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Research Article

Reliability analysis of a reinforced concrete bridge under moving loads

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ABSTRACT

This study presents a reliability analysis procedure for a reinforced concrete bridge exposed to different moving loads. Bridges are one of the important part of transportation infrastructure systems. As bridges age, structural weakening due to heavy traffic and aggressive environmental factors lead to an increase in repair frequency and decrease in load carrying capacity. Therefore, bridges require periodic maintenance and repair in order to function and be reliable throughout their lifetimes. In other words, condition and safety of the bridges must be monitored at regular time intervals to avoid the disadvantages of deterioration. Otherwise, sudden collapse of a bridge may lead to irreversible loss of life and property. Therefore, the importance of the structural assessment of bridges is rapidly increasing in developed countries. In this study, reliability analysis which is one of the structural performance prediction method is applied to a reinforced concrete bridge subjected to the different moving loads. The aim of this study is to observe the safety of the bridge for the effect of the increasing traffic factor over the years.

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1. Introduction

Infrastructure systems are essential facilities for communities and countries because they supply the necessary transportation, water and energy utilities. Because of increasing populations, more facilities are being constructed to meet demand for these systems. Increasing number of infrastructure systems leads to results in encountering the most important problems allocating sufficient funds and making appropriate decision for maintenance and repair to ensure their survival and serviceability during the lifetime period. In addition, aging and environmental factors create more needs for periodic inspection, maintenance and repair of these systems. Furthermore, the process of planning and design are invariably made under conditions of uncertainty and risk is often unavoidable. In addition, there are various causes of performance deterioration of a structural system. Particularly, in reinforced concrete bridges, deterioration is caused by corrosion and the main reason for corrosion in concrete is the chloride diffusion into concrete leading to corrosion of steel reinforcement. The causes of deterioration of performance may be grouped into three main categories. Kong (2001) stated that they include the aging (reduction of resistance and increase in loading), special actions (collusions by vehicles, earthquakes, pollution, etc.) and human errors. Existence of deterioration may have a major impact on the serviceability and load carrying capacity of bridges. For instance, small amount of local corrosion in prestressing steel cables of prestressed reinforced concrete beams may cause a sudden collapse in the structure. Therefore, the infrastructure management systems are needed to monitor the condition and safety of structural systems over the years. One of the infrastructure management systems is the Bridge Management Systems (BMSs) because bridges are one of the crucial part of the transportation infrastructure systems. After unexpected failures of certain bridges have occurred such as the Silver Bridge in the U.S, researchers focused on creating BMSs to establish

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maintenance and repair programs and to record the condition of bridges. Thompson et al. (1998) developed a well-known and most used bridge management system in the world. Also, Hawk and Small (1998) created another bridge management system to enable the allocation of resources for repair and maintenance of bridges. In other words, the aim of BMSs is to implement the best maintenance and repair strategies ensuring an adequate level of reliability at the lowest possible life-cycle cost.

In developed countries for the structural assessment of bridges, structural safety criterion is the most important criterion among the other criteria taken into account which affects the determination of investment budgets of bridge maintenance and repair. Safety is a function of combinations of loads over the lifetime of the structure. Furthermore, structural safety depends on the load carrying capacity of the structure. However, structural deterioration of reinforced concrete bridges decreases their load carrying capacities. Therefore, the assessment of remaining load carrying capacity of bridges is crucial part in management of bridge structures. In addition, performance prediction of an infrastructure system is a difficult process due to existence of many uncertainties. Mori and Ellingwood (1993) studied on uncertainties in the deterioration initiation time of a bridge structure. Hence, deterioration prediction models are produced to overcome this difficulty. The condition-based and safety-based performance prediction methods used by BMSs are two different procedures to give an idea about the structural safety. Unlike the condition-based structural assessment which is generally performed visually, structural assessment normally requires structural engineering formulations or determination of quantified value of resistance degradation and load increase in a bridge member. Reliability index β or probability of failure P_f can be used as a performance indicator to quantify the structural safety. Actually, reliability analysis method is one of the procedure used for safety prediction approaches in developed countries to conduct the structural assessment of bridges. Reliability index β and probability of failure P_f concepts are introduced in the following section.

