u}_0, \quad \mathrm{v}(0) = \mathrm{v}_0, \quad \mathrm{w}(0) = \mathrm{w}_0, \quad \mathrm{z}(0) = \mathrm{z}_0, \tag{2}$$

where D^{α} is the conformable derivative of order $\alpha \in (0, 1)$ defined in the sense of Caputo, with respect to time t. By considering various values of α, we present more detailed results in terms of tables. Moreover, graphs of solutions are also presented. We submit various tables including new numerical results obtained by CFDTM. At the end of this paper, we introduce a conclusion in a detailed manner including novelties of this paper.

2. General properties of CFDTM

In this section of the paper, we present the basic definitions of conformable operator one-dimensional differential transformation as follows for $f(x)$ with continuous function.

Definition 1. Assume $x(t)$ is infinitely α- differentiable function for some $\alpha \in (0, 1]$, conformable fractional differential transform of $x(t)$ is defined as [24]

$$X_{\alpha}(k) = \frac{1}{\alpha^k k!}\left[\left(T^{t_0}_{\propto}x\right)^{(k)}(t)\right]_{t=t_0}, \tag{3}$$

where $\left(T^{t_0}_{\propto}x\right)^{(k)}(t)$ denotes the application of kth iterate of the fractional derivative of the fractional derivative $\left(T^{t_0}_{\propto}x\right)(t)$ for a function $\mathbf{x} : [\mathbf{t}_0, \infty) \to \mathbb{R}$ given by

$$\left(T^{t_0}_{\alpha}x\right)(t) = \lim_{\varepsilon \to 0}\frac{x\left(t + \varepsilon(t - t_0)^{1-\alpha}\right) - x(t)}{\varepsilon}, \tag{4}$$

$0 \leqslant t_0 < t,\ \alpha \in (0, 1]$

Definition 2. Let $X_{\alpha}(k)$ be the conformable fractional differential transform of $x(t)$. Inverse conformable fractional differential transform of $X(k)$ is defined by [23]

$$x(t) = \sum\nolimits_{k=0}^{\infty} X_{\alpha}(k)(t - t_0)^{\alpha k} \equiv D^{-1}X_{\alpha}(k), \tag{5}$$

from here, we obtain

$$x(t) = \sum\nolimits_{k=0}^{\infty}\frac{1}{\alpha^k k!}\left[\left(T^{t_0}_{\propto}x\right)^{(k)}(t)\right]_{t=t_0}(t - t_0)^{\alpha k}. \tag{6}$$

CFDTM of initial conditions for integer order derivatives are defined as

$$X_{\alpha}(k) = \begin{cases} 0 & \alpha k \notin Z^{+} \\ \frac{1}{(\alpha k)!}\left[\frac{d^{\alpha k}x(t)}{dt^{\alpha k}}\right]_{t=t_0} & \alpha k \in Z^{+} \end{cases} \quad \text{for } k = 0, 1, 2, \cdots, \left(\frac{n}{\alpha} - 1\right), \tag{7}$$

where n is the order of the corresponding fractional equation [23]. Some basic properties of the conformable fractional one-dimensional differential transform are introduced in Table 1.

Theorem 1. *If* $x(t) = g(t)h(t)$, *then* $X_{\alpha}(k) = \sum_{l=0}^{k} G_{\alpha}(l)H_{\alpha}(k - l)$.

Proof. By using equation (5), we have

$$x(t) = \sum\nolimits_{k=0}^{\infty} G_{\alpha}(k)(t - t_0)^{\alpha k} \times \sum\nolimits_{k=0}^{\infty} H_{\alpha}(k)(t - t_0)^{\alpha k}$$

$$= \left(G_{\alpha}(0) + G_{\alpha}(1)(x - x_0)^{\alpha} + G_{\alpha}(2)(x - x_0)^{2\alpha} + \cdots\right)$$

$$\times\left(H_{\alpha}(0) + H_{\alpha}(1)(x - x_0)^{\alpha} + H_{\alpha}(2)(x - x_0)^{2\alpha} + \cdots\right)$$

$$= G_{\alpha}(0)H_{\alpha}(0) + (G_{\alpha}(0)H_{\alpha}(1) + G_{\alpha}(1)H_{\alpha}(0))(x - x_0)^{\alpha}$$

$$+ (G_{\alpha}(0)H_{\alpha}(2) + G_{\alpha}(1)H_{\alpha}(1) + G_{\alpha}(2)H_{\alpha}(0))(x - x_0)^{2\alpha} + \cdots$$

$$= \sum\nolimits_{k=0}^{\infty}\sum\nolimits_{l=0}^{k} G_{\alpha}(l)H_{\alpha}(k - l)(x - x_0)^{\alpha k}, \tag{8}$$

By the help of definition 2, we can write

$$X_{\alpha}(k) = \sum\nolimits_{l=0}^{k} G(l)H_{\alpha}(k - l). \tag{9}$$

Theorem 2. If $x(t) = (t - t_0)^{m\alpha}$ then $X_{\alpha}(k) = \delta(k - m) = \begin{cases} 1, k = m \\ 0, k \neq m \end{cases}$.

Proof. We suppose $t_0 = 0$, by using Eq. (8), we have

$$X_{\alpha}(k) = \frac{1}{\alpha^k k!}\left[\left(T^{t_0}_{\propto}x\right)^{(k)}(t)\right]_{t=0} = \frac{1}{\alpha^k k!}\left[\left(T^{t_0}_{\propto}(t)^{m\alpha}\right)^{(k)}(t)\right]_{t=0}$$

$$\{\frac{1}{\alpha^k k!}\left[\left(T^{t_0}_{\propto}(t)^{m\alpha}\right)^{(k)}(t)\right]_{t=0}$$

$$= \frac{1}{\alpha^k k!}[\alpha(m)\alpha(m - 1)\alpha(m - 2)\cdots\alpha(m - (k - 1))]_{t=0}$$

$$= \frac{1}{\alpha^k k!}\alpha^k k! = \begin{cases} 1, k = m \\ 0, k \neq m \end{cases}. \tag{10}$$

3. Analytical and approximate solutions

In this section, CFDTM is used to extract numerical solution of the epidemic system of HIV/AIDS transmission by the conformable derivative shown in Eq. (1) by considering following parameter values shown in Table 2 [4].

By using the basic definitions in Section 2 of the conformable fractional one-dimensional differential transform and taking the corresponding transform of the Eq. (1), we obtain the following system (with $t_0 = 0$):

Table 1 The operations for the conformable fractional differential transform method.

