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TECHNICAL PAPER

A knowledge-based material selection system for interactive pressure vessel design

Mustafa Yurdakul[1] · **Arif Balci**[1] · **Yusuf Tansel Ic**[2]

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Abstract

Continual introduction of new materials and improvements in existing materials increase the variety of materials that can be used for pressure vessel components. Among wide variety of materials, the most suitable one must be selected for a component by matching its functional requirements with various available materials' specifications. This study proposes an interactive knowledge-based decision support system for selecting the most suitable material for a given pressure vessel component and its working environment. The developed decision support system, namely Pressure Vessel SELection (PVSEL), consists of two separate phases. In the first elimination phase of PVSEL, the user obtains a feasible set of alternative materials by answering various questions and providing lower-limit values at materials' critical specifications. PVSEL, then, uses a ranking phase which uses ELECTRE, TOPSIS and VIKOR methods to rank the feasible materials. In the second phase, each alternative material's ranking is determined by combining its performance values at weighted critical specifications (selection criteria), which are considered as important in meeting the functional requirements of the component. Usage of PVSEL is illustrated in the paper and the results show that the proposed PVSEL is an effective selection tool and provides meaningful results for the designers.

Keywords Pressure vessel components · Material selection · Decision support systems · Multi criteria decision making · TOPSIS · VIKOR · ELECTRE

1 Introduction

When a designer needs to make a material selection for a pressure vessel component, the decision has to consider hundreds of different material types which are specified at many specifications such as: strength, operating temperature range, operating pressure range, corrosion resistance, elastic module, hardness, weld ability, etc. [1]. In the literature, there are various studies that proposed models to solve material selection problems. For example, Karande and Chakraborty [2] proposed MOORA (The Multi-Objective Optimization on the basis of Ratio Analysis) method and Chatterjee et al. [3] combined COPRAS (COmplex PRoportional Assessment) and EVAMIX (Evaluation of Mixed Data) methods for cryogenic tank material selection problem. Jahan et al. [4] developed a pin selection model using VIKOR (VIsekriterijumska optimizacija i KOmpromisno Resenje– in Serbian) method. Sapuan et al. [5] presented a composite material selection methodology using AHP (Analytic Hierarchy Process). In another work, Mayyas et al. [6] used AHP for material selection for automobile body panels. Shanian and Savagado [7] used TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) and VIKOR methods in selection of high strength materials for nuclear power plant reactors. Chatterjee ve Chakraborty [8] provided a combined model of PROMETHEE (Preference Ranking Organisation Method for Enrichment Evaluations), EXPROM2 (Ex-tended PROMETHEE II), COPRAS, ORESTE (Organization, Rangement Et Synthese De Donnes Relationnelles) and OCRA (Operational Competitiveness Rating Analysis) methods for gear material selection. Maitya

✉ Yusuf Tansel Ic
yustanic@baskent.edu.tr

Mustafa Yurdakul
yurdakul@gazi.edu.tr

Arif Balci
arifbalci@gazi.edu.tr

1 Department of Mechanical Engineering, Gazi University, 06570 Ankara, Turkey

2 Department of Industrial Engineering, Baskent University, 06810 Ankara, Turkey

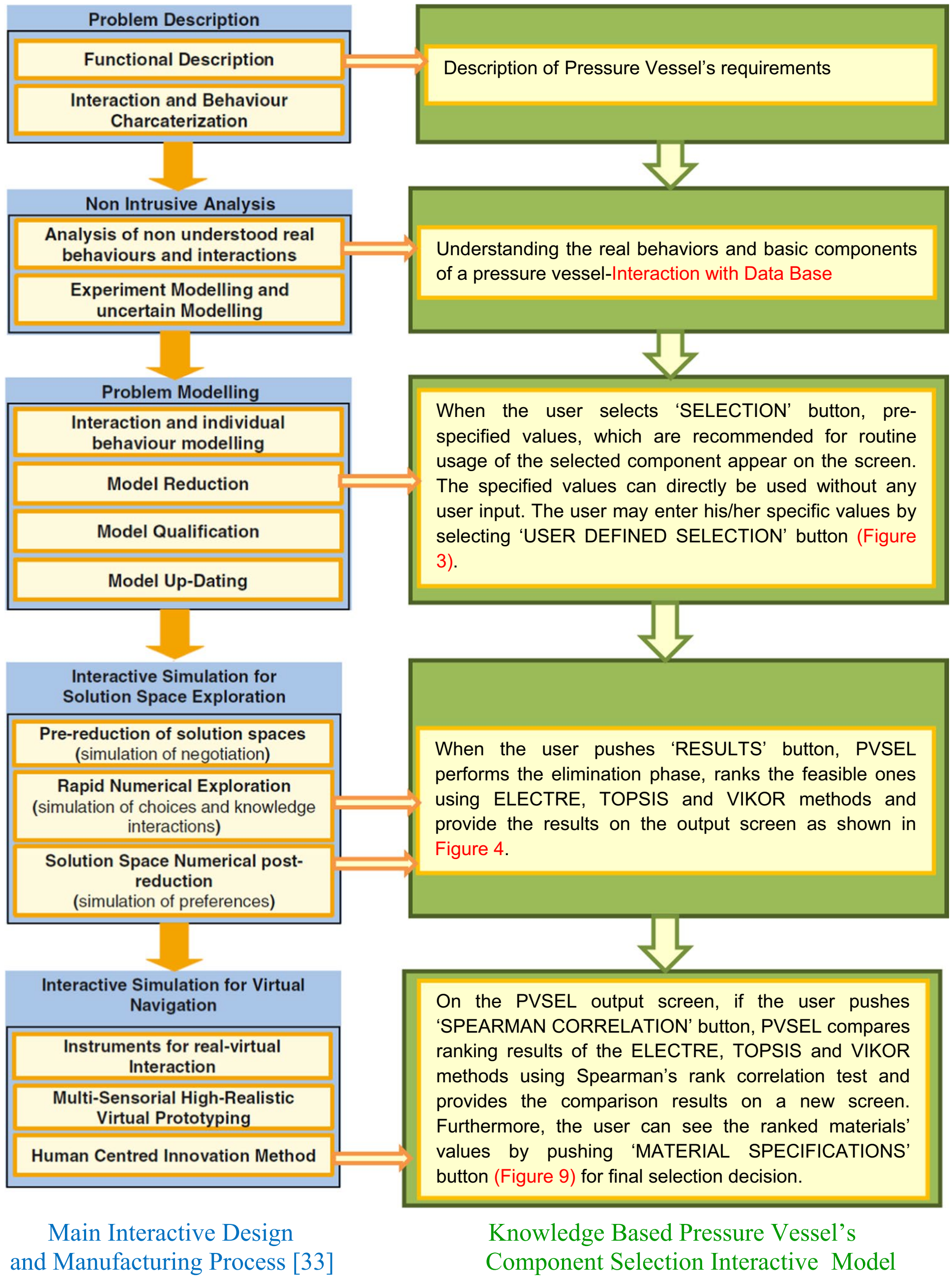


Fig. 1 Interactive design process and relation with PVSEL model

and Chakraborty [9] presented an abrasive material selection model using Fuzzy TOPSIS. Mansor et al. [10] used AHP for natural fiber composite material selection of an automobile breakage system. Bahraminasab and Jahan [11] presented VIKOR-based methodology for dental restoration material selection. Liu et al. [12] proposed an interval 2-tuple linguistic VIKOR (ITL-VIKOR) method for solving the material selection problem under uncertain and incomplete information environment. Chan and Tong [13] presented a grey relational analysis (GRA)-based model for the multi-criteria optimization of material selection problem based on the considerations of material characteristics and the end-of-life product strategies. Serafini et al. [14] and Balci et al. [15] proposed a simple computer aided material selection procedures for structurally optimized products.

In the literature, interactive design principles are also used in development of selection models. For example, Kennan and Vinay [19] proposed an interactive multi-criteria decision making model for selection of CAD/CAM systems. Dey et al. [20] employed an interactive multi-objective

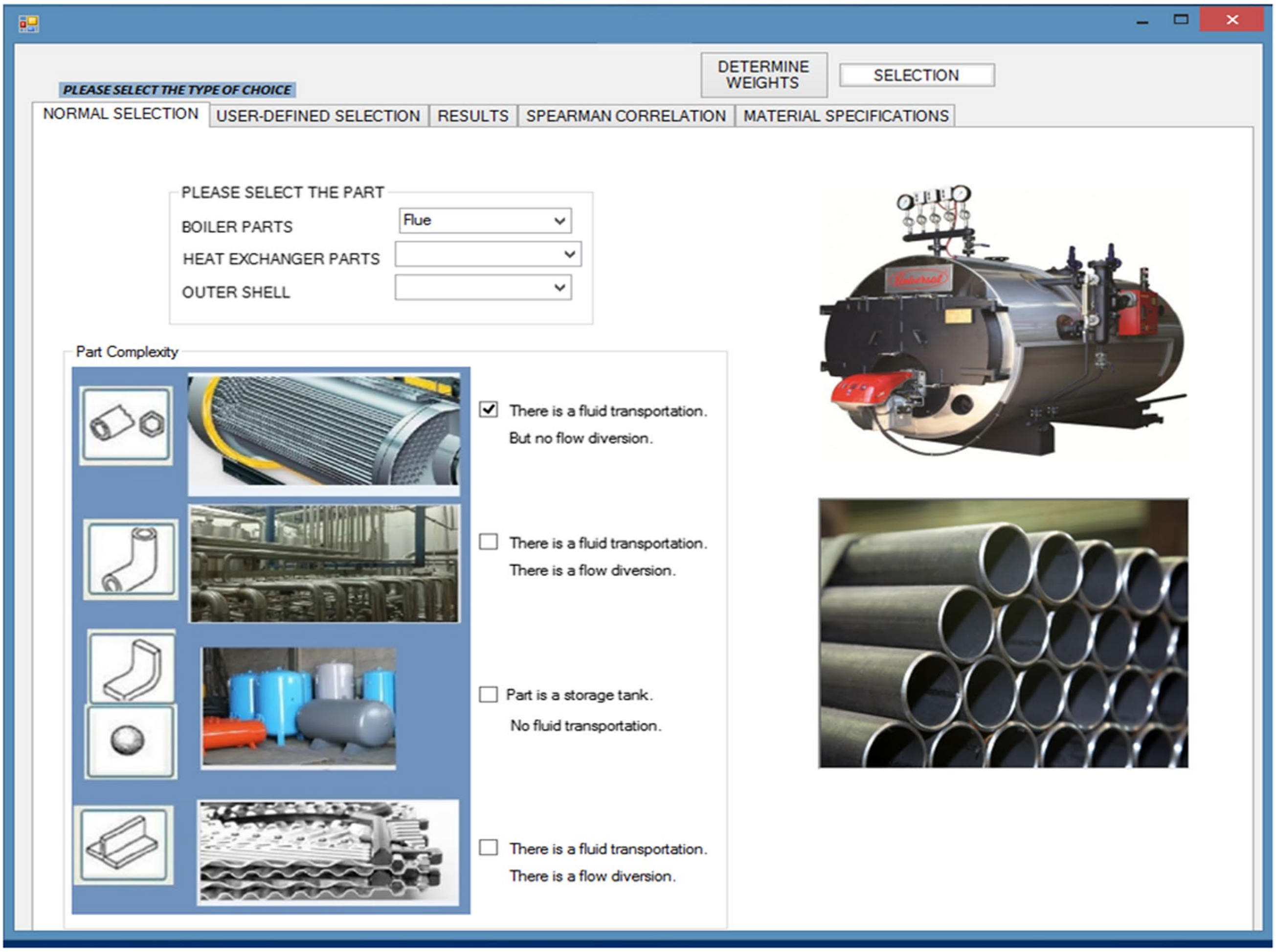


Fig. 2 Component type selection screen of PVSEL

particle swarm optimization based on the data collected from an experimental study. Four process parameters, namely extrusion temperature, density, layer thickness, and build orientation were optimized to achieve lower build time and higher compressive strength. Cheaitou and Khan [21] proposed multiple sourcing models for evaluating and ranking potential suppliers using an interactive MCDM methodology for manufacturing companies. Barajas and Agard [22] proposed an interactive model based on measurement of the relative indifference between characteristics using the Fuzzy Indifference Degree (FID) approach to select the most suitable product that is the closest to customer preferences. In another study, Mengoni et al. [23] proposed a knowledge-based workflow to dynamically manage human interactions in extended enterprises for research and development projects.