2. Reliability Analysis

Reliability analysis methods are subject of the mechanical studies based on probability. In other words, probabilistic measure of assurance of performance is defined as reliability. Prediction of structural reliability is generally based on either the calculation of reliability index or the probability of failure. As an alternative criterion to the probability of failure, reliability index has been more often used as a measure for bridge elements and systems. Reliability can be formulated as the determination of the capacity of the system to meet certain requirements. This way, probabilistic nature of structural capacity and load can be modeled using the supply capacity (resistance) and the demand requirement (load) terms presented by R and Q, respectively. The objective of the reliability analysis is to ensure the event $R>Q$ throughout the lifetime of the structure. This is possible only if the probability event $P(R>Q)$ is satisfied. Fig. 1 shows the relative probability distributions of resistance R of a structural element and load impact Q.

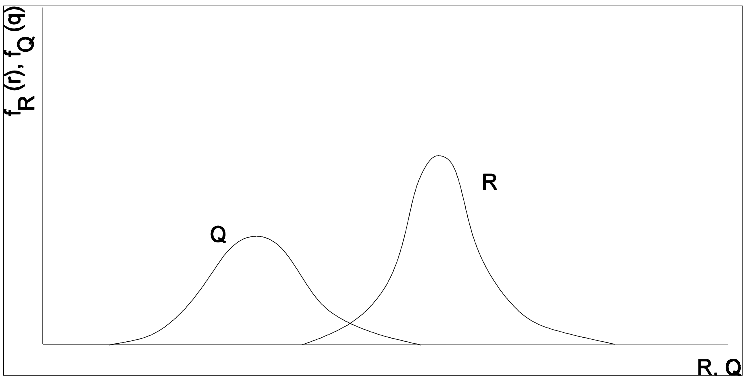

Fig. 1. Probability distributions of strength R and load impact Q.

Load and resistance have a time dependent effect on probability of failure throughout the service life of structure. Indeed, expected resistance of a structure decreases in time because of environmental factors, whereas load impact on a structure increases in time. Probability of failure can be defined as the probability that the resistance is less than the load, which is formulated as Eq. (1).

$$P_f = \int_{-\infty}^{\infty}\left[1 - F_Q(x)\right]f_R(x)dx \quad (1)$$

This integral is not generally solved by analytical. Frangopol et al. (2004) generated a number of method to approximate the probability of failure. In addition, failure probability is used to obtain the reliability index of the structural system. Here, probability of occurrence of $R>Q$ is calculated by integration known as reliability of the structural element (probability of safety, P_s) which is defined by the area under the joint probability distribution function $f_{R,Q}(r,q)$ shown as in Eq. (2).

$$P_s = P(R > Q) = P(R - Q > 0) = \iint_{R>Q} f_{R,Q}(r,q)drdq \quad (2)$$

The term R-Q defines another random variable which is called as Safety Margin and shown by M. The safety margin consists of resistance variable R and load variable Q.

The mean and standard deviation of M are shown by μ_M and σ_M, respectively. If R and Q are normally distributed random variables, there is a direct relationship between the probability of failure and reliability index. Then Eq. (3) can be formulated as:

$$P_f = P(R - Q \leq 0) = P(M \leq 0) = \Phi\left(-\frac{\mu_M}{\sigma_M}\right) \tag{3}$$

In Eq. (3), the value of ϕ which is the cumulative distribution function of standard normal variable can be found in any standard normal distribution table. The value, within the parenthesis of ϕ function, is called the safety index by Cornell (1969). Also, this ratio is called as reliability index β shown in Eq. (4).

$$\beta = \frac{\mu_M}{\sigma_M} \tag{4}$$

The probability of occurrence of any event in statistics is between 0 and 1. Therefore, probability of safety in terms of the probability of failure is defined as shown in Eq. (5). In addition, probability of safety is defined in terms of reliability index as shown in Eq. (6).

$$P_S = 1 - P_f \tag{5}$$

$$P_s = \Phi(\beta) \tag{6}$$

Using the description of safety margin, for normally distributed random variables, the reliability index formula can be extended as in Eq. (7).