Original function	Transformed function
$x(t) = g(t) + h(t)$	$X_\alpha(k) = G_\alpha(k) + H_\alpha(k)$
$x(t) = \lambda g(t)$	$X_\alpha(k) = \lambda G_\alpha(k)$
$x(t) = e^{\gamma\frac{(t-t_0)^\alpha}{\alpha}}$	$X_\alpha(k) = \frac{\gamma^k}{\alpha^k k!}$
$x(t) = D^\alpha_{x_0} g(t)$	$X_\alpha(k) = \alpha(k+1)G_\alpha(k+1)$

$$U_\alpha(k+1)=\frac{1}{\alpha(k+1)}\left[a-b\sum_{l=0}^{k}V(l)U(k-l)-ba_2\sum_{l=0}^{k}W(l)U(k-l)-ba_1\sum_{l=0}^{k}Z(l)U(k-l)-mU(k)\right]$$

$$V_\alpha(k+1)=\frac{1}{\alpha(k+1)}\left[b\sum_{l=0}^{k}V(l)U(k-l)+ba_2\sum_{l=0}^{k}W(l)U(k-l)+ba_1\sum_{l=0}^{k}Z(l)U(k-l)-(r+p+m)V(k)+cW(k)+yZ(k)\right]$$

$$W_\alpha(k+1)=\frac{1}{\alpha(k+1)}[pV(k)-(c+m)W(k)],$$

$$Z_\alpha(k+1)=\frac{1}{\alpha(k+1)}[rV(k)-(y+m+\delta)Z(k)]. \tag{11}$$

Using initial conditions given by Eq. (2) as

$$U(0)=u_0,\quad V(0)=v_0,\quad W(0)=w_0,\quad Z(0)=z_0, \tag{12}$$

and the values of parameters presented in Table 2, we obtain following iteration structure

$$U_\alpha(k+1)=\frac{1}{\alpha(k+1)}\left[2.10-0.01\sum_{l=0}^{k}V(l)U-0.00015\sum_{l=0}^{k}W(l)U(k-l)-0.013\sum_{l=0}^{k}Z(l)U(k-l)-0.0144U(k)\right]$$

$$V_\alpha(k+1)=\frac{1}{\alpha(k+1)}\left[0.01\sum_{l=0}^{k}V(l)U(k-l)+0.00015\sum_{l=0}^{k}W(l)U(k-l)+0.013\sum_{l=0}^{k}Z(l)U(k-l)-1.1144V(k)+0.09W(k)+0.33Z(k)\right]$$

$$W_\alpha(k+1)=\frac{1}{\alpha(k+1)}[V(k)-0.1044W(k)],$$

$$Z_\alpha(k+1)=\frac{1}{\alpha(k+1)}[0.1V(k)-1.3444Z(k)], \tag{13}$$

with

$$U_\alpha(0)=1.8,\quad V_\alpha(0)=0.05,\quad W_\alpha(0)=2.5,\quad Z_\alpha(0)=1.4. \tag{14}$$

From Eqs. (13) and (14), the numerical approximate values of $U_\alpha(k), V_\alpha(k), W_\alpha(k), Z_\alpha(k)$ for $\mathbf{k}=1,2,\cdots,12,$ can be obtained for different values of αnumerical comparisons as below Tables 3-12.

Case 1. When we consider $\alpha = 0.25$, **it may be obtained the following numerical results**.

Case 2. When $\alpha = 0.35$, **it produces the following numerical results**.

Case 3. If $\alpha = 0.45$, **it gives the following approximate solutions to the Eq.**(1).

Case 4. When $\alpha = 0.5$, **we gain the following numerical results**.

Case 5. When we consider $\alpha = 0.55$, **it may be obtained the following numerical results**.

Case 6. If $\alpha = 0.65$, **it gives the following approximate solutions to the Eq.**(1).

Case 7. For $\alpha = 0.75$, **the results are given as following**

Case 8. Using $\alpha = 0.85$, **the results are given as beng in** Table 10.

Case 9. By considering $\alpha = 0.95$, we obtain following numerical results.

Case 10. If it is considered as $\alpha = 1$, it may be observed by Table 12.

By using the differential inverse reduced transform of $U_\alpha(k)$, $V_\alpha(k)$, $W_\alpha(k)$ and $Z_\alpha(k)$, we get the solution given

$$u(t)=\sum_{k=0}^{\infty}U_\alpha(k)t^{k\alpha}=U_\alpha(0)+U_\alpha(1)t^\alpha+U_\alpha(2)t^{2\alpha}+U_\alpha(3)t^{3\alpha}+\cdots$$

$$v(t)=\sum_{k=0}^{\infty}V_\alpha(k)t^{k\alpha}=V_\alpha(0)+V_\alpha(1)t^\alpha+V_\alpha(2)t^{2\alpha}+V_\alpha(3)t^{3\alpha}+\cdots w(t)=\sum_{k=0}^{\infty}W_\alpha(k)t^{k\alpha}=W_\alpha(0)+W_\alpha(1)t^\alpha+W_\alpha(2)t^{2\alpha}+W_\alpha(3)t^{3\alpha}+\cdots$$

$$z(t)=\sum_{k=0}^{\infty}Z_\alpha(k)t^{k\alpha}=Z_\alpha(0)+Z_\alpha(1)t^\alpha+Z_\alpha(2)t^{2\alpha}+Z_\alpha(3)t^{3\alpha}+\cdots \tag{14}$$

Table 2 Parameters values for the epidemic system of HIV/AIDS transmission.

Parameter	m	y	a	δ	a_1	p	a_2	c	r	b	u_0	v_0	w_0	z_0
Value	0.0144	0.33	2.1	1	1.3	1	0.015	0.09	0.1	0.01	1.8	0.05	2.5	1.4

Table 3 If we set $\alpha = 0.25$ the values of $U_\alpha(k), V_\alpha(k), W_\alpha(k)$ and $Z_\alpha(k)$.

k	$U_\alpha(k)$	$V_\alpha(k)$	$W_\alpha(k)$	$Z_\alpha(k)$
k = 1	8.2963200	2.5251200	−0.8440000	−7.5086400
k = 2	3.6445614	−10.4315497	5.2264672	20.6942552
k = 3	3.4391728	24.5225482	−14.6362572	−38.4860156
k = 4	0.8760702	−40.1858351	26.0505735	54.1928542
k = 5	3.0810912	50.6225889	−34.3244119	−61.5003654
k = 6	−0.0762560	−51.7896202	36.1373717	58.4955667
k = 7	2.5282029	44.5820045	−31.7499210	−47.8973725
k = 8	−0.1492663	−33.0329377	23.9483481	34.4257140
k = 9	1.9797057	21.3591259	−15.7925090	−22.0378772
k = 10	−0.1165147	−12.0866844	9.2031455	12.7054539
k = 11	1.6207275	5.8966508	−4.7445428	−6.6508657
k = 12	−0.0838543	−2.3152505	2.1306604	3.1770296
k = 13	1.3379018	0.5082430	−0.7808281	−1.3854534
k = 14	−0.0157260	0.2757421	0.1685033	0.5466937
k = 15	1.0906885	−0.5412637	0.0688401	−0.1886402

Table 4 If we set $\alpha = 0.35$ the values of $U_\alpha(k), V_\alpha(k), W_\alpha(k)$ and $Z_\alpha(k)$.

k	$U_\alpha(k)$	$V_\alpha(k)$	$W_\alpha(k)$	$Z_\alpha(k)$
k = 1	5.9259429	1,8036571	−0.6028571	−5.3633143
k = 2	2.7166129	−5,3222192	2.6665649	10.5582935
k = 3	2.2056082	8,9517479	−5.3339130	−14.0255159
k = 4	1.1903319	−10,4917351	6.7918632	14.1079131
k = 5	1.4284157	9,4564662	−6.4004603	−11.4376297
k = 6	0.8146474	−6,9192484	4.8212734	7.7725695
k = 7	0.9645325	4,2624104	−3.0296283	−4.5474968
k = 8	0.6721325	−2,2596050	1.6352513	2.3356771
k = 9	0.7083095	1,0475451	−0.7715318	−1.0685856
k = 10	0.5670360	−0,4252605	0.3223123	0.4403889
k = 11	0.5617969	0,1504615	−0.1191974	−0.1648272
k = 12	0.4853360	−0,0432184	0.0387871	0.0563428
k = 13	0.4673945	0,0082436	−0.0103885	−0.0175976
k = 14	0.4219593	0,0017383	0.0019037	0.0049964
k = 15	0.4015760	−0,0027044	0.0002932	−0.0012464