This study also presents an interactive decision model for pressure vessel component selection problem (Fig. 1). In the model, MCDM approaches and rules are used together in the decision structure to represent the complex behavior of the selection decision making process. The authors' literature survey showed that the best way to form material selection decision structure in multi-phase is to separate the selection divide the decision making structure into two separate phases. In a typical application of such a two-phase selection model, unsuitable materials are eliminated in the first phase leaving a feasible set of materials; and the feasible materials are then ranked in the second phase [15]. The material selection system developed in this paper, namely Pressure Vessel SELection (PVSEL), uses a similar two-phase approach in an interactive and flexible way to fulfill different needs of the users.

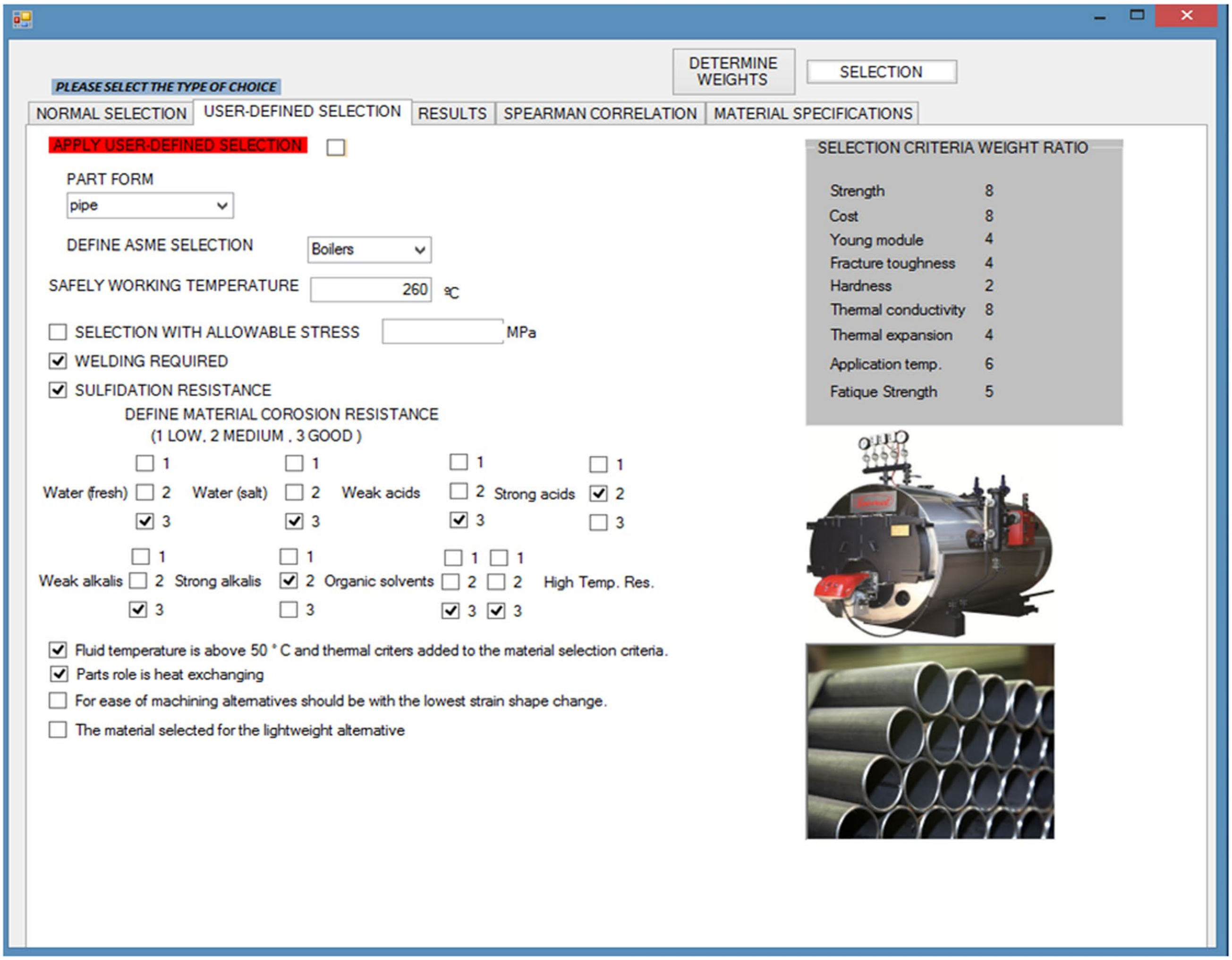
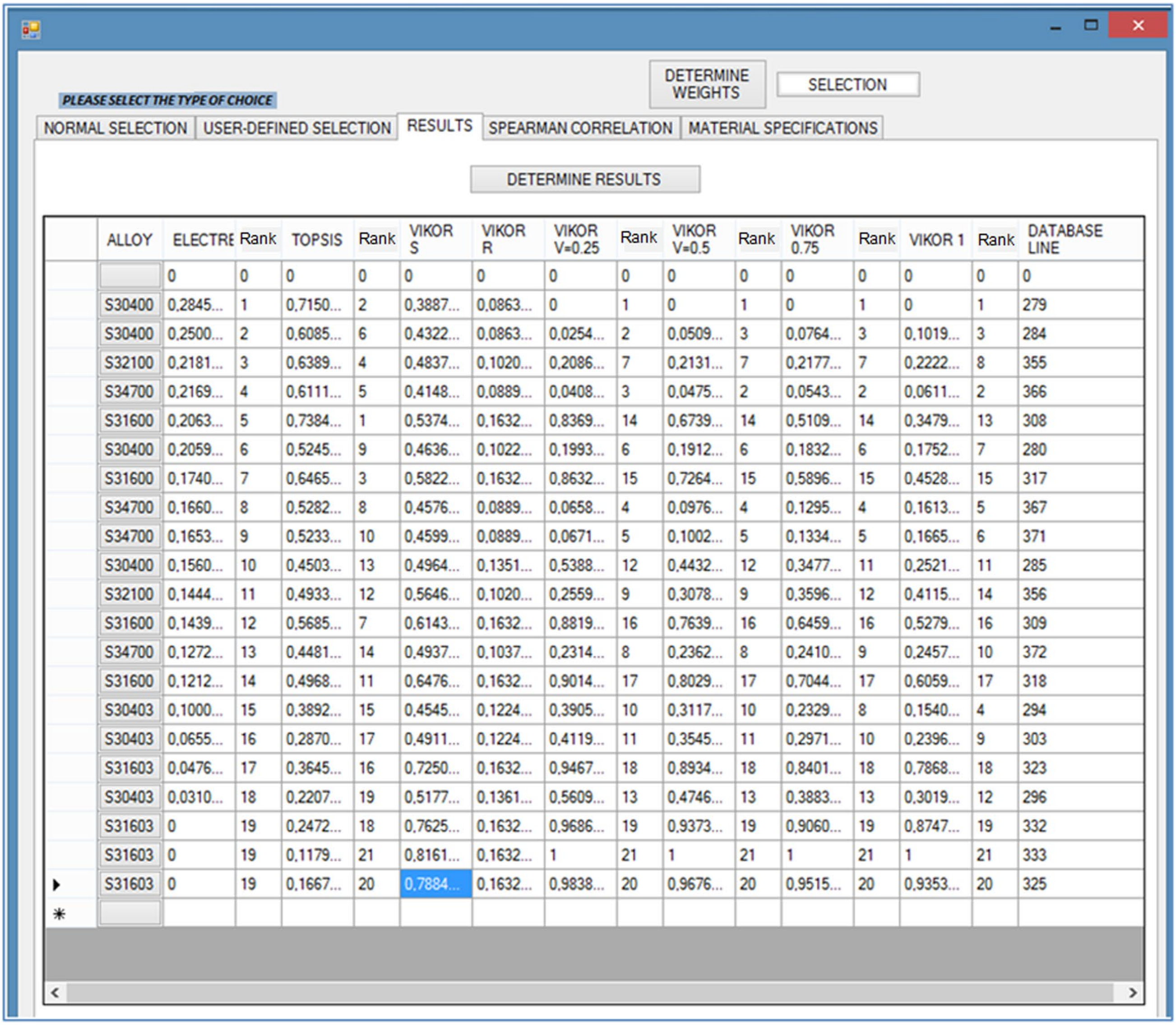


Fig. 3 User-selection screen of PVSEL

In the typical application of PVSEL, PVSEL scans the material database based on the requirements entered by the user and outputs a feasible set of materials in its first phase. PVSEL, then, ranks the feasible materials using ELECTRE, TOPSIS, and VIKOR methods in its second phase. The detailed explanations and application steps of ELECTRE, TOPSIS and VIKOR methods are presented in "Appendix A" [16–18]. Usage of PVSEL is provided through various examples in the following sections.

2 Description of PVSEL

In the operation of PVSEL, the window (Fig. 2), in which the component's type and shape complexity selections are made, comes to the screen. After entering the selections in the window, the user can continue directly by pushing 'SELECTION' or 'USER DEFINED SELECTION' buttons. When the user selects 'SELECTION' button, pre-specified values, which are recommended for regular usage of the selected component, are directly used without any user input. On the other hand, by selecting 'USER DEFINED SELECTION' button the pre-specified values are displayed in a new screen (Fig. 3). The user can alter the pre-specified values for the component on the screen, the selection criteria and their weights ranging from 1 to 10 also appear. The user can also change weights of the selection criteria. Once all necessary changes are made by the user, 'RESULTS' button is pushed. PVSEL performs the elimination phase, ranks the feasible ones using ELECTRE, TOPSIS and VIKOR methods and provide the results on the output screen as shown in Fig. 4. In the output screen, the feasible materials are listed