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R - \sigma_Q}} \tag{7}$$

Safety margin equation described above M=R-Q is called as performance function g(x) which defines limit state M=0 as well is defined as thus. The vector X contains the random variables in limit state function. g(x) describes the limit state of the system, g(x)<0 defines failure state, and however g(x)>0 represents safety state. In reliability analysis, Christensen (1998) introduced performance functions considering failure modes for bridges.

Structural systems are composed of structural members. In addition, capacity of the systems is affected by the capacity and formation of the members. There are three types of structural systems according to combination of topologies and configuration of structural components, namely; series, parallel, and the combination of series and parallel systems. The safety or failure of these systems are determined by formulas according to system types. Enright and Frangopol (1998) studied on system reliability for reinforced concrete highway girder bridges with time-dependent resistance and loads. The study considered environmental factors to predict the reliability of reinforced concrete bridges. Estes and Frangopol (1999) proposed a system reliability approach for optimizing the lifetime repair strategy for highway bridges. In their study, the bridge was modeled as a series-parallel combination of failure modes, limit-state equations were developed for each failure mode in terms of certain random variables, and the reliability with respect to occurrence of each possible failure mode was computed separately based on these limit-state equations using First Order Reliability Method (FORM). In addition, Hong (2000) studied on taking into account the correlation between the failures of structural elements for the system reliability. Furthermore, Estes and Frangopol (2001) demonstrated that a component whose reliability index is below the target reliability level may not cause the reliability of the system to fall below the target reliability. If components of the systems are connected in series, such systems are called series systems and the failure of these systems requires failures of any one of the components. In other words, the reliability or safety of the system requires that none of the components fail. If components of the systems are connected in parallel, such systems are called parallel systems and the total failure of these systems requires failures of all components. In other words, the system remains safe if any one of the components survives. On the other hand, many structures in reality include a combination of series and parallel systems as illustrated in Fig. 2, where each number designates a failure mode or a component of a system. Also, bridges are examples of combination of series and parallel structural systems. In multi span bridges, the spans are in series connections with each other. Furthermore, each span consists of elements that form a parallel system in itself.

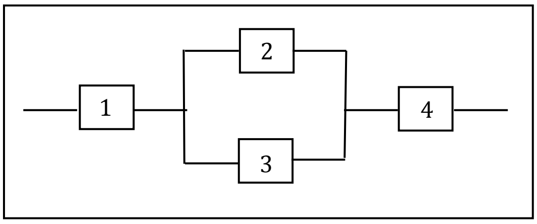

Fig. 2. A combined series-parallel system.

Failure of combination of series and parallel systems are formulated shown in Eq. (8). Here, the terms n and m show the number of series and parallel components in the structural system, respectively. Eq. (8) shows that the total failure of a system requires failure of any one of the series elements or failure of all parallel components. For instance, in Fig. 2, system fails when the components numbered 1 or 4 fails. In addition, system fails when both components numbered 2 and 3 fail together. Otherwise, the system remains safe and continue to survive.

$$P_f = P\left(\bigcup_{k=1}^{n} \bigcap_{j=1}^{m_i} \{g_{ij}(x) < 0\}\right) \tag{8}$$

When the bridge superstructure elements are considered, the limit state equations are different for the deck and girders. In addition, limit state equations are determined for every structural components according to their failure modes. For instance, different limit state equations are determined by considering the bending moment and shear force. Akgül and Frangopol (2004) and (2005) developed the limit state equations of bridge elements for different bridge types. According to solution of limit state functions, there are two different reliability method. These are the First Order Reliability Method (FORM) and the Second Order Reliability Method (SORM). In FORM, limit state functions are linearized and then solved. Also, FORM can be solved in an iterative manner. The formulations consist of theoretical essentials of FORM were presented by Ang and Tang (1984). On the other hand, SORM is a method based on a quadratic approximation to limit state function.