Table 5 If we set $\alpha = 0.45$ the values of $U_\alpha(k), V_\alpha(k), W_\alpha(k)$ and $Z_\alpha(k)$.

k	$U_\alpha(k)$	$V_\alpha(k)$	$W_\alpha(k)$	$Z_\alpha(k)$
k = 1	4.6090667	1,8036571	−0.4688889	−4.1714667
k = 2	2.1619017	−5,3222192	1.6131072	6.3871158
k = 3	1.6394384	8,9517479	−2.5096463	−6.5991110
k = 4	1.0537546	−10,4917351	2.4893947	5.1631862
k = 5	0.9880038	9,4564662	−1.8262839	−3.2561387
k = 6	0.7359334	−6,9192484	1.0716333	1.7214176
k = 7	0.6801638	4,2624104	−0.5240860	−0.7835470
k = 8	0.5710883	−2,2596050	0.2204610	0.3131375
k = 9	0.5202230	1,0475451	−0.0808918	−0.1114671
k = 10	0.4623520	−0,4252605	0.0264411	0.0357579
k = 11	0.4235943	0,1504615	−0.0075639	−0.0104123
k = 12	0.3868809	−0,0432184	0.0019876	0.0027764
k = 13	0.3580095	0,0082436	−0.0003798	−0.0006725
k = 14	0.3321409	0,0017383	0.0000926	0.0001521
k = 15	0.3102545	−0,0027044	0.0000267	−0.0000275

Table 6 If we set $\alpha = 0.50$ the values of $U_\alpha(k), V_\alpha(k), W_\alpha(k)$ and $Z_\alpha(k)$.

k	$U_\alpha(k)$	$V_\alpha(k)$	$W_\alpha(k)$	$Z_\alpha(k)$
k = 1	4.1481600	1.2625600	−0.4220000	−3.7543200
k = 2	1.9611403	−2.6078874	1.3066168	5.1735638
k = 3	1.4564641	3.0781460	−1.8295322	−4.8107520
k = 4	0.9753502	−2.5283178	1.6345746	3.3876948
k = 5	0.8687503	1.5990593	−1.0795870	−1.9228994
k = 6	0.6766691	−0.8194285	0.5705894	0.9150173
k = 7	0.6045444	0.3546377	−0.2511423	−0.3748835
k = 8	0.5185467	−0.1313442	0.0952142	0.1348643
k = 9	0.4660980	0.0432013	−0.0313966	−0.0432102
k = 10	0.4175136	−0.0119976	0.0092958	0.0124824
k = 11	0.3807086	0.0033030	−0.0023578	−0.0032693
k = 12	0.3486490	−0.0004443	0.0005915	0.0007876
k = 13	0.3221238	0.0002671	−0.0000779	−0.0001697
k = 14	0.2990966	0.0001536	0.0000393	0.0000364
k = 15	0.2792537	0.0001218	0.0000199	−0.0000045

Table 7 If we set $\alpha = 0.55$ the values of $U_\alpha(k), V_\alpha(k), W_\alpha(k)$ and $Z_\alpha(k)$.

k	$U_\alpha(k)$	$V_\alpha(k)$	$W_\alpha(k)$	$Z_\alpha(k)$
k = 1	3.7710545	1.1477818	−0.3836364	−3.4130182
k = 2	1.7943309	−2.1552789	1.0798486	4.2756726
k = 3	1.3116286	2.3145842	−1.3745546	−3.6143891
k = 4	0.9028505	−1.7286311	1.1173126	2.3139287
k = 5	0.7790271	0.9947866	−0.6710104	−1.1940760
k = 6	0.6221481	−0.4632613	0.3226788	0.5166044
k = 7	0.5464044	0.1826474	−0.1290777	−0.1924283
k = 8	0.4732787	−0.0612765	0.0445734	0.0629467
k = 9	0.4230619	0.0185576	−0.0133192	−0.0183340
k = 10	0.3800702	−0.0045052	0.0036269	0.0048189
k = 11	0.3460070	0.0012992	−0.0008072	−0.0011453
k = 12	0.3171246	−0.0000263	0.0002096	0.0002530
k = 13	0.2928670	0.0001864	−0.0000067	−0.0000479
k = 14	0.2719839	0.0001378	0.0000243	0.0000108
k = 15	0.2539096	0.0001186	0.0000164	−0.0000001

Table 8 If we set $\alpha = 0.65$ the values of $U_\alpha(k), V_\alpha(k), W_\alpha(k)$ and $Z_\alpha(k)$.

k	$U_\alpha(k)$	$V_\alpha(k)$	$W_\alpha(k)$	$Z_\alpha(k)$
k = 1	0.1908923	0.9712000	−0.3246154	−2.8879385
k = 2	1.5332191	−1.5431287	0.7731460	3.0612804
k = 3	1.0962242	1.4045710	−0.8327411	−2.1896914
k = 4	0.7799741	−0.8876654	0.5736574	1.1862608
k = 5	0.6502553	0.4331420	−0.2915554	−0.5180233
k = 6	0.5318677	−0.1703490	0.1188668	0.1896781
k = 7	0.4603989	0.0572367	−0.0401667	−0.0597886
k = 8	0.4016991	−0.0159640	0.0118135	0.0165584
k = 9	0.3577459	0.0043501	−0.0029397	−0.0040782
k = 10	0.3219339	−0.0006860	0.0007165	0.0009104
k = 11	0.2928238	0.0003590	−0.0001064	−0.0001808
k = 12	0.2684743	0.0001274	0.0000474	0.0000358
k = 13	0.2478862	0.0001384	0.0000145	−0.0000042
k = 14	0.2302222	0.0001174	0.0000150	0.0000021
k = 15	0.2149107	0.0001031	0.0000119	0.0000009

Table 9 If we set $\alpha = 0.75$ the values of $U_\alpha(k), V_\alpha(k), W_\alpha(k)$ and $Z_\alpha(k)$.

k	$U_\alpha(k)$	$V_\alpha(k)$	$W_\alpha(k)$	$Z_\alpha(k)$
k = 1	2.7654400	0.8417067	−0.2813333	−2.5028800
k = 2	1.3382846	−1.1590611	0.5807186	2.2993617
k = 3	0.9431205	0.9158440	−0.5420836	−1.4254080
k = 4	0.6833047	−0.5014533	0.3241458	0.6693010
k = 5	0.5603862	0.2126288	−0.1427451	−0.2533209
k = 6	0.4628292	−0.0721723	0.0505625	0.0804061
k = 7	0.3986068	0.0213116	−0.0147526	−0.0219648
k = 8	0.3485244	−0.0049225	0.0038086	0.0052768
k = 9	0.3100864	0.0013654	−0.0007882	−0.0011239
k = 10	0.2791384	−0.0000268	0.0001930	0.0002197
k = 11	0.2538503	0.0001973	−0.0000057	−0.0000361
k = 12	0.2327491	0.0001312	0.0000220	0.0000076
k = 13	0.2148904	0.0001182	0.0000132	0.0000003
k = 14	0.1995753	0.0001023	0.0000111	0.0000011
k = 15	0.1862982	0.0000898	0.0000090	0.0000008

Table 10 If we set $\alpha = 0.85$ the values of $U_\alpha(k), V_\alpha(k), W_\alpha(k)$ and $Z_\alpha(k)$.