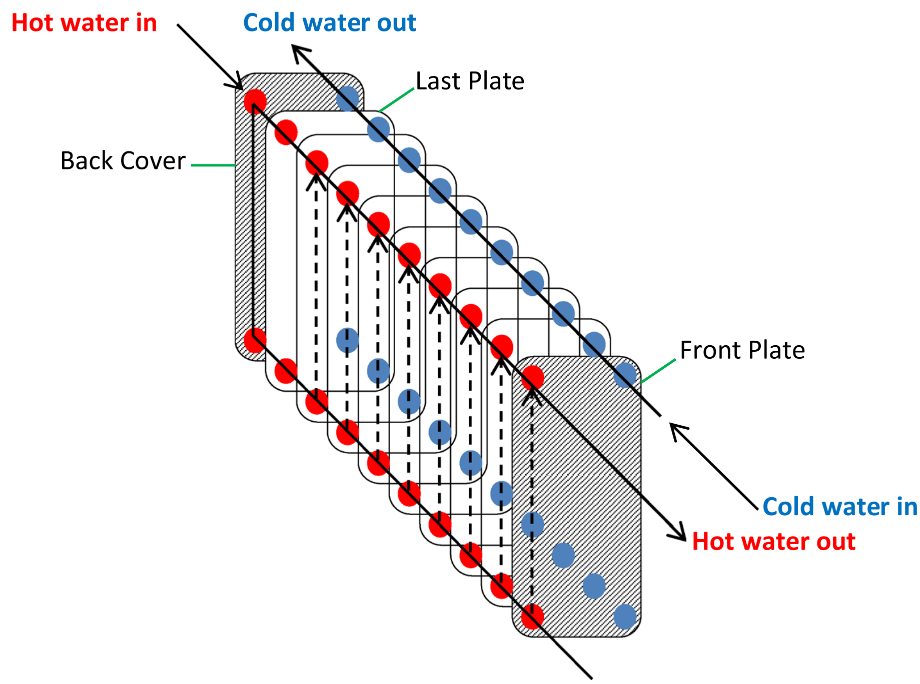
ALLOY	ELECTRE	Rank	TOPSIS	Rank	VIKOR S	VIKOR R	VIKOR V=0.25	Rank	VIKOR V=0.5	Rank	VIKOR 0.75	Rank	VIKOR 1	Rank	DATABASE LINE
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S30400	0,2845...	1	0,7150...	2	0,3887...	0,0863...	0	1	0	1	0	1	0	1	279
S30400	0,2500...	2	0,6085...	6	0,4322...	0,0863...	0,0254...	2	0,0509...	3	0,0764...	3	0,1019...	3	284
S32100	0,2181...	3	0,6389...	4	0,4837...	0,1020...	0,2086...	7	0,2131...	7	0,2177...	7	0,2222...	8	355
S34700	0,2169...	4	0,6111...	5	0,4148...	0,0889...	0,0408...	3	0,0475...	2	0,0543...	2	0,0611...	2	366
S31600	0,2063...	5	0,7384...	1	0,5374...	0,1632...	0,8369...	14	0,6739...	14	0,5109...	14	0,3479...	13	308
S30400	0,2059...	6	0,5245...	9	0,4636...	0,1022...	0,1993...	6	0,1912...	6	0,1832...	6	0,1752...	7	280
S31600	0,1740...	7	0,6465...	3	0,5822...	0,1632...	0,8632...	15	0,7264...	15	0,5896...	15	0,4528...	15	317
S34700	0,1660...	8	0,5282...	8	0,4576...	0,0889...	0,0658...	4	0,0976...	4	0,1295...	4	0,1613...	5	367
S34700	0,1653...	9	0,5233...	10	0,4599...	0,0889...	0,0671...	5	0,1002...	5	0,1334...	5	0,1665...	6	371
S30400	0,1560...	10	0,4503...	13	0,4964...	0,1351...	0,5388...	12	0,4432...	12	0,3477...	11	0,2521...	11	285
S32100	0,1444...	11	0,4933...	12	0,5646...	0,1020...	0,2559...	9	0,3078...	9	0,3596...	12	0,4115...	14	356
S31600	0,1439...	12	0,5685...	7	0,6143...	0,1632...	0,8819...	16	0,7639...	16	0,6459...	16	0,5279...	16	309
S34700	0,1272...	13	0,4481...	14	0,4937...	0,1037...	0,2314...	8	0,2362...	8	0,2410...	9	0,2457...	10	372
S31600	0,1212...	14	0,4968...	11	0,6476...	0,1632...	0,9014...	17	0,8029...	17	0,7044...	17	0,6059...	17	318
S30403	0,1000...	15	0,3892...	15	0,4545...	0,1224...	0,3905...	10	0,3117...	10	0,2329...	8	0,1540...	4	294
S30403	0,0655...	16	0,2870...	17	0,4911...	0,1224...	0,4119...	11	0,3545...	11	0,2971...	10	0,2396...	9	303
S31603	0,0476...	17	0,3645...	16	0,7250...	0,1632...	0,9467...	18	0,8934...	18	0,8401...	18	0,7868...	18	323
S30403	0,0310...	18	0,2207...	19	0,5177...	0,1361...	0,5609...	13	0,4746...	13	0,3883...	13	0,3019...	12	296
S31603	0	19	0,2472...	18	0,7625...	0,1632...	0,9686...	19	0,9373...	19	0,9060...	19	0,8747...	19	332
S31603	0	19	0,1179...	21	0,8161...	0,1632...	1	21	1	21	1	21	1	21	333
S31603	0	19	0,1667...	20	0,7884...	0,1632...	0,9838...	20	0,9676...	20	0,9515...	20	0,9353...	20	325

Fig. 4 An example of output screen of the PVSEL

according to their ranking scores. Each method provides different ranking results. For each method, the highest ranked feasible material is listed in the first row and its ranking value is considered as '1'. The other feasible materials' ranking scores are simply their row numbers. To obtain a single material recommendation, the rankings of feasible materials obtained with different methods can be summed to obtain a single overall ranking value. The material which has the lowest overall ranking sum is recommended by PVSEL.

The reason for using different ranking methods is their versatility. For example, ELECTRE method is based on multi-attribute utility theory [16]. ELECTRE methodology is described as "…it is a procedure that sequentially reduces the number of alternatives the decision maker is faced within a set of non-dominated alternatives. The objective of this outranking method is to find all the alternatives that dominate other alternatives while they themselves cannot be dominated by any other alternative. The ELECTRE method is used to develop a partial ranking and choose a set of the promising alternatives…". On the other hand, TOPSIS method is based on ranking functions of the distances from negative-ideal (worst) solution and ideal solution for alternative solutions and makes its ranking based on the 'closeness to the ideal solution' and 'farthest from the negative-ideal

Fig. 5 The thermal transfer plate



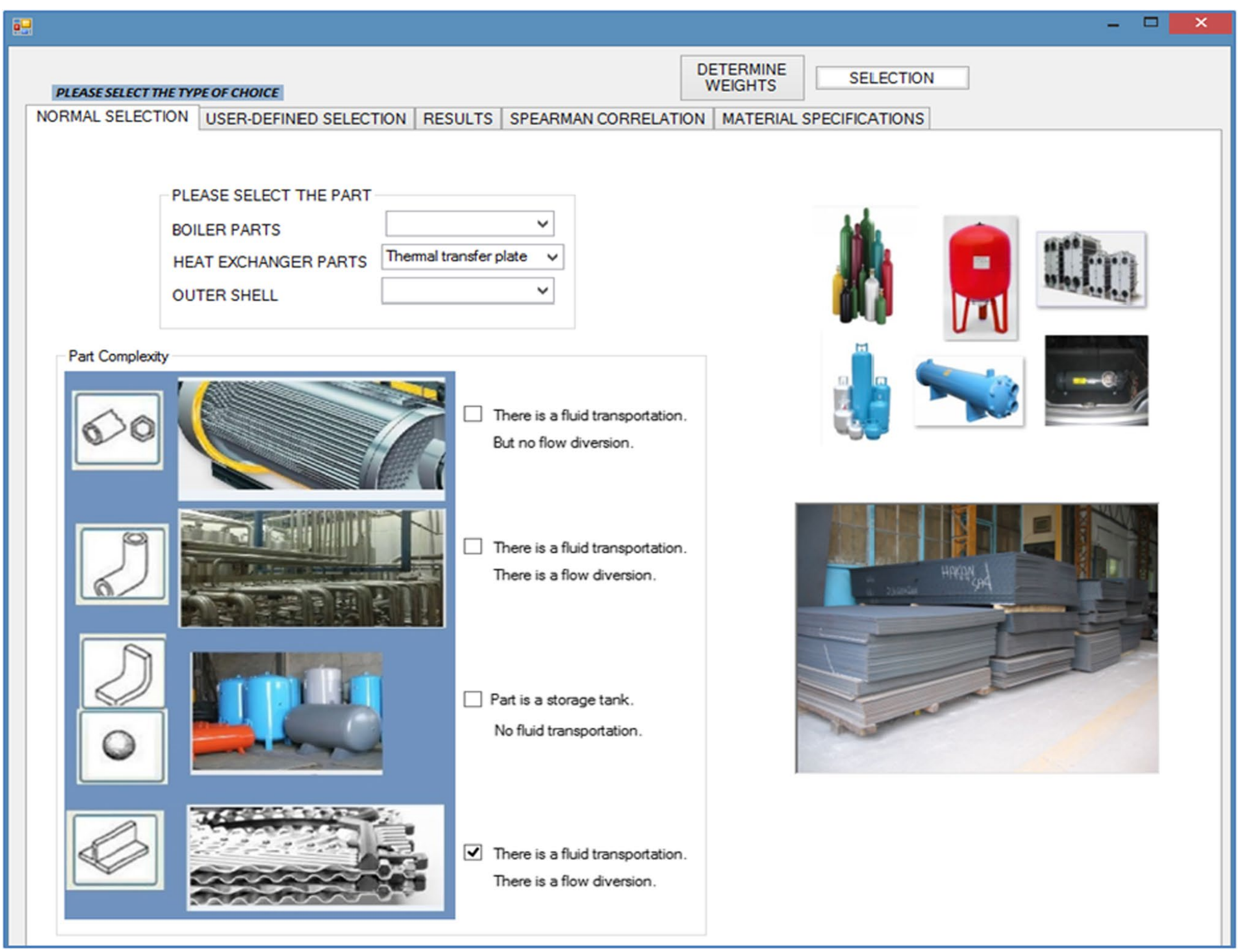


Fig. 6 Component type selection screen for the illustrative example

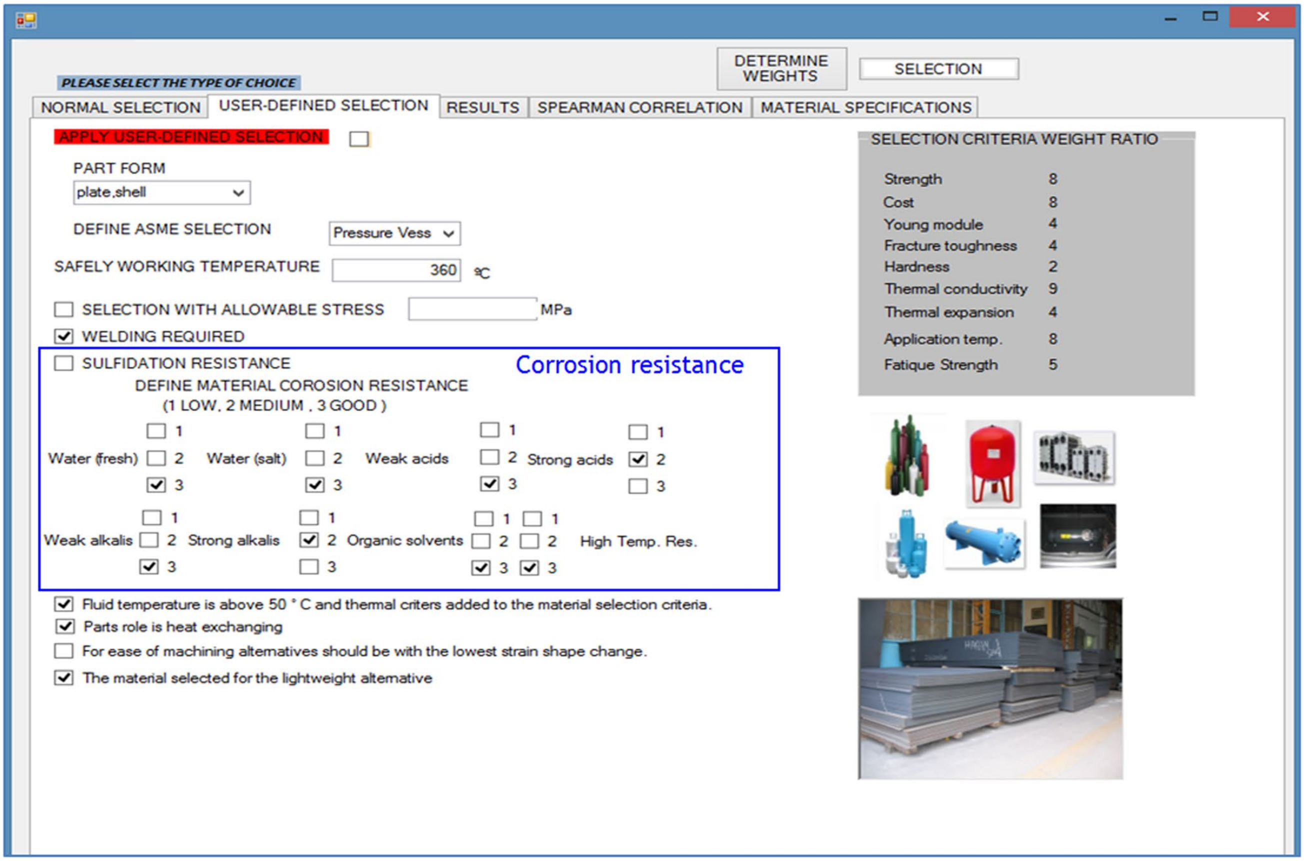


Fig. 7 User selection screen for the illustrative example

solution' [24]. Compromise ranking for the VIKOR method determines "…a compromise solution, providing a maximum 'group utility' for the 'majority' and a minimum of an individual regret for the 'opponent'. VIKOR method uses linear normalization and TOPSIS method uses vector normalization to eliminate the units of criterion functions.