3. Analysis of a Reinforced Concrete Bridge

Behavior of civil infrastructure systems is predicted using structural assessment methods. In this study, reliability analysis method is applied to an example reinforced concrete bridge aiming to predict the performance level of the bridge. The concrete material is found easily in the most areas of the world and has high flexibility. Furthermore, reinforced concrete bridges provide sufficiently earthquake-resistance performance and appropriate long-term maintenance cost. Also, reinforcing the concrete gives an important ability to bridges for crossing the appropriate openings. Therefore, the reinforced concrete bridge is one of the most preferred bridge type. In this study, a multi span reinforced concrete bridge is chosen as an exemplary bridge built by Turkish General Directorate of Highways to cross the stream in the end of 1960s. The chosen bridge was designed with 4 spans and 8 number of I profiles were used in cross-section of the bridge. Total length of the bridge is 44 meters. Both length of inner spans are 12 meters. Also, length of outer spans is 10 meters. In addition, steel ST 37 and concrete Wb28 225 have been used in construction of the bridge. In this study, the system failure model is constituted by considering the superstructure of the bridge consisting of decks and girders. Also, the First Order Reliability Method is used to calculate the reliability index for the reinforced concrete bridge. The reliability index β calculated is compared to the target reliability index β_T which is the minimum reliability index for the structure to be safe. The limit state functions and bridge span length affect the value of target reliability index shown as in Table 1. In this study, target reliability index for analysed bridge is selected as 3.5 according to ASSHTO (1992).

Table 1. Target reliability indices according to bridge properties.

Bridge Type	Span length	Girder spacing	Target Reliability Index
All type	~10 m	~1.2 m	between 2 and 4
Steel girders and reinforced concrete T-beams	~(20-60 m)	-	between 3 and 4
Prestressed concrete girders	-	-	~5

In order to perform reliability analysis, firstly, structural analysis was conducted and the loads to which the bridge was exposed were calculated. The bridge was designed by the Turkish General Directorate of Highways according to H15-S12 truck load. In this study, two different moving loads are acted on the bridge to observe the safety of the bridge for the effect of the increasing traffic factor over the years. The selected moving loads are H15-S12 and H20-S16 truck loads. Load characteristics of both vehicles are given in the Table 2. Analysis carried out for the design load of the bridge and the results obtained from the design load as H15-S12 truck were represented in Figs. 3 and 4. The bridge was analyzed by loads at supports called as C, D, E and at spans called as F, G because of having a structural symmetry. Moving load resulted by truck passing over the bridge was distributed to girders of the bridge using moving load distribution factor and impact factor according to American Association of State Highway and Transportation Officials – Live and Resistance Factor Design, Live Load Distribution Specifications, AASHTO (1994).

Table 2. Vehicles load characteristics.

Load class	H15-S12	H20-S16
Weight, kN	150	200
Concentrated load for moment, kN	67.5	90
Concentrated load for shear, kN	97.5	135
Distributed load, kN/m	7.5	10