k	$U_\alpha(k)$	$V_\alpha(k)$	$W_\alpha(k)$	$Z_\alpha(k)$
k = 1	2.4400941	0.7426824	−0.2482353	−2.2084235
k = 2	1.1872458	−0.9023832	0.4521165	1.7901605
k = 3	0.8283449	0.6301861	−0.3723859	−0.9791883
k = 4	0.6066619	−0.3042261	0.1967833	0.4057174
k = 5	0.4932178	0.1142196	−0.0764165	−0.1354986
k = 6	0.4091655	−0.0339555	0.0239603	0.0379581
k = 7	0.3516568	0.0090754	−0.0061272	−0.0091473
k = 8	0.3076934	−0.0016668	0.0014287	0.0019419
k = 9	0.2736695	0.0005750	−0.0002374	−0.0003631
k = 10	0.2463726	0.0001066	0.0000706	0.0000642
k = 11	0.2240389	0.0001513	0.0000106	−0.0000081
k = 12	0.2054137	0.0001194	0.0000147	0.0000025
k = 13	0.1896481	0.0001038	0.0000107	0.0000008
k = 14	0.1761295	0.0000901	0.0000086	0.0000008
k = 15	0.1644099	0.0000790	0.0000070	0.0000006

Table 11 If we set $\alpha = 0.95$ the values of $U_\alpha(k), V_\alpha(k), W_\alpha(k)$ and $Z_\alpha(k)$.

k	$U_\alpha(k)$	$V_\alpha(k)$	$W_\alpha(k)$	$Z_\alpha(k)$
k = 1	2.1832421	0.6645053	−0.2221053	−1.9759579
k = 2	1.0667979	−0.7224065	0.3619437	1.4331202
k = 3	0.7389249	0.4521411	−0.2667345	−0.7013780
k = 4	0.5448928	−0.1950738	0.1263127	0.2600386
k = 5	0.4407898	0.0658258	−0.0438444	−0.0777060
k = 6	0.3664831	−0.0173014	0.0123514	0.0194825
k = 7	0.3146705	0.0043200	−0.0027956	−0.0041989
k = 8	0.2754032	−0.0005638	0.0006068	0.0007996
k = 9	0.2449170	0.0003201	−0.0000734	−0.0001323
k = 10	0.2204890	0.0001304	0.0000345	0.0000221
k = 11	0.2004954	0.0001292	0.0000121	−0.0000016
k = 12	0.1838247	0.0001070	0.0000112	0.0000013
k = 13	0.1697134	0.0000922	0.0000086	0.0000007
k = 14	0.1576139	0.0000800	0.0000069	0.0000006
k = 15	0.1471246	0.0000701	0.0000056	0.0000005

Table 12 If we set $\alpha = 1$ the values of $U_\alpha(k), V_\alpha(k), W_\alpha(k)$ and $Z_\alpha(k)$.

k	$U_\alpha(k)$	$V_\alpha(k)$	$W_\alpha(k)$	$Z_\alpha(k)$
k = 1	2.0740800	0.6312800	−0.2110000	−1.8771600
k = 2	1.0152851	−0.6519719	0.3266542	1.2933910
k = 3	0.7011999	0.3879751	−0.2286915	−0.6013440
k = 4	0.5183683	−0.1589052	0.1029626	0.2118111
k = 5	0.4186183	0.0510781	−0.0339309	−0.0601299
k = 6	0.3482855	−0.0126506	0.0091034	0.0143244
k = 7	0.2989608	0.0030937	−0.0019430	−0.0029318
k = 8	0.2616699	−0.0003098	0.0004121	0.0005314
k = 9	0.2326949	0.0002603	−0.0000392	−0.0000828
k = 10	0.2094849	0.0001306	0.0000264	0.0000137
k = 11	0.1904869	0.0001212	0.0000116	−0.0000005
k = 12	0.1746472	0.0001013	0.0000100	0.0000011
k = 13	0.1612394	0.0000871	0.0000077	0.0000007
k = 14	0.1497432	0.0000756	0.0000062	0.0000006
k = 15	0.1397771	0.0000662	0.0000050	0.0000005

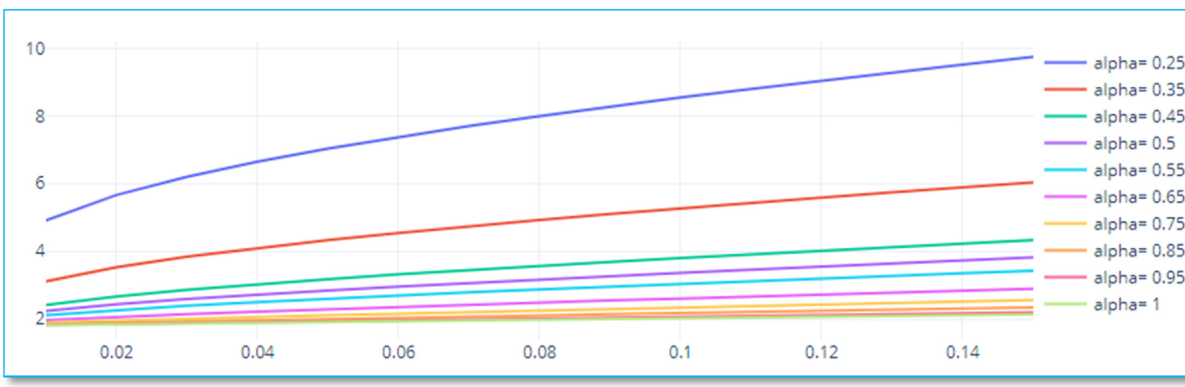

Fig. 1 This is a figure of $u(t)$ for different values of $\alpha, 0.01 < t < 0.15$.

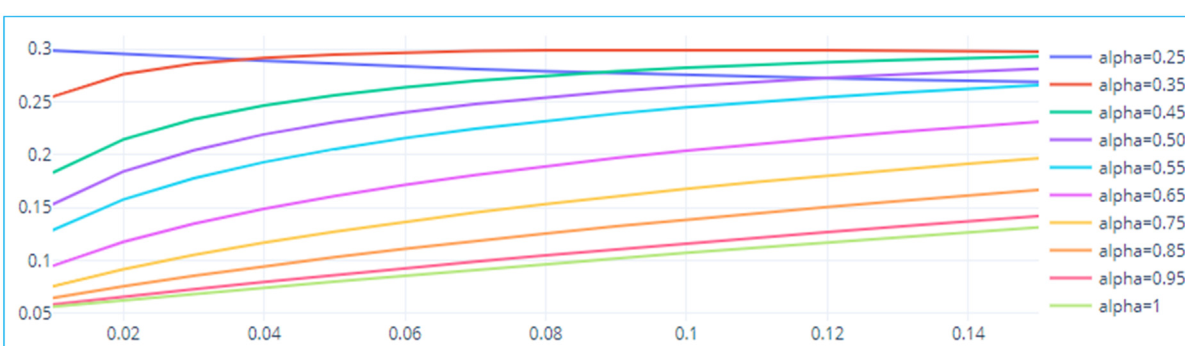

Fig. 2 This is a figure of $v(t)$ for different values of $\alpha, 0.01 < t < 0.15$.