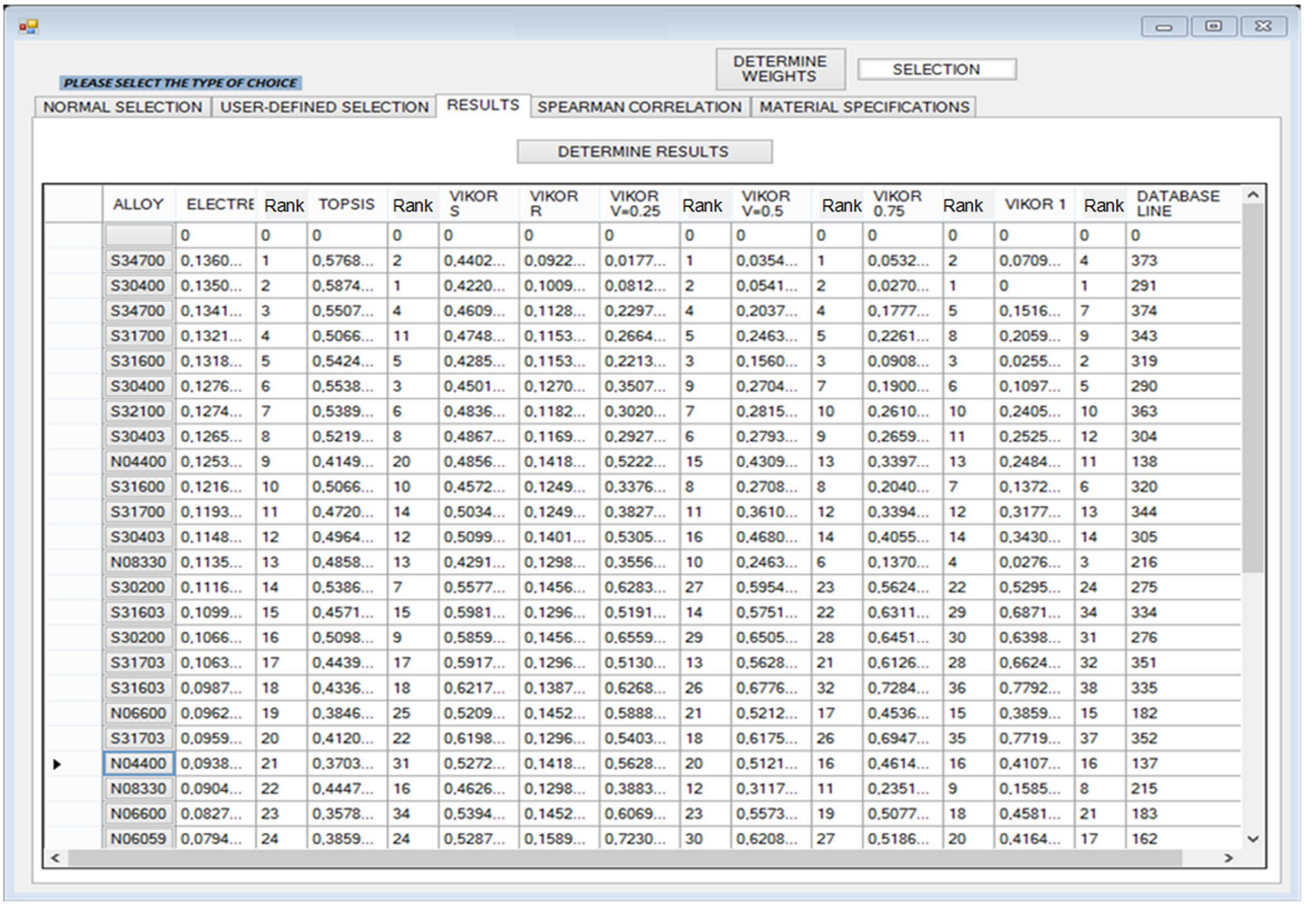
It can be summarized that ranking methods differ in their application procedure in terms of the type of data normalization and the way of combining the data and the attribute weights into the methodology such that they yield different ranking results [24–26].

On the PVSEL output screen, if 'SPEARMAN CORRELATION' button is pushed, PVSEL compares the ranking results of the ELECTRE, TOPSIS and VIKOR methods using Spearman's rank correlation test and provides the comparison results on a new screen. Furthermore, the user can see the ranked materials' values by pushing 'MATERIAL SPECIFICATIONS' button.

3 Testing PVSEL and Illustration of Critical Issues in its Usage

Among many case studies developed to verify PVSEL in the selection of pressure vessel components, thermal transfer (heat exchanger) plate material (Fig. 5) selection problem is provided below to illustrate the application of PVSEL. In this example, the designer provides the following requirements for the material based on its working environment:

1. Working pressure requirement is 70 psi.
2. Working temperature range is between − 195 °C and + 360 °C.
3. Resistance to organic solvent and water corrosion should be high.
4. Welding will be used in the construction of the component.
5. It is desirable to use light weight material for easy handling of the component.
6. The material should have high formability.



ALLOY	ELECTRE	Rank	TOPSIS	Rank	VIKOR S	VIKOR R	VIKOR V=0.25	Rank	VIKOR V=0.5	Rank	VIKOR 0.75	Rank	VIKOR 1	Rank	DATABASE LINE
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S34700	0,1360...	1	0,5768...	2	0,4402...	0,0922...	0,0177...	1	0,0354...	1	0,0532...	2	0,0709...	4	373
S30400	0,1350...	2	0,5874...	1	0,4220...	0,1009...	0,0812...	2	0,0541...	2	0,0270...	1	0	1	291
S34700	0,1341...	3	0,5507...	4	0,4609...	0,1128...	0,2297...	4	0,2037...	4	0,1777...	5	0,1516...	7	374
S31700	0,1321...	4	0,5066...	11	0,4748...	0,1153...	0,2664...	5	0,2463...	5	0,2261...	8	0,2059...	9	343
S31600	0,1318...	5	0,5424...	5	0,4285...	0,1153...	0,2213...	3	0,1560...	3	0,0908...	3	0,0255...	2	319
S30400	0,1276...	6	0,5538...	3	0,4501...	0,1270...	0,3507...	9	0,2704...	7	0,1900...	6	0,1097...	5	290
S32100	0,1274...	7	0,5389...	6	0,4836...	0,1182...	0,3020...	7	0,2815...	10	0,2610...	10	0,2405...	10	363
S30403	0,1265...	8	0,5219...	8	0,4867...	0,1169...	0,2927...	6	0,2793...	9	0,2659...	11	0,2525...	12	304
N04400	0,1253...	9	0,4149...	20	0,4856...	0,1418...	0,5222...	15	0,4309...	13	0,3397...	13	0,2484...	11	138
S31600	0,1216...	10	0,5066...	10	0,4572...	0,1249...	0,3376...	8	0,2708...	8	0,2040...	7	0,1372...	6	320
S31700	0,1193...	11	0,4720...	14	0,5034...	0,1249...	0,3827...	11	0,3610...	12	0,3394...	12	0,3177...	13	344
S30403	0,1148...	12	0,4964...	12	0,5099...	0,1401...	0,5305...	16	0,4680...	14	0,4055...	14	0,3430...	14	305
N08330	0,1135...	13	0,4858...	13	0,4291...	0,1298...	0,3556...	10	0,2463...	6	0,1370...	4	0,0276...	3	216
S30200	0,1116...	14	0,5386...	7	0,5577...	0,1456...	0,6283...	27	0,5954...	23	0,5624...	22	0,5295...	24	275
S31603	0,1099...	15	0,4571...	15	0,5981...	0,1296...	0,5191...	14	0,5751...	22	0,6311...	29	0,6871...	34	334
S30200	0,1066...	16	0,5098...	9	0,5859...	0,1456...	0,6559...	29	0,6505...	28	0,6451...	30	0,6398...	31	276
S31703	0,1063...	17	0,4439...	17	0,5917...	0,1296...	0,5130...	13	0,5628...	21	0,6126...	28	0,6624...	32	351
S31603	0,0987...	18	0,4336...	18	0,6217...	0,1387...	0,6268...	26	0,6776...	32	0,7284...	36	0,7792...	38	335
N06600	0,0962...	19	0,3846...	25	0,5209...	0,1452...	0,5888...	21	0,5212...	17	0,4536...	15	0,3859...	15	182
S31703	0,0959...	20	0,4120...	22	0,6198...	0,1296...	0,5403...	18	0,6175...	26	0,6947...	35	0,7719...	37	352
N04400	0,0938...	21	0,3703...	31	0,5272...	0,1418...	0,5628...	20	0,5121...	16	0,4614...	16	0,4107...	16	137
N08330	0,0904...	22	0,4447...	16	0,4626...	0,1298...	0,3883...	12	0,3117...	11	0,2351...	9	0,1585...	8	215
N06600	0,0827...	23	0,3578...	34	0,5394...	0,1452...	0,6069...	23	0,5573...	19	0,5077...	18	0,4581...	21	183
N06059	0,0794...	24	0,3859...	24	0,5287...	0,1589...	0,7230...	30	0,6208...	27	0,5186...	20	0,4164...	17	162

Fig. 8 Output screen of the MADM module for the illustrative example

Based on the requirements provided by the designer, the user enters necessary input data as shown in Figs. 6 and 7.

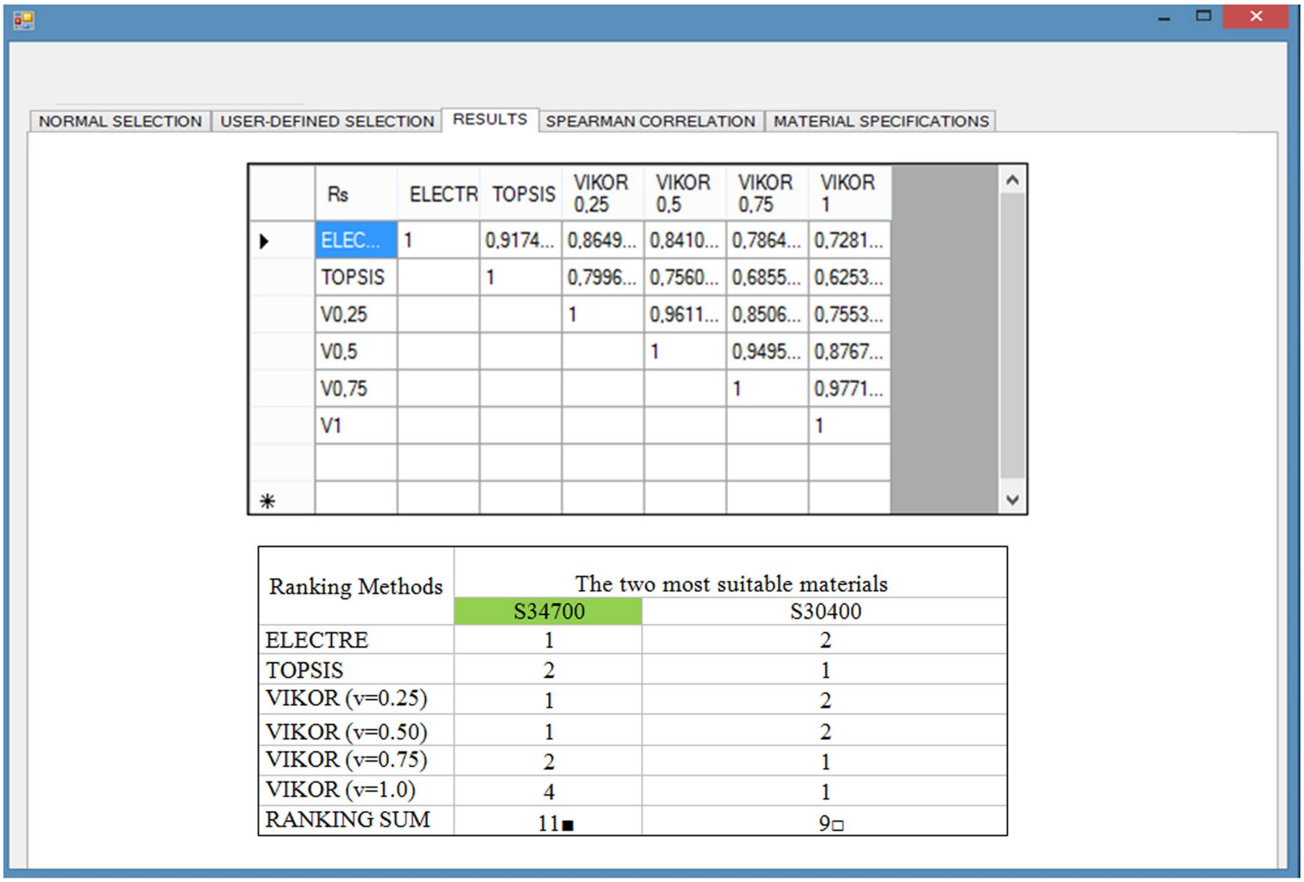
The output screen, which provides rankings of the feasible materials, is presented in Fig. 8.