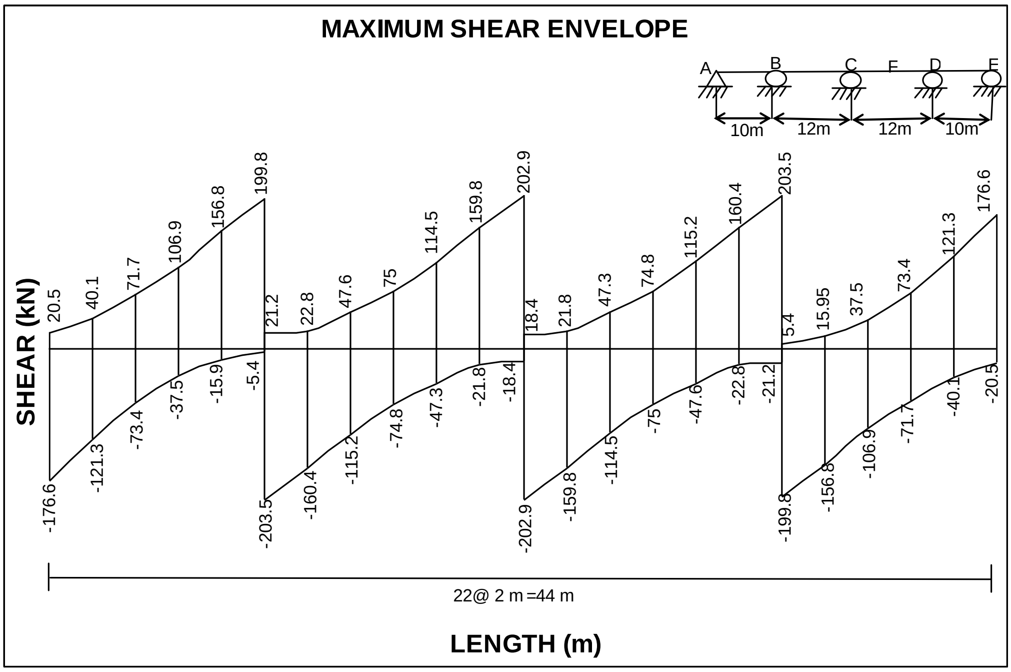

Fig. 3. Maximum shear envelope for H15-S12 truck load.

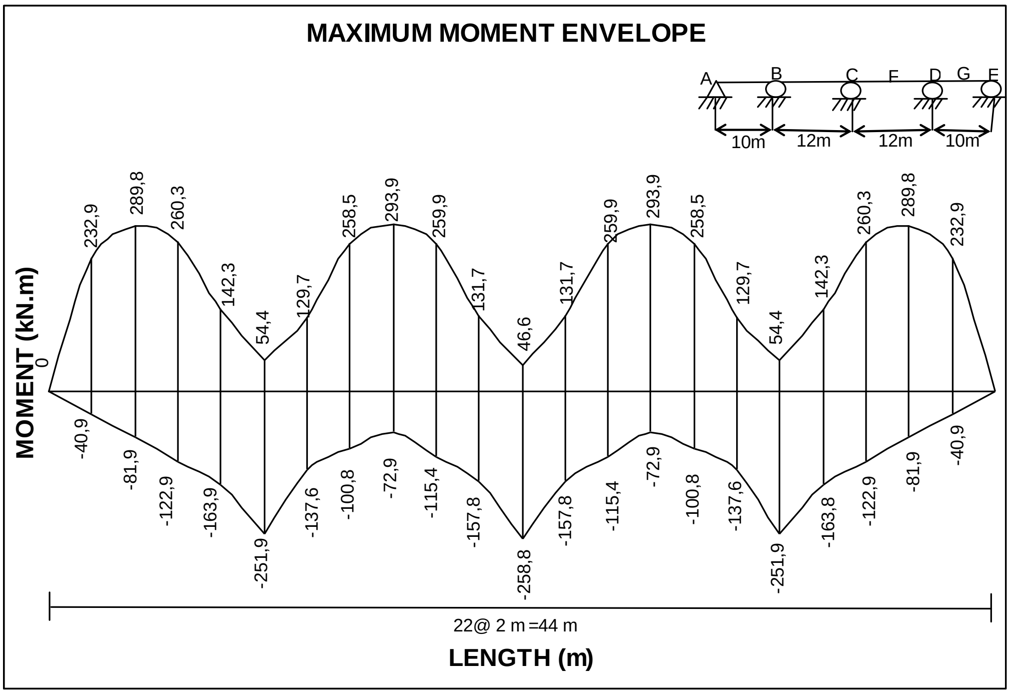

Fig. 4. Maximum moment envelope for H15-S12 truck load.