Table 13 The numerical results of $u(t), v(t), w(t)$ and $z(t)$for $\alpha = 0.25$.

t	$u(t)$	$v(t)$	w(t)	z(t)
0.01	4.9160182	0.2986297	2.4729453	0.2716013
0.02	5.6616786	0.2959231	2.4822694	0.2029665
0.03	6.2024510	0.2922803	2.4878443	0.1678250
0.04	6.6467476	0.2889136	2.4916541	0.1453519
0.05	7.0332482	0.2859308	2.4944405	0.1293755
0.06	7.3807530	0.2832977	2.4965641	0.1172774
0.07	7.6999894	0.2809630	2.4982272	0.1077203
0.08	7.9977258	0.2788803	2.4995546	0.0999374
0.09	8.2785397	0.2770118	2.5006281	0.0934522
0.1	8.5456899	0.2753268	2.5015043	0.0879504
0.11	8.8015894	0.2738011	2.5022240	0.0832153
0.12	9.0480815	0.2724149	2.5028171	0.0790919
0.13	9.2866108	0.2711525	2.5033064	0.0754659
0.14	9.5183339	0.2700008	2.5037094	0.0722511
0.15	9.7441933	0.2689494	2.5040400	0.0693813

Table 14 The numerical results of $u(t), v(t), w(t)$ and $z(t)$ for $\alpha = 0.35$.

t	$u(t)$	$v(t)$	$w(t)$	z(t)
0.01	3.1104550	0.2550758	2.4525135	0.6579839
0.02	3.5257827	0.2762727	2.4541373	0.5368909
0.03	3.8382991	0.2861524	2.4569258	0.4654935
0.04	4.1009797	0.2916686	2.4598016	0.4155805
0.05	4.3332899	0.2949744	2.4625453	0.3777056
0.06	4.5448729	0.2969982	2.4651071	0.3475144
0.07	4.7413109	0.2982149	2.4674837	0.3226361
0.08	4.9261676	0.2988953	2.4696863	0.3016379
0.09	5.1018773	0.2992053	2.4717296	0.2835883
0.1	5.2701890	0.2992527	2.4736286	0.2678487
0.11	5.4324082	0.2991100	2.4753973	0.2539629
0.12	5.5895406	0.2988280	2.4770483	0.2415939
0.13	5.7423804	0.2984430	2.4785927	0.2304862
0.14	5.8915683	0.2979815	2.4800403	0.2204417
0.15	6.0376308	0.2974634	2.4813998	0.2113039

Table 15 The numerical results of $u(t), v(t), w(t)$ and $z(t)$ for $\alpha = 0.45$.

t	$u(t)$	$v(t)$	$w(t)$	$z(t)$
0.01	2.4180813	0.1831108	2.4621006	0.9641059
0.02	2.6660109	0.2144935	2.4562349	0.8420184
0.03	2.8601643	0.2334019	2.4537834	0.7613593
0.04	3.0274226	0.2465504	2.4527716	0.7006213
0.05	3.1779911	0.2563486	2.4525244	0.6518963
0.06	3.3170343	0.2639578	2.4527311	0.6112896
0.07	3.4475901	0.2700309	2.4532249	0.5765666
0.08	3.5716229	0.2749724	2.4539071	0.5463158
0.09	3.6904898	0.2790503	2.4547154	0.5195843
0.1	3.8051750	0.2824511	2.4556087	0.4956966
0.11	3.9164200	0.2853098	2.4565587	0.4741552
0.12	4.0248004	0.2877269	2.4575455	0.4545827
0.13	4.1307744	0.2897792	2.4585548	0.4366854
0.14	4.2347145	0.2915268	2.4595762	0.4202307
0.15	4.3369291	0.2930169	2.4606018	0.4050305

Table 16 The numerical results of $u(t), v(t), w(t)$ and $z(t)$ for $\alpha = 0.5$.

t	$u(t)$	$v(t)$	$w(t)$	$z(t)$
0.01	2.4180813	0.1530176	2.4691898	1.0718133
0.02	2.6660109	0.1841745	2.4618749	0.9601766
0.03	2.8601643	0.2043929	2.4579166	0.8827144
0.04	3.0274226	0.2192398	2.4555318	0.8224516
0.05	3.1779911	0.2308172	2.4540618	0.7728999
0.06	3.3170343	0.2401772	2.4531836	0.7307732
0.07	3.4475901	0.2479304	2.4527139	0.6941430
0.08	3.5716229	0.2544657	2.4525385	0.6617678
0.09	3.6904898	0.2600468	2.4525815	0.6327952
0.1	3.8051750	0.2648616	2.4527903	0.6066117
0.11	3.9164200	0.2690489	2.4531267	0.5827594
0.12	4.0248004	0.2727140	2.4535626	0.5608865
0.13	4.1307744	0.2759388	2.4540767	0.5407165
0.14	4.2347145	0.2787883	2.4546524	0.5220275
0.15	4.3369291	0.2813149	2.4552767	0.5046387

Table 17 The numerical results of $u(t), v(t), w(t)$ and $z(t)$ for $\alpha = 0.55$.

t	$u(t)$	$v(t)$	$w(t)$	$z(t)$
0.01	2.1115629	0.1286669	2.4756936	1.1541490
0.02	2.2650749	0.1576776	2.4680181	1.0556200
0.03	2.3905213	0.1776931	2.4632890	0.9841081
0.04	2.5014723	0.1930507	2.4600503	0.9267733
0.05	2.6032649	0.2054602	2.4577320	0.8785355
0.06	2.6986633	0.2158054	2.4560417	0.8367559
0.07	2.7893225	0.2246132	2.4548067	0.7998538
0.08	2.8763285	0.2322270	2.4539157	0.7667934
0.09	2.9604407	0.2388849	2.4532921	0.7368525
0.1	3.0422162	0.2447595	2.4528812	0.7095040
0.11	3.1220788	0.2499806	2.4526422	0.6843495
0.12	3.2003607	0.2546482	2.4525441	0.6610794
0.13	3.2773288	0.2588417	2.4525625	0.6394475
0.14	3.3532022	0.2626245	2.4526783	0.6192545
0.15	3.4281636	0.2660488	2.4528759	0.6003363

Table 18 The numerical results of $u(t), v(t), w(t)$ and $z(t)$ for $\alpha = 0.65$.

t	$u(t)$	$v(t)$	$w(t)$	z(t)
0.01	1.9639179	0.0949705	2.4855714	1.2626815
0.02	2.0609949	0.1174860	2.4788687	1.1907917
0.03	2.1439522	0.1346572	2.4740399	1.1342424
0.04	2.2193893	0.1487977	2.4702747	1.0863754
0.05	2.2900004	0.1608861	2.4672275	1.0443944
0.06	2.3572163	0.1714560	2.4647084	1.0067813
0.07	2.4219104	0.1808402	2.4625988	0.9725932
0.08	2.4846651	0.1892641	2.4608178	0.9411926
0.09	2.5458935	0.1968898	2.4593074	0.9121224
0.1	2.6059026	0.2038384	2.4580242	0.8850406
0.11	2.6649288	0.2102041	2.4569343	0.8596824
0.12	2.7231594	0.2160613	2.4560112	0.8358376
0.13	2.7807469	0.2214705	2.4552330	0.8133359
0.14	2.8378175	0.2264818	2.4545820	0.7920365
0.15	2.8944781	0.2311367	2.4540433	0.7718215

Table 19 The numerical results of $u(t), v(t), w(t)$ and $z(t)$ for $\alpha = 0.75$.