The ranking results of the ELECTRE, TOPSIS and VIKOR methods can also be compared statistically by PVSEL using Spearman's rank correlation test. The outputs of the steps of the methods are provided in "Appendix B".

The comparison of the ranking results is presented in Fig. 9. The outcome of the spearman rank correlation test, the correlation coefficient values of the differences in the rankings (R_s) [27] are provided in the first matrix of Fig. 9. Since all R_s values are higher than 0.6, it can be stated that the ranking provided by ELECTRE, TOPSIS and VIKOR methods are statistically similar to each other [27–32].

Although the spearman rank correlation test shows that the ranking results given in Fig. 8 are statistically similar, there are differences in the ranking results obtained with the three methods. For example, there is no obvious selection for the most appropriate material. The ranking results in Fig. 9 show that S34700 and S30400 materials are determined as the most appropriate alternative material three times each. The ranking results may be combined to obtain a single ranking list by adding the rankings obtained with the applications of the three methods. The alternative with the lowest ranking sum can be recommended as the most appropriate material for the component. S30400 material whose ranking sum is the lowest with 9 is recommended as the most appropriate material for the pressure vessel component selection problem.

Two other examples for nitrogen tube and membrane tank material selection problems are also provided in "Appendix C" for illustrative purposes. The ranking results show that S31600 is the most appropriate material in both examples.



Rs	ELECTR	TOPSIS	VIKOR 0,25	VIKOR 0,5	VIKOR 0,75	VIKOR 1
ELEC...	1	0,9174...	0,8649...	0,8410...	0,7864...	0,7281...
TOPSIS		1	0,7996...	0,7560...	0,6855...	0,6253...
V0,25			1	0,9611...	0,8506...	0,7553...
V0,5				1	0,9495...	0,8767...
V0,75					1	0,9771...
V1						1

Ranking Methods	The two most suitable materials	
	S34700	S30400
ELECTRE	1	2
TOPSIS	2	1
VIKOR (v=0.25)	1	2
VIKOR (v=0.50)	1	2
VIKOR (v=0.75)	2	1
VIKOR (v=1.0)	4	1
RANKING SUM	11■	9□

Fig. 9 Spearman's rank correlation test output screen for the illustrative example

4 Conclusion

A knowledge based interactive decision support system (PVSEL) is developed in this paper to select the most appropriate material for a described pressure vessel component. The developed PVSEL uses different ranking approaches namely ELECTRE, VIKOR and TOPSIS along with an elimination phase. Application of PVSEL brings uniform structure and to the complex pressure vessel component material selection problem. In addition, PVSEL can easily be modified by adding new components, material types, requirements, criteria and eliminating or modifying existing ones.

The study provided in this paper has various limitations that may require further research. For example, using fuzzy logic techniques, such as hesitant or intuitionistic fuzzy approaches recommended for modeling of uncertainties and linguistic preferences, in the TOPSIS, VIKOR, and ELECTRE applications may provide more realistic ranking results.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Appendix A

(a) TOPSIS method [26, 30, 31]

Step 1 Development of the decision matrix (y_{ij}; $i=1,2,\ldots$,number of alternatives (m), $j=1,2,\ldots$,number of criteria (n) are placed in matrix form as shown in Eq. (1)).

$$D = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \cdots & & & \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{bmatrix} \quad (1)$$

Step 2 The normalized decision matrix is constructed using Eq. (2).

$$y_{ij}^{*} = \frac{y_{ij}}{\sqrt{\sum_{i=1}^{m} y_{ij}^{2}}} \quad \mathrm{i} = 1, 2, \ldots, \mathrm{m}, \; \mathrm{j} = 1, 2, \ldots \mathrm{n}. \quad (2)$$

Step 3 The weighted normalized decision matrix is obtained:

$$V = \left[X_{ij}\right]_{mxr} \quad \mathrm{i} = 1, 2, \ldots, \mathrm{m}, \; \mathrm{j} = 1, 2, \ldots \mathrm{n}. \quad (3)$$

$$X_{ij} = y_{ij}^{*} w_j \quad (4)$$

$$W = \left[w_1, w_2, \ldots, w_n\right] \quad (5)$$

Step 4 Determine the ideal and negative-ideal solutions:

$$A^{*} = \left(X_1^{*}, X_2^{*}, X_3^{*}\right) \quad (6)$$

$$X_j^{*} = \left\{ \left(\max_i \quad X_{ij} | j \in J \right) i = 1, \ldots m \right\} \quad (7)$$

$$A^{-} = \left(X_1^{-}, X_2^{-}, X_3^{-}\right) \quad (8)$$

$$X_j^{-} = \left\{ \left(\min_i \quad X_{ij} | j \in J \right) i = 1, \ldots m \right\} \quad (9)$$

Step 5 The distance of an alternative *i* to the ideal solution (S_i^{*}), and from the negative ideal solution (S_i^{-}) are calculated using Eqs. (10) and (11).

$$S_i^{*} = \sqrt{\sum_{j=1}^{n} (X_{ij} - X_j^{*})^2} \quad \mathrm{i} = 1, 2, \ldots, \mathrm{m}, \; \mathrm{j} = 1, 2, \ldots \mathrm{n} \quad (10)$$

$$S_i^{-} = \sqrt{\sum_{j=1}^{n} (X_{ij} - X_j^{-})^2} \quad \mathrm{i} = 1, 2, \ldots, \mathrm{m}, \; \mathrm{j} = 1, 2, \ldots \mathrm{n} \quad (11)$$

Step 6 The ranking score (C_i^{*}) is calculated using Eq. (12).

$$C_i^{*} = S_i^{-} / (S_i^{-} + S_i^{*}), \quad \mathrm{i} = 1, 2, \ldots, \mathrm{m}, \; \mathrm{j} = 1, 2, \ldots \mathrm{n} \quad (12)$$

(b) VIKOR Method [24, 26]

VIKOR method focuses on proposing a compromise solution. The compromise ranking is developed from the L_P-metric used in the compromise programming method.

$$L_{p,i} = L_{1,i} = \left\{ \sum_{j=1}^{n} \left(w_j \left[(y_{ij})\mathrm{max} - y_{ij} \right] / \left[(y_{ij})\mathrm{max} - (y_{ij})\mathrm{min} \right] \right)^p \right\}^{1/p}$$
$$1 \le p \le \infty; \quad i = 1, 2, \ldots, m. \quad (13)$$

where *n* is the number of criteria and *m* is the number of alternatives. The y_{ij} values (for i = 1,2,…,m; j = 1,2,…,n) denote the values of criteria for different alternatives. In the VIKOR method, $L_{1,i}$ and $L_{\infty,i}$ are used to formulate the ranking measures.

The VIKOR values (P_i) are computed in the following steps:

Step 1 Development of the decision matrix, determine the best, $(y_{ij})_{max}$ and the worst, $(y_{ij})_{min}$ values for all the criteria.

Step 2 Calculate E_i and F_i values

$$E_i = L_{1,i} = \sum_{j=1}^{n} w_j \left[(y_{ij})\mathrm{max} - y_{ij} \right] / \left[(y_{ij})\mathrm{max} - (y_{ij})\mathrm{min} \right] \quad (14)$$

$$F_i = L_{\infty,i} = \overset{y}{\mathrm{max}}\, of \left\{ w_j \left[(y_{ij})\mathrm{max} - y_{ij} \right] / \left[(y_{ij})\mathrm{max} - (y_{ij})\mathrm{min} \right] \right\}$$
$$\mathrm{j} = 1, 2, \ldots, n \quad (15)$$

where w_j is the weight for the j^{th} response, assigned by the decision maker, and

$$w = \sum_{j=1}^{n} w_j = 1 \quad (16)$$

Equation (14) is only applicable to benefit criteria. In the classical application of VIKOR method for cost criteria, the term $[(y_{ij})_{max} - y_{ij}]$ in Eq. (14), is to be replaced by $[y_{ij} - (y_{ij})_{min}]$. For cost criteria, Eq. (14) can be rewritten as;

$$E_i = L_{1,i} = \sum_{j=1}^{n} w_j \left[(y_{ij}) - (y_{ij})\mathrm{min} \right] / \left[(y_{ij})\mathrm{max} - (y_{ij})\mathrm{min} \right] \quad (17)$$

Step 3 Calculate P_i values as follows

$$P_i = v[(E_i) - (E_i)\mathrm{min}]/[(E_i)\mathrm{max} - (E_i)\mathrm{min}]$$
$$+ (1 - v)[(F_i) - (F_i)\mathrm{min}]/[(F_i)\mathrm{max} - (F_i)\mathrm{min}] \quad (18)$$

where $(E_i)_{max}$ and $(E_i)_{min}$ are the maximum and minimum values of E_i, respectively, and $(F_i)_{max}$ and $(F_i)_{min}$ are the maximum and minimum values of F_i, respectively. The value of v changes from 0 to 1. The compromise can be selected with 'voting by majority' ($v > 0.5$), with 'consensus' ($v \approx 0.5$) or with 'veto' ($v < 0.5$).

Step 4 Rank the alternatives in ascending order, according to the P_i values. The best alternative is the one having the minimum P_i value.

(c) ELECTRE (The ELimination and Et Choice Translating REality) Method) [32]

Step 1 Development of the decision matrix (y_{ij}; $i = 1, 2, \ldots$, number of alternatives (m), $j = 1, 2, \ldots$, number of criteria (n) are placed in matrix form as shown in Eq. (1)).