3.1. Calculation of reliability index

The shear force capacity and moment capacity are calculated by multiplying nominal capacity values of the cross-section with 1,1 shown in Eqs. (9) and (10). Also, the structural analysis results for shear force and moments capacity are presented in Table 3.

$$\overline{M_R} = 1{,}1 \times M_{n,cap} \tag{9}$$

$$\overline{V_R} = 1{,}1 \times V_{n,cap} \tag{10}$$

The moving load due to the vehicles passing over the bridge is distributed to the girders on the bridge by using the live load distribution factor and the impact factor as shown in Eqs. (11-13).

$$D_f = \frac{S}{5{,}5} \tag{11}$$

In Eq. (11), D_f is the live load distribution factor and its approximate value for the analysed bridge is 0.182. S is the distance between the two adjacent girders. In addition, live load shear force and moment which applied to girders are calculated as in Eqs. (12) and (13). Here, I is the impact factor and assumed as 1.3 in calculations.

$$\overline{M_{LL}} = M_{LL} \times I \times D_f \quad (12)$$

$$\overline{V_{LL}} = V_{LL} \times I \times D_f \quad (13)$$

In the probabilistic reliability analysis, the coefficient of variation (COV) values generally varies from 0.05 to 0.30 according to the variables denoted by Frangopol (1999). In this study, COV values used for variables are shown in Table 4.

Finally, reliability index of the bridge for shear force and moment were calculated by using Eq. (7) taking into account unknowns of capacity, dead load and moving load shown in Tables 5 and 6.

Table 3. Structural analysis results for H15-S12 truck load.

	Support C	Span F	Support D	Span G
Shear force capacity (kN)	5156.8	5156.8	5156.8	5156.8
Dead load shear (kN)	48.45	0.253	-50.5	-9.94
Moving load shear (kN)	202.9	-75	203.5	176.6
Moment capacity (kN.M)	683.28	683.28	683.28	683.28
Dead load moment (kN.M)	-96.4	48.19	-99.43	51.75
Moving load moment (kN.M)	-258.8	293.9	-251.9	289.8

Table 4. Coefficient of variation (COV) for the variables (%).

COV values	Used values	Range
Capacity	10	5-15
Dead load	5	5-10
Moving load	20	15-30

Table 5. Reliability Indices at supports and spans based on design load for shear force.

	Support C	Span F	Support D	Span G
Shear force capacity (kN)	5156.8	5156.8	5156.8	5156.8
Dead load shear (kN)	48.45	0.253	-50.5	-9.94
Moving load shear force (kN)	48	-17.75	48.15	41.78
Variance of shear force capacity (kN)	265925.86	265925.86	265925.86	265925.86
Sum of variance of loads (kN)	315.26	112.5	321.5	264.4
Reliability index (β)	11.21	13.55	10.75	12.23

Table 6. Reliability Indices at supports and spans based on design load for moment.

	Support C	Span F	Support D	Span G
Moment capacity (kN.M)	683.28	683.28	683.28	683.28
Dead load moment (kN.M)	-96.40	48.19	-99.43	51.75
Moving load moment (kN.M)	-61.23	69.55	-59.60	63.78
Variance of moment capacity (kN.M)	4668.99	4668.99	4668.99	4668.99
Sum of variance of loads (kN)	173.20	199.30	166.80	169.50
Reliability index (β)	7.55	8.11	7.54	8.16

Reliability index β for the shear force capacity was calculated based on the analysis results for truck H15-S12 regarding standard normal distribution table. The smallest reliability index for shear capacity is obtained at support D as 10.75. The value 10.75 of reliability index pairs with the value 0.25E-15 of probability of failure and this value of the probability of failure is very small. Also, reliability index value is much bigger than selected target reliability value. This situation implies that the bridge is in safe according to shear capacity.