t	$u(t)$	$v(t)$	$w(t)$	$z(t)$
0.01	1.8888197	0.0754865	2.4916674	1.3231069
0.02	1.9510071	0.0916200	2.4866014	1.2731838
0.03	2.0066717	0.1049811	2.4825432	1.2310131
0.04	2.0587768	0.1166365	2.4791146	1.1935520
0.05	2.1085806	0.1270637	2.4761431	1.1594556
0.06	2.1567698	0.1365350	2.4735291	1.1279656
0.07	2.2037727	0.1452268	2.4712072	1.0985956
0.08	2.2498801	0.1532625	2.4691313	1.0710075
0.09	2.2953019	0.1607336	2.4672668	1.0449528
0.1	2.3401966	0.1677106	2.4655869	1.0202412
0.11	2.3846880	0.1742495	2.4640702	0.9967223
0.12	2.4288759	0.1803960	2.4626992	0.9742741
0.13	2.4728425	0.1861879	2.4614591	0.9527960
0.14	2.5166574	0.1916573	2.4603376	0.9322033
0.15	2.5603799	0.1968314	2.4593240	0.9124237

Table 20 The numerical results of $u(t), v(t), w(t)$ and $z(t)$ for $\alpha = 0.85$.

t	$u(t)$	$v(t)$	$w(t)$	$z(t)$
0.01	1.849166	0.064464	2.495224	1.356641
0.02	1.889332	0.075572	2.491640	1.322845
0.03	1.927040	0.085456	2.488516	1.292380
0.04	1.963408	0.094520	2.485710	1.264099
0.05	1.998933	0.102951	2.483151	1.237473
0.06	2.033893	0.110864	2.480799	1.212191
0.07	2.068468	0.118332	2.478625	1.188049
0.08	2.102781	0.125412	2.476608	1.164900
0.09	2.136924	0.132145	2.474731	1.142634
0.1	2.170969	0.138563	2.472981	1.121165
0.11	2.204972	0.144694	2.471348	1.100422
0.12	2.238980	0.150560	2.469823	1.080349
0.13	2.273033	0.156181	2.468397	1.060896
0.14	2.307165	0.161573	2.467063	1.042022
0.15	2.341407	0.166751	2.465817	1.023690

Table 21 The numerical results of $u(t), v(t), w(t)$ and $z(t)$ for $\alpha = 0.95$.

t	$u(t)$	$v(t)$	w(t)	$z(t)$
0.01	1.8276560	0.0582520	2.4972607	1.3753499
0.02	1.8537401	0.0657405	2.4948085	1.3527807
0.03	1.8794470	0.0728526	2.4925105	1.3311612
0.04	1.9050135	0.0796728	2.4903364	1.3102523
0.05	1.9305510	0.0862436	2.4882703	1.2899374
0.06	1.9561254	0.0925915	2.4863021	1.2701452
0.07	1.9817814	0.0987354	2.4844242	1.2508266
0.08	2.0075516	0.1046896	2.4826307	1.2319452
0.09	2.0334617	0.1104656	2.4809168	1.2134724
0.1	2.0595327	0.1160730	2.4792785	1.1953849
0.11	2.0857828	0.1215200	2.4777120	1.1776635
0.12	2.1122277	0.1268137	2.4762143	1.1602913
0.13	2.1388816	0.1319605	2.4747825	1.1432540
0.14	2.1657579	0.1369660	2.4734138	1.1265384
0.15	2.1928689	0.1418356	2.4721060	1.1101332

Table 22 The numerical results of $u(t), v(t), w(t)$ and $z(t)$ for $\alpha = 1$.

t	$u(t)$	$v(t)$	$w(t)$	$z(t)$
0.01	1.8208430	0.0562480	2.4979224	1.3813571
0.02	1.8418934	0.0623679	2.4959088	1.3629694
0.03	1.8631555	0.0683620	2.4939579	1.3448332
0.04	1.8846339	0.0742325	2.4920683	1.3269451
0.05	1.9063332	0.0799816	2.4902387	1.3093016
0.06	1.9282583	0.0856115	2.4884679	1.2918994
0.07	1.9504142	0.0911243	2.4867546	1.2747351
0.08	1.9728059	0.0965221	2.4850976	1.2578055
0.09	1.9954389	0.1018069	2.4834957	1.2411072
0.1	2.0183185	0.1069809	2.4819478	1.2246372
0.11	2.0414504	0.1120459	2.4804527	1.2083921
0.12	2.0648404	0.1170039	2.4790092	1.1923690
0.13	2.0884947	0.1218569	2.4776162	1.1765647
0.14	2.1124195	0.1266068	2.4762727	1.1609762
0.15	2.1366212	0.1312553	2.4749775	1.1456006

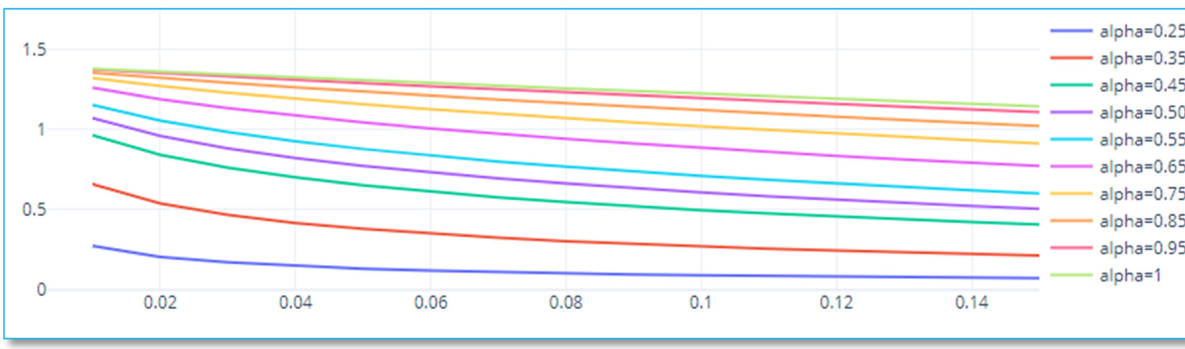

Fig. 3 This is a figure of $w(t)$ for different values of $\alpha, 0.01 < t < 0.15$.

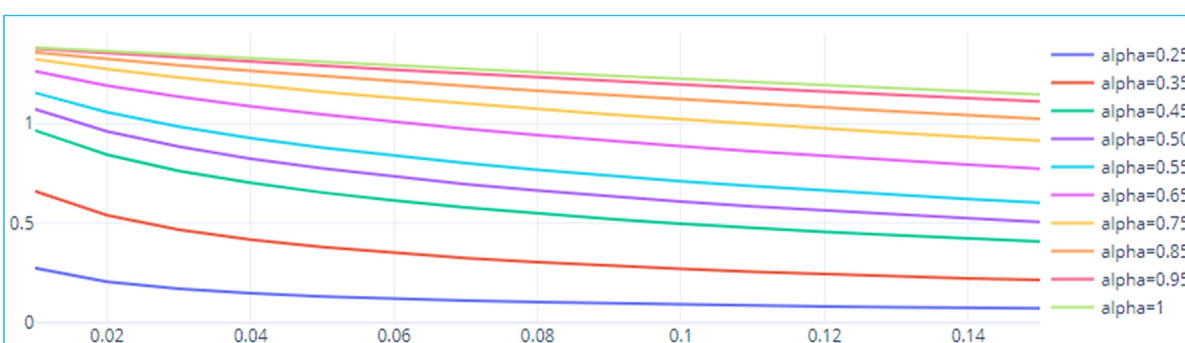

Fig. 4 This is a figure of $z(t)$ for different values of, $0.01 < t < 0.15$.