Step 2 The weighted normalized decision (WNDM) matrix is obtained as follows:

$$V_{ij} = \begin{bmatrix} w_1y_{11} & w_2y_{12} & \cdots & w_ny_{1n} \\ w_1y_{21} & w_2y_{22} & \cdots & w_ny_{2n} \\ . & & & . \\ . & & & . \\ . & & & . \\ w_1y_{m1} & w_2y_{m2} & \cdots & w_ny_{mn} \end{bmatrix} \quad w = \sum_{j=1}^{n} w_j = 1 \tag{19}$$

Step 3 Evaluate randomly different row k and row l in the WNDM V to make sure of the concordance and discordance set. If value v of row k is higher than value v of row l, the element j can be clustered as the concordance set C_{kl}, or the discordance set D_{kl}. The concordance set C_{kl}, or the discordance set D_{kl} is shown as Eqs. (20) and (21).

$$C_{kl} = \left\{ j, v_{kj} \geq v_{lj} \right\} \tag{20}$$

$$D_{kl} = \left\{ j, v_{kj} < v_{lj} \right\} \tag{21}$$

Step 4 The sum of each element's weight forms a concordance matrix C, as shown in (22).

$$c_{kl} = \frac{\sum\limits_{j \in C_{kl}} w_j}{\sum\limits_{k \in 1}^{n} w_j}, \text{and } C = \begin{bmatrix} - & c_{12} & c_{13} & \cdots & c_{1m} \\ c_{21} & - & c_{23} & \cdots & c_{2m} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ c_{m1} & c_{m2} & c_{m3} & \cdots & - \end{bmatrix} \tag{22}$$

Step 5 We use a formula to get the discordance matrix as shown in (23).

$$d_{kl} = \frac{\max\limits_{j \in D_{kl}} \left| v_{kj} - v_{lj} \right|}{\max\limits_{j} \left| v_{kj} - v_{lj} \right|} \tag{23}$$

Therefore, a discordance matrix can be presented as $D = [d_{kl}]_{m \times m}$.

$$D = \begin{bmatrix} - & d_{12} & d_{13} & \cdots & d_{1m} \\ d_{21} & - & d_{23} & \cdots & d_{2m} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ d_{m1} & d_{m2} & d_{m3} & \cdots & - \end{bmatrix} \tag{24}$$

Step 6 To indicate the large element value of the alternative solution, when the expected value is larger, we combine each element C_{kl} of the concordance set with the discordance matrix to get the modified global matrix A as shown in (25).

$$A = [a_{kl}]_{m \times m}, a_{kl} = c_{kl} \circ d'_{kl}. \tag{25}$$

Step 7 Determination of the modified superiority matrix:

$$A_k = \max\{a_{kl} | k = 1, 2, \ldots, m\}, \quad l = 1, 2, \ldots, m. \tag{26}$$

To obtain an appropriate solution, we have to rank a_k from smallest to largest: $a1, a2, \ldots, a_m$. The threshold $\bar{a}$ is set behind the smallest value a'_1. If the value a_{kl} is smaller than threshold $\bar{a}$, it is replaced as 0, or 1:

$$E = \left[e'_{kl} \right], \begin{cases} e'_{kl} = 1, a_{kl} \geq \bar{a} \\ e'_{kl} = 0, a_{kl} < \bar{a} \end{cases} \tag{27}$$

Step 8 We obtain $e'_{kl} = 1$ from the matrix E. It indicates that solution k is better than solution l.

Appendix B

(a) Decision Matrix

Alternatives		Strength (MPa)	Cost (USD/kg)	Young module (GPa)	Fracture toughness (MPa)	Hardness (HV)	Thermal conductivity (W/mK)	Thermal expansion (μstrain/°C)	Application Temperature (°C)	Fatigue strength (MPa)
1	N04400	11.543	0.0397	170	120	300	20	0.0826	482	230
2	N04400	16.914	0.0397	170	120	300	20	0.0826	482	230
3	N06022	20.857	0.027	206	120	176	9.3	0.0806	677	280
4	N06022	16.454	0.027	206	120	176	9.3	0.0806	677	280
5	N06059	17.442	0.028	205	120	196	9.98	0.0855	760	220
6	N06059	21.047	0.028	205	120	196	9.98	0.0855	760	220
7	N06455	16.279	0.0271	207	120	170	9	0.1053	427	280
8	N06455	21.395	0.0271	207	120	170	9	0.1053	427	280
9	N06600	18.877	0.0345	207	120	135	14.7	0.0806	649	220
10	N06600	16.487	0.0345	207	120	135	14.7	0.0806	649	220
11	N06625	24.405	0.0217	205	120	145	9.9	0.0781	649	145
12	N06625	26.667	0.0217	205	120	145	9.9	0.0781	649	145
13	N06690	15.718	0.037	195	120	175	13	0.0909	454	250
14	N06690	19.554	0.037	195	120	175	13	0.0909	454	250
15	N08020	15.223	0.1142	192	139	170	11.7	0.0699	427	269
16	N08020	18.812	0.1142	192	139	170	11.7	0.0699	427	269
17	N08330	12.38	0.1264	192	126	130	12	0.0769	899	256
18	N08330	16.709	0.1264	192	126	130	12	0.0769	899	256
19	N08800	17.58	0.0599	190	120	160	11	0.0741	816	200
20	N08800	14.395	0.0599	190	120	160	11	0.0741	816	200
21	N10276	20.114	0.026	200	120	160	9.5	0.1	677	280
22	N10276	14.886	0.026	200	120	160	9.5	0.1	677	280
23	R60702	6.7969	0.036	95	120	140	21	0.1786	371	120
24	S30200	13.956	0.2519	189	60	170	15	0.0625	399	210
25	S30200	10.307	0.2519	189	60	170	15	0.0625	399	210
26	S30400	10.255	0.2506	190	80	170	14	0.0625	816	229
27	S30400	13.885	0.2506	190	80	170	14	0.0625	816	229
28	S30403	11.561	0.2198	200	54	210	15	0.0556	649	262
29	S30403	8.5643	0.2198	200	54	210	15	0.0556	649	262
30	S31600	14.231	0.1988	189	118	190	13	0.0667	816	228
31	S31600	10.534	0.1988	189	118	190	13	0.0667	816	228
32	S31603	11.792	0.1988	190	53	170	13	0.0667	454	256

Alternatives		Strength (MPa)	Cost (USD/kg)	Young module (GPa)	Fracture toughness (MPa)	Hardness (HV)	Thermal conductivity (W/mK)	Thermal expansion (μstrain/°C)	Application Temperature (°C)	Fatigue strength (MPa)
33	S31603	8.7421	0.1988	190	53	170	13	0.0667	454	256
34	S31700	14.231	0.1776	189	64	190	13	0.0667	816	255
35	S31700	10.534	0.1776	189	64	190	13	0.0667	816	255
36	S31703	14.104	0.1733	196	55	160	13	0.0667	454	259
37	S31703	10.483	0.1733	196	55	160	13	0.0667	454	259
38	S32100	11.389	0.2299	189	53	170	14	0.0625	816	223
39	S34700	14.758	0.2247	189	53	185	15	0.0625	816	228
40	S34700	12.087	0.2247	189	53	185	15	0.0625	816	228

(b) Normalized decision matrix

	Strength (MPa)	Cost (USD/kg)	Young module (GPa)	Fracture toughness (MPa)	Hardness (HV)	Thermal conductivity (W/mK)	Thermal expansion (μstrain/°C)	Application temperature (°C)	Fatigue strength (MPa)
1	0.117	0.043	0.139	0.184	0.265	0.239	0.162	0.116	0.152
2	0.171	0.043	0.139	0.184	0.265	0.239	0.162	0.116	0.152
3	0.211	0.029	0.169	0.184	0.155	0.111	0.159	0.163	0.185
4	0.166	0.029	0.169	0.184	0.155	0.111	0.159	0.163	0.185
5	0.176	0.03	0.168	0.184	0.173	0.119	0.168	0.182	0.145
6	0.212	0.03	0.168	0.184	0.173	0.119	0.168	0.182	0.145
7	0.164	0.029	0.17	0.184	0.15	0.107	0.207	0.103	0.185
8	0.216	0.029	0.17	0.184	0.15	0.107	0.207	0.103	0.185
9	0.191	0.037	0.17	0.184	0.119	0.176	0.159	0.156	0.145
10	0.166	0.037	0.17	0.184	0.119	0.176	0.159	0.156	0.145
11	0.246	0.023	0.168	0.184	0.128	0.118	0.154	0.156	0.096
12	0.269	0.023	0.168	0.184	0.128	0.118	0.154	0.156	0.096
13	0.159	0.04	0.16	0.184	0.154	0.155	0.179	0.109	0.165
14	0.197	0.04	0.16	0.184	0.154	0.155	0.179	0.109	0.165
15	0.154	0.123	0.157	0.213	0.15	0.14	0.137	0.103	0.178
16	0.19	0.123	0.157	0.213	0.15	0.14	0.137	0.103	0.178
17	0.125	0.136	0.157	0.193	0.115	0.143	0.151	0.216	0.169
18	0.169	0.136	0.157	0.193	0.115	0.143	0.151	0.216	0.169
19	0.177	0.064	0.156	0.184	0.141	0.131	0.146	0.196	0.132
20	0.145	0.064	0.156	0.184	0.141	0.131	0.146	0.196	0.132
21	0.203	0.028	0.164	0.184	0.141	0.113	0.197	0.163	0.185

	Strength (MPa)	Cost (USD/kg)	Young module (GPa)	Fracture toughness (MPa)	Hardness (HV)	Thermal conductivity (W/mK)	Thermal expansion (μstrain/°C)	Application temperature (°C)	Fatigue strength (MPa)
22	0.15	0.028	0.164	0.184	0.141	0.113	0.197	0.163	0.185
23	0.069	0.039	0.078	0.184	0.124	0.251	0.351	0.089	0.079
24	0.141	0.271	0.155	0.092	0.15	0.179	0.123	0.096	0.139
25	0.104	0.271	0.155	0.092	0.15	0.179	0.123	0.096	0.139
26	0.103	0.269	0.156	0.123	0.15	0.167	0.123	0.196	0.151
27	0.14	0.269	0.156	0.123	0.15	0.167	0.123	0.196	0.151
28	0.117	0.236	0.164	0.083	0.185	0.179	0.109	0.156	0.173
29	0.086	0.236	0.164	0.083	0.185	0.179	0.109	0.156	0.173
30	0.144	0.214	0.155	0.181	0.168	0.155	0.131	0.196	0.151
31	0.106	0.214	0.155	0.181	0.168	0.155	0.131	0.196	0.151
32	0.119	0.214	0.156	0.081	0.15	0.155	0.131	0.109	0.169
33	0.088	0.214	0.156	0.081	0.15	0.155	0.131	0.109	0.169
34	0.144	0.191	0.155	0.098	0.168	0.155	0.131	0.196	0.168
35	0.106	0.191	0.155	0.098	0.168	0.155	0.131	0.196	0.168
36	0.142	0.186	0.161	0.084	0.141	0.155	0.131	0.109	0.171
37	0.106	0.186	0.161	0.084	0.141	0.155	0.131	0.109	0.171
38	0.115	0.247	0.155	0.081	0.15	0.167	0.123	0.196	0.147
39	0.149	0.242	0.155	0.081	0.163	0.179	0.123	0.196	0.151
40	0.122	0.242	0.155	0.081	0.163	0.179	0.123	0.196	0.151

(c) Weighted normalized decision matrix

	Strength (MPa)	Cost (USD/kg)	Young module (GPa)	Fracture toughness (MPa)	Hardness (HV)	Thermal conductivity (W/mK)	Thermal expansion (μstrain/°C)	Application temperature (°C)	Fatigue strength (MPa)
1	0.0179	0.0066	0.0107	0.0141	0.0102	0.0413	0.0125	0.0178	0.0146
2	0.0263	0.0066	0.0107	0.0141	0.0102	0.0413	0.0125	0.0178	0.0146
3	0.0324	0.0045	0.013	0.0141	0.006	0.0192	0.0122	0.025	0.0178
4	0.0255	0.0045	0.013	0.0141	0.006	0.0192	0.0122	0.025	0.0178
5	0.0271	0.0046	0.0129	0.0141	0.0067	0.0206	0.0129	0.0281	0.014
6	0.0327	0.0046	0.0129	0.0141	0.0067	0.0206	0.0129	0.0281	0.014
7	0.0253	0.0045	0.013	0.0141	0.0058	0.0186	0.0159	0.0158	0.0178
8	0.0332	0.0045	0.013	0.0141	0.0058	0.0186	0.0159	0.0158	0.0178
9	0.0293	0.0057	0.013	0.0141	0.0046	0.0304	0.0122	0.024	0.014
10	0.0256	0.0057	0.013	0.0141	0.0046	0.0304	0.0122	0.024	0.014
11	0.0379	0.0036	0.0129	0.0141	0.0049	0.0205	0.0118	0.024	0.0092