As shown in Table 6, reliability index β for the moment capacity was calculated based on the analysis results for truck H15-S12 and probability of failure of the bridge was calculated based on β values and regarding standard normal distribution table. As seen, the smallest reliability index is obtained at support D as 7.54. The value 7.54 of reliability index pairs with the value 0.20E-13 of probability of failure and this value of the probability of failure is also very small. In addition, reliability index value is much bigger than the selected target reliability value. Therefore, under the design load, the bridge is in safe for moment capacity.

The bridge decreasing its strength in time is exposed to the more load than the design one as well. Therefore, the second analysis was performed for heavier load than the design load of the bridge and the structural analysis results of H20-S16 truck acted to the bridge are represented in Table 7.

The analysis procedure for the results of H15-S12 truck is repeated this time for the results of H20-S16 truck by using Eqs. (9)-(13). Finally, the new reliability indices of the bridge for shear force and moment according to H20-S16 truck load were calculated by using Eq. (7) taking into account unknowns of capacity, dead load and moving load shown in Tables 8 and 9.

Reliability index β for shear force capacity was calculated based on the result of analysis for truck H20-S16. The smallest reliability index is obtained at support C and support D as 9.66. As seen, reliability index value obtained from analysis is much bigger than the selected target reliability index. So, this situation implies that the bridge subjected to H20-S16 truck load is in safe according to shear force.

Table 6. Reliability Indices at supports and spans based on design load for moment.

	Support C	Span F	Support D	Span G
Moment capacity (kN.M)	683.28	683.28	683.28	683.28
Dead load moment (kN.M)	-96.40	48.19	-99.43	51.75
Moving load moment (kN.M)	-61.23	69.55	-59.60	63.78
Variance of moment capacity (kN.M)	4668.99	4668.99	4668.99	4668.99
Sum of variance of loads (kN)	173.20	199.30	166.80	169.50
Reliability index (β)	7.55	8.11	7.54	8.16

Table 7. Structural analysis results for H20-S16 truck load.

	Support C	Span F	Support D	Span G
Shear force capacity (kN)	5156.8	5156.8	5156.8	5156.8
Dead load shear (kN)	48.45	0.253	-50.5	-9.94
Moving load shear (kN)	508.4	-243.5	-509.5	-278.83
Moment capacity (kN.M)	683.28	683.28	683.28	683.28
Dead load moment (kN.M)	-96.4	48.19	-99.43	51.75
Moving load moment (kN.M)	-860.3	893.1	-816	886

Table 8. Reliability indices for shear force capacity at supports and spans based on H20-S16 truck load.

	Support C	Span F	Support D	Span G
Shear force capacity (kN)	5156.8	5156.8	5156.8	5156.8
Dead load shear (kN)	48.45	0.253	-50.5	-9.94
Moving load shear force (kN)	120.29	-57.61	-59.60	63.78
Variance of shear force capacity (kN)	265925.86	265925.86	265925.86	265925.86
Sum of variance of loads (kN)	584.76	132.7	587.7	174.5
Reliability index (β)	9.66	9.88	9.66	9.85

Table 9. Reliability indices for moment capacity at supports and spans based on H20-S16 truck load.

	Support C	Span F	Support D	Span G
Moment capacity	683.28	683.28	683.28	683.28
Dead load moment	-96.40	48.19	-99.43	51.75
Moving load moment	-203.55	211.31	-191.10	209.63
Variance of moment capacity	4668.99	4668.99	4668.99	4668.99
Sum of variance of loads	1680.53	1791.90	1516.20	1764.83
Reliability index (β)	4.81	5.27	4.96	5.26

Furthermore, reliability index β for moment capacity was calculated based on the result of analysis for truck H20-S16. The smallest reliability index is obtained at support C as 4.81. Reliability index value obtained from analysis is slightly bigger than the value of selected target reliability index. So, according to the result, the bridge subjected to H20-S16 truck load is in safe for the moment capacity.