From Eqs. (13) and (15), it may be calculated by using the inverse transformation rule in Eq. (7), $u(t), v(t), w(t)$ and $z(t)$for different values of α.Numerical results in a detailed manner are given in Tables. 13-22. The results show that the values of $u(t)$, $v(t)$, $w(t)$ and $z(t)$ decreases very faster with the increasing value ofα. In the special case when $\alpha = 1$, we obtain the approximate solutions.

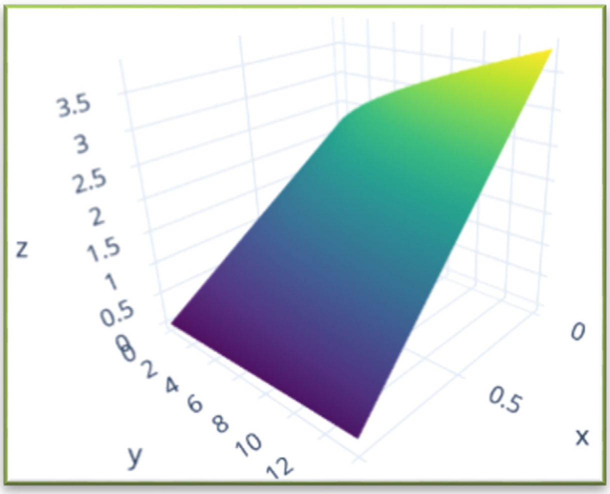

Fig. 5 3D surfaces of $u(t)$ for $\alpha = 0.5, 0 < x < 1, 0 < y < 12$.

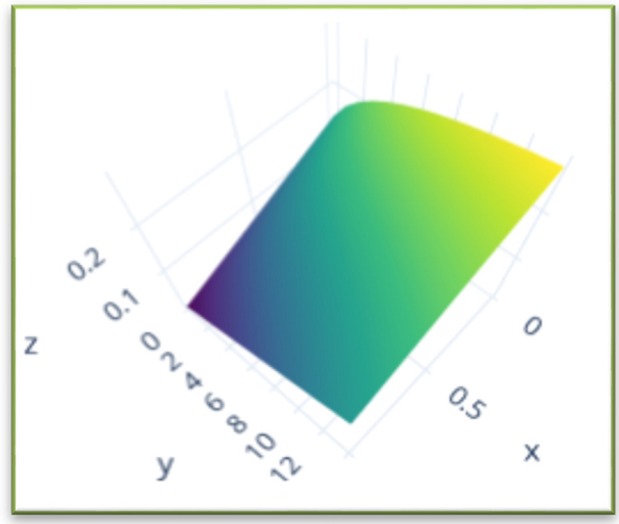

Fig. 8 3D surfaces of $v(t)$ for $\alpha = 0.5, 0 < x < 1, 0 < y < 12$.

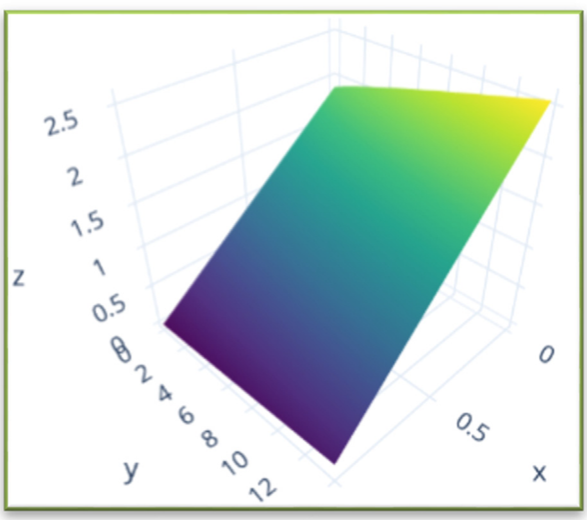

Fig. 6 3D surfaces of $u(t)$ for $\alpha = 0.75, 0 < x < 1, 0 < y < 12$.

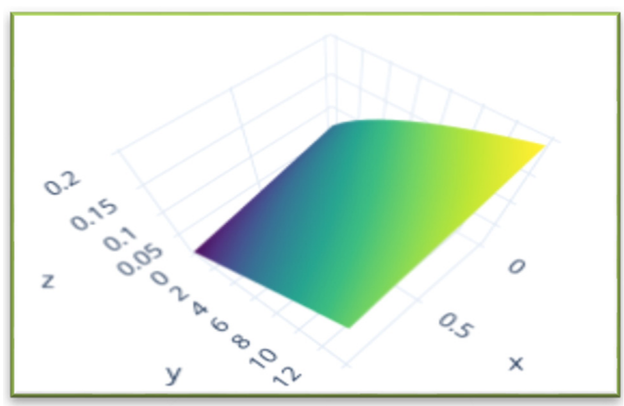

Fig. 9 3D surfaces of $v(t)$ for $\alpha = 0.75, 0 < x < 1, 0 < y < 12$.

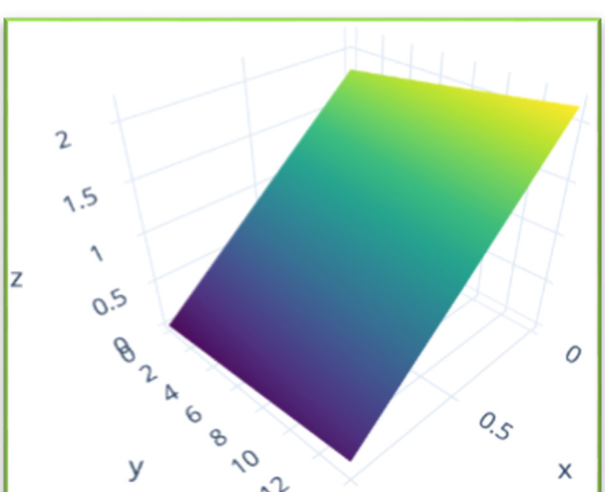

Fig. 7 3D surfaces of $u(t)$ for $\alpha = 1, 0 < x < 1, 0 < y < 12$.

$u(t)$, $v(t)$, $w(t)$ and $z(t)$ are calculated for different values of α and numerical comparisons are also presented and shown in the graphs 1–16.

4. Conclusions

In this paper, we have successfully applied the conformable fractional differential transform method to extract numerical solutions of an Epidemic system of HIV/AIDS transmission with conformable derivative. This method is based on the differential transform method. Considering suitable values of parameters, we have presented numerical solutions as being in Tables 3-22 which change the values of $\boldsymbol{\alpha}$ parameter. Moreover, we have also introduced several simulations as being in Figs. 1-16 to observe numerical behaviors of results. From these numerical results presented in Tables 3-22 and Figs. 1-16, it is said that these results may be useful to explain some important properties of an Epidemic system of HIV/AIDS

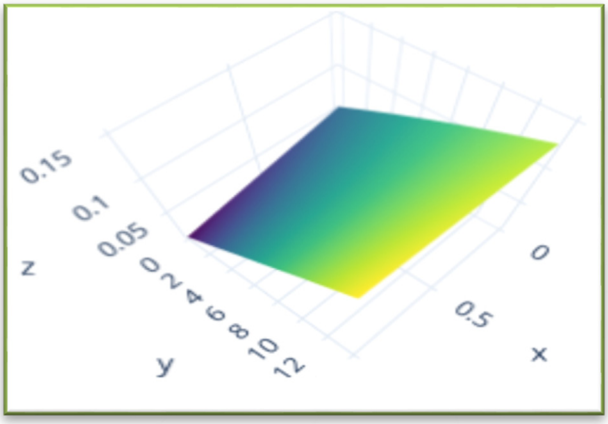

Fig. 10 3D surfaces of $v(t)$ for $\alpha = 1, 0 < x < 1, 0 < y < 12$.