	Strength (MPa)	Cost (USD/kg)	Young module (GPa)	Fracture toughness (MPa)	Hardness (HV)	Thermal conductivity (W/mK)	Thermal expansion (µstrain/°C)	Application temperature (°C)	Fatigue strength (MPa)
12	0.0414	0.0036	0.0129	0.0141	0.0049	0.0205	0.0118	0.024	0.0092
13	0.0244	0.0061	0.0123	0.0141	0.0059	0.0269	0.0137	0.0168	0.0159
14	0.0304	0.0061	0.0123	0.0141	0.0059	0.0269	0.0137	0.0168	0.0159
15	0.0236	0.0189	0.0121	0.0164	0.0058	0.0242	0.0106	0.0158	0.0171
16	0.0292	0.0189	0.0121	0.0164	0.0058	0.0242	0.0106	0.0158	0.0171
17	0.0192	0.0209	0.0121	0.0149	0.0044	0.0248	0.0116	0.0332	0.0163
18	0.0259	0.0209	0.0121	0.0149	0.0044	0.0248	0.0116	0.0332	0.0163
19	0.0273	0.0099	0.012	0.0141	0.0054	0.0227	0.0112	0.0301	0.0127
20	0.0224	0.0099	0.012	0.0141	0.0054	0.0227	0.0112	0.0301	0.0127
21	0.0312	0.0043	0.0126	0.0141	0.0054	0.0196	0.0151	0.025	0.0178
22	0.0231	0.0043	0.0126	0.0141	0.0054	0.0196	0.0151	0.025	0.0178
23	0.0106	0.0059	0.006	0.0141	0.0048	0.0434	0.027	0.0137	0.0076
24	0.0217	0.0416	0.0119	0.0071	0.0058	0.031	0.0095	0.0147	0.0133
25	0.016	0.0416	0.0119	0.0071	0.0058	0.031	0.0095	0.0147	0.0133
26	0.0159	0.0414	0.012	0.0094	0.0058	0.0289	0.0095	0.0301	0.0145
27	0.0216	0.0414	0.012	0.0094	0.0058	0.0289	0.0095	0.0301	0.0145
28	0.018	0.0363	0.0126	0.0064	0.0071	0.031	0.0084	0.024	0.0166
29	0.0133	0.0363	0.0126	0.0064	0.0071	0.031	0.0084	0.024	0.0166
30	0.0221	0.0329	0.0119	0.0139	0.0064	0.0269	0.0101	0.0301	0.0145
31	0.0164	0.0329	0.0119	0.0139	0.0064	0.0269	0.0101	0.0301	0.0145
32	0.0183	0.0329	0.012	0.0062	0.0058	0.0269	0.0101	0.0168	0.0163
33	0.0136	0.0329	0.012	0.0062	0.0058	0.0269	0.0101	0.0168	0.0163
34	0.0221	0.0294	0.0119	0.0075	0.0064	0.0269	0.0101	0.0301	0.0162
35	0.0164	0.0294	0.0119	0.0075	0.0064	0.0269	0.0101	0.0301	0.0162
36	0.0219	0.0287	0.0124	0.0065	0.0054	0.0269	0.0101	0.0168	0.0165
37	0.0163	0.0287	0.0124	0.0065	0.0054	0.0269	0.0101	0.0168	0.0165
38	0.0177	0.038	0.0119	0.0062	0.0058	0.0289	0.0095	0.0301	0.0142
39	0.0229	0.0372	0.0119	0.0062	0.0063	0.031	0.0095	0.0301	0.0145
40	0.0188	0.0372	0.0119	0.0062	0.0063	0.031	0.0095	0.0301	0.0145

(d) Concordance matrix for ELECTRE

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	0	0	0,44	0,44	0,46	0,46	0,52	0,52	0,54	0,54	0,54	0,54	0,52	0,52	0,44	0,44	0,29	0,29	0,38	0,38	0,37	0,37	0,67	0,62	0,77	0,62	0,46	0,37	0,52	0,46	0,62	0,52	0,67	0,37	0,52	0,52	0,67	0,62	0,46	0,46
2	0,15	0	0,44	0,6	0,46	0,46	0,67	0,52	0,54	0,69	0,54	0,54	0,67	0,52	0,6	0,44	0,44	0,44	0,38	0,54	0,37	0,52	0,67	0,77	0,77	0,62	0,62	0,52	0,52	0,62	0,62	0,67	0,67	0,52	0,52	0,67	0,67	0,62	0,62	0,62
3	0,48	0,48	0	0,15	0,33	0,17	0,52	0,37	0,44	0,44	0,6	0,6	0,52	0,52	0,6	0,6	0,44	0,44	0,44	0,44	0,42	0,42	0,52	0,67	0,67	0,52	0,52	0,63	0,63	0,48	0,48	0,67	0,67	0,48	0,48	0,67	0,67	0,52	0,48	0,48
4	0,48	0,33	0	0	0,17	0,17	0,52	0,37	0,29	0,29	0,6	0,6	0,52	0,37	0,6	0,44	0,44	0,29	0,29	0,44	0,27	0,42	0,52	0,67	0,67	0,52	0,52	0,63	0,63	0,48	0,48	0,67	0,67	0,48	0,48	0,67	0,67	0,52	0,48	0,48
5	0,46	0,46	0,6	0,75	0	0	0,67	0,52	0,27	0,42	0,69	0,69	0,42	0,27	0,5	0,35	0,35	0,35	0,29	0,44	0,6	0,75	0,52	0,67	0,67	0,42	0,42	0,54	0,54	0,42	0,42	0,58	0,58	0,42	0,42	0,58	0,58	0,42	0,42	0,42
6	0,46	0,46	0,75	0,75	0,15	0	0,67	0,52	0,42	0,42	0,69	0,69	0,42	0,42	0,5	0,5	0,35	0,35	0,44	0,44	0,75	0,75	0,52	0,67	0,67	0,42	0,42	0,54	0,54	0,42	0,42	0,58	0,58	0,42	0,42	0,58	0,58	0,42	0,42	0,42
7	0,4	0,25	0,31	0,31	0,25	0,25	0	0	0,21	0,21	0,44	0,44	0,4	0,25	0,4	0,25	0,44	0,29	0,29	0,44	0,35	0,5	0,52	0,63	0,63	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,52	0,52	0,48	0,48	0,48
8	0,4	0,4	0,46	0,46	0,4	0,4	0,15	0	0,37	0,37	0,44	0,44	0,4	0,4	0,4	0,4	0,44	0,44	0,44	0,44	0,5	0,5	0,52	0,63	0,63	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,48	0,52	0,52	0,48	0,48	0,48
9	0,38	0,38	0,4	0,56	0,56	0,4	0,63	0,48	0	0,15	0,58	0,58	0,56	0,4	0,63	0,63	0,52	0,52	0,58	0,58	0,4	0,56	0,48	0,63	0,63	0,56	0,56	0,38	0,38	0,56	0,56	0,71	0,71	0,56	0,56	0,71	0,71	0,56	0,38	0,38
10	0,38	0,23	0,4	0,56	0,4	0,4	0,63	0,48	0	0	0,58	0,58	0,56	0,4	0,63	0,48	0,52	0,37	0,42	0,58	0,4	0,56	0,48	0,63	0,63	0,56	0,56	0,38	0,38	0,56	0,56	0,71	0,71	0,56	0,56	0,71	0,71	0,56	0,38	0,38
11	0,38	0,38	0,33	0,33	0,15	0,15	0,48	0,48	0,19	0,19	0	0	0,38	0,38	0,46	0,46	0,35	0,35	0,31	0,31	0,4	0,4	0,52	0,54	0,54	0,38	0,38	0,38	0,38	0,38	0,38	0,54	0,54	0,38	0,38	0,54	0,54	0,38	0,38	0,38
12	0,38	0,38	0,33	0,33	0,15	0,15	0,48	0,48	0,19	0,19	0,15	0	0,38	0,38	0,46	0,46	0,35	0,35	0,31	0,31	0,4	0,4	0,52	0,54	0,54	0,38	0,38	0,38	0,38	0,38	0,38	0,54	0,54	0,38	0,38	0,54	0,54	0,38	0,38	0,38
13	0,4	0,25	0,4	0,4	0,5	0,5	0,52	0,52	0,37	0,37	0,54	0,54	0	0	0,67	0,52	0,52	0,37	0,46	0,62	0,37	0,52	0,67	0,67	0,67	0,52	0,52	0,31	0,31	0,48	0,48	0,42	0,42	0,38	0,38	0,35	0,35	0,52	0,48	0,48
14	0,4	0,4	0,4	0,56	0,65	0,5	0,67	0,52	0,52	0,52	0,54	0,54	0,15	0	0,67	0,67	0,52	0,52	0,62	0,62	0,37	0,52	0,67	0,67	0,67	0,52	0,52	0,31	0,31	0,48	0,48	0,42	0,42	0,38	0,38	0,35	0,35	0,52	0,48	0,48
15	0,56	0,4	0,4	0,4	0,5	0,5	0,4	0,4	0,37	0,37	0,54	0,54	0,33	0,33	0	0	0,37	0,21	0,62	0,77	0,44	0,6	0,75	0,63	0,63	0,48	0,48	0,4	0,4	0,48	0,48	0,48	0,48	0,48	0,48	0,44	0,44	0,48	0,48	0,48
16	0,56	0,56	0,4	0,56	0,65	0,5	0,56	0,4	0,37	0,52	0,54	0,54	0,48	0,33	0,15	0	0,37	0,37	0,77	0,77	0,44	0,6	0,75	0,63	0,63	0,48	0,48	0,4	0,4	0,48	0,48	0,48	0,48	0,48	0,48	0,44	0,44	0,48	0,48	0,48
17	0,71	0,56	0,56	0,56	0,65	0,65	0,56	0,56	0,48	0,48	0,65	0,65	0,48	0,48	0,56	0,56	0	0	0,81	0,81	0,56	0,56	0,71	0,48	0,63	0,63	0,48	0,46	0,46	0,48	0,63	0,54	0,54	0,48	0,63	0,31	0,46	0,63	0,48	0,63
18	0,71	0,56	0,56	0,71	0,65	0,65	0,71	0,56	0,48	0,63	0,65	0,65	0,63	0,48	0,71	0,56	0,15	0	0,81	0,96	0,56	0,71	0,71	0,63	0,63	0,63	0,63	0,46	0,46	0,63	0,63	0,54	0,54	0,63	0,63	0,46	0,46	0,63	0,63	0,63
19	0,54	0,54	0,48	0,63	0,63	0,48	0,63	0,48	0,35	0,5	0,62	0,62	0,46	0,31	0,38	0,23	0,19	0,19	0	0,15	0,48	0,63	0,67	0,54	0,54	0,31	0,31	0,46	0,46	0,38	0,38	0,46	0,46	0,38	0,38	0,46	0,46	0,38	0,38	0,38
20	0,54	0,38	0,48	0,48	0,48	0,48	0,48	0,48	0,35	0,35	0,62	0,62	0,31	0,31	0,23	0,23	0,19	0,04	0	0	0,48	0,48	0,67	0,54	0,54	0,31	0,31	0,46	0,46	0,38	0,38	0,46	0,46	0,38	0,38	0,46	0,46	0,38	0,23	0,38
21	0,56	0,56	0,25	0,4	0,33	0,17	0,48	0,33	0,52	0,52	0,52	0,52	0,56	0,56	0,56	0,56	0,44	0,44	0,4	0,4	0	0,15	0,52	0,63	0,63	0,48	0,48	0,56	0,56	0,48	0,48	0,63	0,63	0,48	0,48	0,63	0,63	0,48	0,48	0,48
22	0,56	0,4	0,25	0,25	0,17	0,17	0,33	0,33	0,37	0,37	0,52	0,52	0,4	0,4	0,4	0,4	0,44	0,29	0,25	0,4	0	0	0,52	0,63	0,63	0,48	0,48	0,56	0,56	0,48	0,48	0,63	0,63	0,48	0,48	0,63	0,63	0,48	0,48	0,48
23	0,25	0,25	0,4	0,4	0,4	0,4	0,4	0,4	0,44	0,44	0,4	0,4	0,25	0,25	0,25	0,25	0,29	0,29	0,25	0,25	0,4	0,4	0	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33
24	0,38	0,23	0,33	0,33	0,33	0,33	0,33	0,33	0,37	0,37	0,46	0,46	0,33	0,33	0,33	0,33	0,52	0,37	0,46	0,46	0,37	0,37	0,67	0	0,15	0,48	0,48	0,46	0,46	0,33	0,48	0,56	0,56	0,33	0,48	0,44	0,6	0,56	0,23	0,38
25	0,23	0,23	0,33	0,33	0,33	0,33	0,33	0,33	0,37	0,37	0,46	0,46	0,33	0,33	0,33	0,33	0,37	0,37	0,46	0,46	0,37	0,37	0,67	0	0	0,48	0,33	0,31	0,46	0,33	0,33	0,4	0,56	0,33	0,33	0,44	0,44	0,4	0,23	0,23
26	0,38	0,38	0,48	0,48	0,58	0,58	0,48	0,48	0,44	0,44	0,62	0,62	0,48	0,48	0,48	0,48	0,37	0,37	0,46	0,46	0,52	0,52	0,67	0,4	0,4	0	0	0,46	0,62	0,5	0,5	0,56	0,71	0,48	0,48	0,6	0,6	0,4	0,4	0,4
27	0,54	0,38	0,48	0,48	0,58	0,58	0,48	0,48	0,44	0,44	0,62	0,62	0,48	0,48	0,48	0,48	0,52	0,37	0,46	0,46	0,52	0,52	0,67	0,4	0,56	0,15	0	0,62	0,62	0,5	0,65	0,71	0,71	0,48	0,63	0,6	0,75	0,56	0,4	0,56
28	0,63	0,48	0,37	0,37	0,46	0,46	0,52	0,52	0,46	0,46	0,46	0,46	0,69	0,69	0,6	0,6	0,54	0,54	0,54	0,54	0,37	0,37	0,67	0,37	0,52	0,54	0,38	0	0,15	0,54	0,69	0,77	0,92	0,54	0,69	0,69	0,85	0,62	0,29	0,29
29	0,48	0,48	0,37	0,37	0,46	0,46	0,52	0,52	0,46	0,46	0,46	0,46	0,69	0,69	0,6	0,6	0,54	0,54	0,54	0,54	0,37	0,37	0,67	0,37	0,37	0,38	0,38	0	0	0,54	0,54	0,77	0,77	0,54	0,54	0,69	0,69	0,46	0,29	0,29
30	0,54	0,38	0,52	0,52	0,58	0,58	0,52	0,52	0,44	0,44	0,62	0,62	0,35	0,35	0,52	0,52	0,52	0,37	0,46	0,46	0,52	0,52	0,67	0,6	0,6	0,35	0,35	0,46	0,46	0	0,15	0,42	0,42	0,23	0,38	0,58	0,58	0,44	0,19	0,35
31	0,38	0,38	0,52	0,52	0,58	0,58	0,52	0,52	0,44	0,44	0,62	0,62	0,35	0,35	0,52	0,52	0,37	0,37	0,46	0,46	0,52	0,52	0,67	0,44	0,6	0,35	0,19	0,31	0,46	0	0	0,27	0,42	0,23	0,23	0,42	0,58	0,29	0,19	0,19
32	0,48	0,33	0,33	0,33	0,42	0,42	0,48	0,48	0,29	0,29	0,46	0,46	0,25	0,25	0,48	0,48	0,37	0,37	0,46	0,46	0,37	0,37	0,67	0,4	0,56	0,33	0,17	0,23	0,23	0,17	0,33	0	0,15	0,33	0,48	0,19	0,35	0,4	0,25	0,25
33	0,33	0,33	0,33	0,33	0,42	0,42	0,48	0,48	0,29	0,29	0,46	0,46	0,25	0,25	0,48	0,48	0,37	0,37	0,46	0,46	0,37	0,37	0,67	0,4	0,4	0,17	0,17	0,08	0,23	0,17	0,17	0	0	0,33	0,33	0,19	0,19	0,25	0,25	0,25
34	0,63	0,48	0,52	0,52	0,58	0,58	0,52	0,52	0,44	0,44	0,62	0,62	0,44	0,44	0,52	0,52	0,52	0,37	0,46	0,46	0,52	0,52	0,67	0,6	0,6	0,37	0,37	0,46	0,46	0,1	0,25	0,42	0,42	0	0,15	0,58	0,58	0,44	0,29	0,44
35	0,48	0,48	0,52	0,52	0,58	0,58	0,52	0,52	0,44	0,44	0,62	0,62	0,44	0,44	0,52	0,52	0,37	0,37	0,46	0,46	0,52	0,52	0,67	0,44	0,6	0,37	0,21	0,31	0,46	0,1	0,1	0,27	0,42	0	0	0,42	0,58	0,29	0,29	0,29
36	0,48	0,33	0,33	0,33	0,42	0,42	0,48	0,48	0,29	0,29	0,46	0,46	0,33	0,33	0,56	0,56	0,69	0,54	0,5	0,5	0,33	0,33	0,67	0,56	0,56	0,4	0,4	0,31	0,31	0,17	0,33	0,4	0,4	0,17	0,33	0	0,15	0,48	0,33	0,48
37	0,33	0,33	0,33	0,33	0,42	0,42	0,48	0,48	0,29	0,29	0,46	0,46	0,33	0,33	0,56	0,56	0,54	0,54	0,5	0,5	0,33	0,33	0,67	0,4	0,56	0,4	0,25	0,15	0,31	0,17	0,17	0,25	0,4	0,17	0,17	0	0	0,33	0,33	0,33
38	0,38	0,38	0,48	0,48	0,58	0,58	0,48	0,48	0,44	0,44	0,62	0,62	0,48	0,48	0,48	0,48	0,37	0,37	0,46	0,46	0,52	0,52	0,67	0,25	0,4	0,15	0	0,38	0,54	0,33	0,48	0,48	0,63	0,33	0,48	0,52	0,67	0	0,15	0,15
39	0,54	0,38	0,52	0,52	0,58	0,58	0,52	0,52	0,62	0,62	0,62	0,62	0,52	0,52	0,52	0,52	0,52	0,37	0,46	0,62	0,52	0,52	0,67	0,44	0,44	0,37	0,37	0,54	0,54	0,48	0,48	0,67	0,67	0,48	0,48	0,67	0,67	0,46	0	0,15
40	0,54	0,38	0,52	0,52	0,58	0,58	0,52	0,52	0,62	0,62	0,62	0,62	0,52	0,52	0,52	0,52	0,37	0,37	0,46	0,46	0,52	0,52	0,67	0,29	0,44	0,37	0,21	0,54	0,54	0,33	0,48	0,67	0,67	0,33	0,48	0,52	0,67	0,46	0	0