4. Conclusions

Reliability index is used as performance indicator for the structures. In this study, reliability index of a reinforced concrete bridge subjected to different moving loads is calculated and the obtained results are compared and discussed. As bridges age, reliability index of bridge decreases due to aggressive environmental factors and heavy traffic load. The values of probability of failure of bridges not applied any maintenance and repair actions increase more and more in time. In addition, decreasing of resistance capacity resulted from aging of bridge is the most important factor causing the failure of a bridge. One of the most important reason of decreasing of the resistance capacity is the structural deterioration over the years. Material degradation models and statistical load increment models are improved in order to estimate how safety index decrease over time. In this study, any material degradation model was not used. Analysis was carried out for the design load and the load heavier than the design load to illustrate the effect of increasing traffic load in time.

The difference between reliability index obtained from the H20-S16 truck load and the reliability index obtained from the design load is compared with each other. The smallest reliability index obtained from design load is 7.54 and it has changed dramatically and reached to 4.81 which is very close to the target reliability index under H20-S16 truck load. The results of conducted analysis indicate that if the bridge is not subjected to any maintenance or repair activities for many years, the load carrying capacity of the bridge may decrease leading to sudden collapse of the bridge. Also, the analysis results revealed how important Bridge Management Systems are, especially in countries with high number of bridges. As a future work, a material degradation model and a performance deterioration model may be generated and applied to obtain the more accurate value of the reliability index of a bridge system.

References

Akgül F, Frangopol DM (2004). Lifetime performance analysis of existing steel girder bridge superstructures. *Journal of Structural Engineering, American Society of Civil Engineers,* 130(12), 1875-1888.

Akgül F, Frangopol DM (2005). Lifetime performance prediction of existing reinforced concrete bridges, I: Theory. *Journal of Infrastructure Systems, American Society of Civil Engineers*, 11(2), 122-128.

Ang AH-S, Tang WH (1984). Probability Concepts in Engineering Planning and Design, Vol. II, Decision, Risk, Reliability. John Wiley & Sons, Inc., New York, USA.

ASSHTO (1992). Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials, Wash, DC, USA.

ASSHTO (1994). LRFD Bridge Design Specifications, American Association of State Highway and Transportation Officials, Wash, DC, USA.

Christensen PT (1998). Assessment of the reliability profiles for concrete bridges. *Journal of Engineering Structures*, 20, 1004-1009.

Cornell CA (1969). A probability based structural code. *Journal of American Concrete Institute*, 66(12), 974-985.

Enright MP, Frangopol DM (1998). Probabilistic analysis of resistance degradation of reinforced concrete bridge beams under corrosion, *Engineering Structures,* 20(11), 960-971.

Estes AC, Frangopol DM (1999). Repair optimization of highway bridges using system reliability approach. *Journal of Structural Engineering, ASCE*, 125(7), 766-775.

Estes AC, Frangopol DM (2001). Bridge lifetime system reliability under multiple limit states. *Journal of Bridge Engineering*, 125(7), 766-775.

Frangopol DM (1999). Bridge Safety and Reliability. American Society and Civil Engineers, Reston, USA.

Frangopol DM, Kallen MJ, and van Noortwijk JM (2004). Probabilistic models for life-cycle performance of deteriorating structures: review and future directions. *Progress in Structural Engineering and Materials*, 6, 197-212.

Hawk H, Small EP (1998). The bridgit bridge management system. *Structural Engineering International*, 8(4), 308-314.

Hong HP (2000). Assessment of reliability of aging reinforced concrete structures. *Journal of Structural Engineering*, 126(12), 1458-1465.

Kong JS (2001). Lifetime maintenance strategies for deteriorating structures. *Ph.D thesis*, University of Colorado, Boulder, Colorado, USA.

Mori Y, Ellingwood BR (1993). Reliability-based service-life assessment aging concrete structures. *Journal of Structural Engineering*, 119(5), 1600-1621.

Thompson PD, Small EP, Johnson M, Marshall A (1998). The Pontis bridge management system. *Structural Engineering International*, 8(4), 303-308.