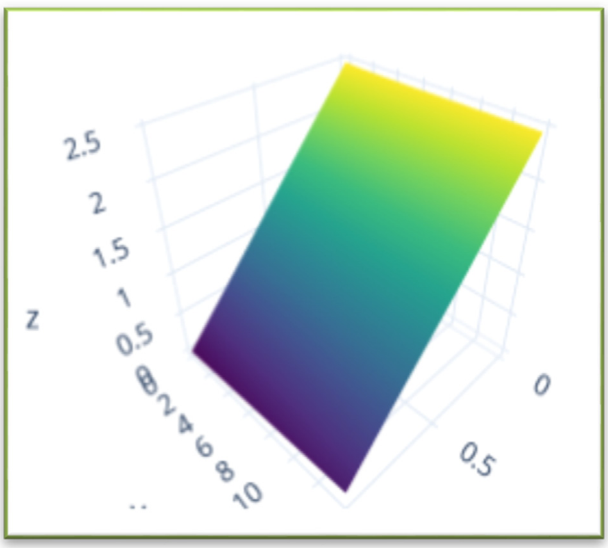

Fig. 11 3D surfaces of $w(t)$ for $\alpha = 0.5, 0 < x < 1, 0 < y < 12$.

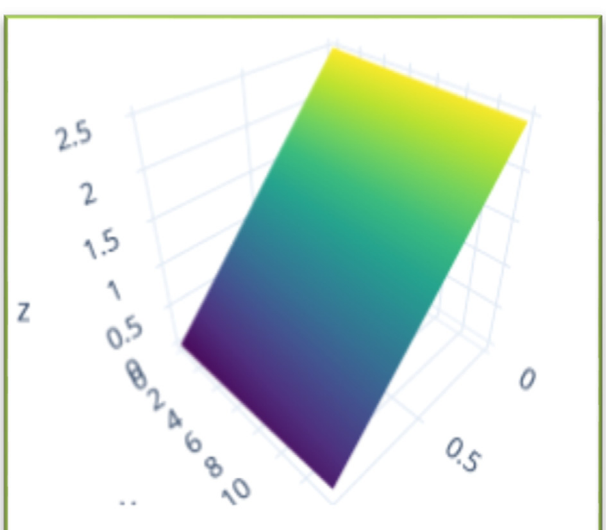

Fig. 12 3D surfaces of $w(t)$ for $\alpha = 0.75, 0 < x < 1, 0 < y < 12$.

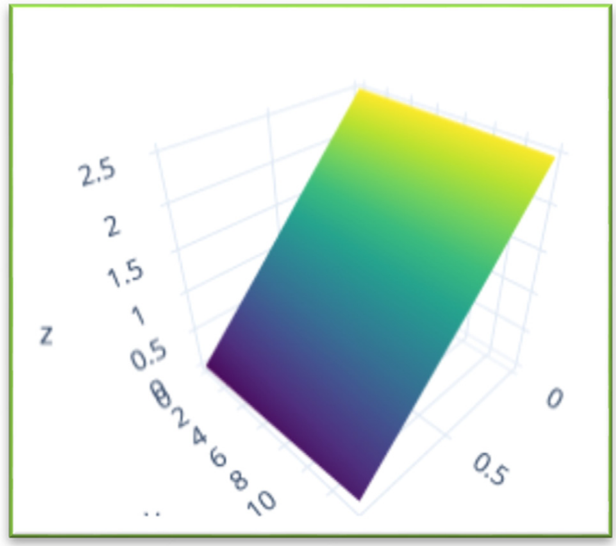

Fig. 13 3D surfaces of $w(t)$ for $\alpha = 1, 0 < x < 1, 0 < y < 12$.

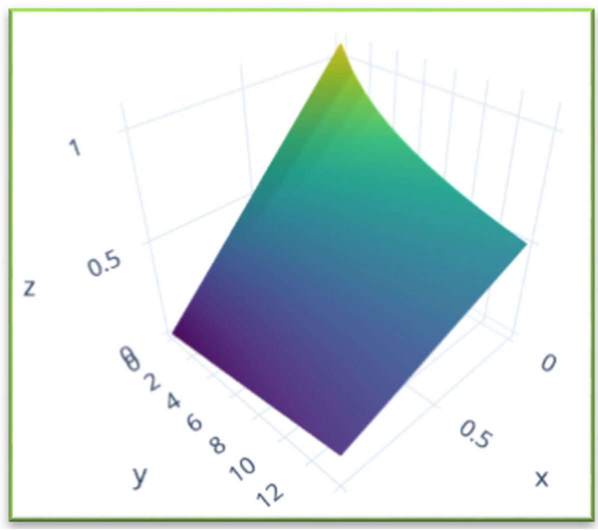

Fig. 14 3D surfaces of $z(t)$ for $\alpha = 0.5, 0 < x < 1, 0 < y < 12$.

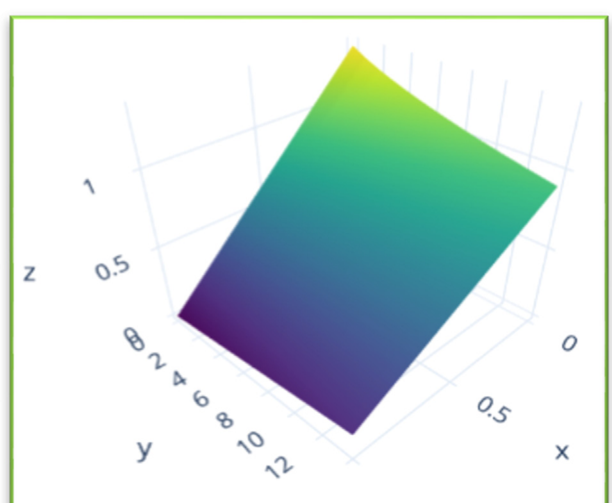

Fig. 15 3D surfaces of $z(t)$ for $\alpha = 0.75, 0 < x < 1, 0 < y < 12$.

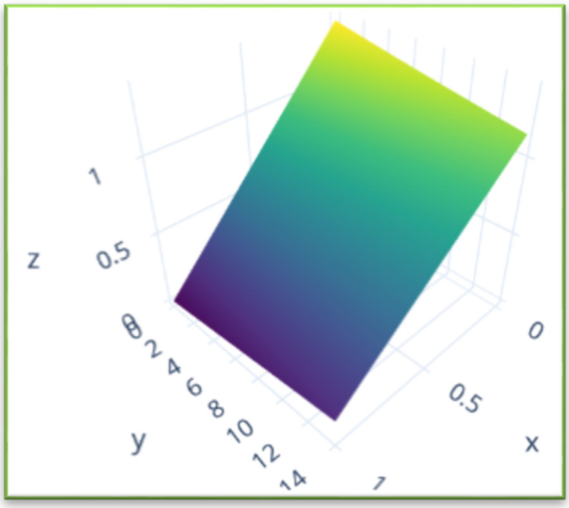

Fig. 16 3D surfaces of $z(t)$ for $\alpha = 1, 0 < x < 1, 0 < y < 12$.

transmission. Furthermore, this study shows that the method is an effective and suitable technique to observe some important properties of such kind systems.

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