(e) Discordance matrix for ELECTRE

D	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	0	1	0,65	0,34	0,5	0,71	0,32	0,67	1	0,7	0,96	1	0,45	0,86	0,72	0,72	0,93	0,93	0,66	0,66	0,61	0,33	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0,33	0,33	0,5	0,5	0,15	0,31	0,56	0,56	0,56	0,73	0,11	0,28	0,72	0,72	0,93	0,93	0,66	0,66	0,33	0,33	0,92	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	0	0	0,58	0,8	0,4	0,4	1	1	0,64	1	0,93	0,93	1	1	1	1	1	0,54	1	0,32	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	0	0,8	1	0,4	0,83	1	1	1	1	0,93	0,93	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	0	1	0,31	0,5	1	1	1	1	0,55	0,55	1	1	1	1	1	1	1	0,96	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	0,53	0	0	0,31	0,31	1	1	1	1	0,55	0,55	1	1	1	1	0,98	0,51	1	0,4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	0	0	1	1	0,96	0,96	0,94	1	1	1	1	1	1	1	1	0,91	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	0,96	1	0,34	0,34	0,42	0,42	0,32	0,33	0	0	0,87	1	0,26	0,26	1	1	1	1	0,81	0,81	0,35	0,35	0,79	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	0,61	0,34	0,42	0,73	0,32	0,65	1	0	1	1	0,26	0,66	1	1	1	1	0,81	0,81	0,52	0,35	0,98	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11	1	1	1	0,69	0,44	0,91	0,68	1	1	0,81	0	1	0,49	0,89	1	1	0,93	1	0,6	0,41	1	0,58	0,84	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
12	0,89	1	0,95	0,54	0,33	0,55	0,53	1	0,82	0,63	0	0	0,39	0,6	0,86	1	0,78	1	0,45	0,33	0,84	0,47	0,74	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	0,26	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	0,26	0,35	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0,83	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
15	1	1	0,64	0,64	0,86	0,86	0,37	0,67	0,62	0,62	0,93	1	0,25	0,53	0	1	1	1	1	1	0,63	0,63	1	1	1	1	1	1	1	1	1	1	1	1	1	0,99	0,99	1	1	1
16	1	1	0,64	0,64	0,86	0,86	0,37	0,37	0,62	0,62	0,57	0,8	0,25	0,25	0	0	1	1	1	1	0,63	0,63	1	1	1	1	1	1	1	1	1	1	0,9	1	1	0,99	0,76	1	1	1
17	1	1	0,8	0,38	0,48	0,83	0,35	0,8	0,66	0,42	1	1	0,32	0,68	0,25	0,57	0	1	0,73	0,28	0,72	0,23	0,95	1	1	1	1	1	1	1	1	0,73	0,73	1	1	0,47	0,47	1	1	1
18	1	1	0,39	0,09	0,14	0,41	0,25	0,42	0,37	0,37	0,69	0,89	0,13	0,27	0,09	0,19	0	0	0,12	0,09	0,32	0,21	0,95	1	1	1	1	1	1	1	1	0,73	0,73	1	0,88	0,47	0,47	1	1	1
19	1	1	0,94	0,94	0,33	1	0,35	0,41	1	1	1	1	0,31	0,31	0,62	0,62	1	1	0	0	0,91	0,91	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
20	1	1	1	0,94	0,9	1	0,35	0,76	1	1	1	1	0,31	0,6	0,62	0,62	1	1	1	0	1	0,91	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	1	1	0,39	0,1	0,74	0,8	0,09	0,22	1	1	0,78	1	0,88	0,88	1	1	1	1	1	0,63	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	1	1	1	0,83	1	1	0,23	1	1	1	1	1	0,88	0,88	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	0,51	1	0,9	0,62	0,73	0,97	0,59	0,91	1	1	1	1	0,84	1	0,68	0,97	1	1	0,81	0,8	0,87	0,53	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24	0,29	0,29	0,29	0,28	0,36	0,36	0,19	0,31	0,26	0,26	0,43	0,52	0,2	0,24	0,41	0,41	0,89	0,89	0,49	0,49	0,28	0,28	0,49	0	0	1	1	1	1	1	1	0,33	0,33	1	1	0,24	0,24	1	1	1
25	0,29	0,29	0,44	0,28	0,36	0,45	0,25	0,46	0,37	0,27	0,58	0,67	0,24	0,4	0,41	0,58	0,89	0,89	0,49	0,49	0,41	0,28	0,49	1	0	1	1	1	1	1	1	0,33	0,33	1	1	0,45	0,24	1	1	1
26	0,36	0,36	0,45	0,26	0,3	0,46	0,25	0,47	0,37	0,27	0,58	0,67	0,24	0,41	0,34	0,59	0,26	0,49	0,36	0,2	0,41	0,19	0,49	0,37	0,13	0	1	0,34	0,34	0,72	0,52	0,18	0,13	0,51	0,14	0,45	0,14	0,51	1	0,66
27	0,36	0,36	0,29	0,13	0,15	0,3	0,17	0,32	0,22	0,13	0,43	0,52	0,13	0,25	0,31	0,34	0,26	0,26	0,18	0,15	0,26	0,15	0,49	0,13	0,13	0	0	0,34	0,25	0,52	0,52	0,13	0,13	0,14	0,14	0,14	0,14	0	0,48	0,48
28	0,35	0,35	0,45	0,24	0,29	0,46	0,24	0,48	0,37	0,25	0,61	0,72	0,26	0,41	0,57	0,64	0,6	0,6	0,35	0,29	0,41	0,24	0,61	0,57	0,57	1	1	0	0	1	1	0,23	0,23	0,89	0,89	0,51	0,22	1	1	1
29	0,35	0,44	0,6	0,38	0,43	0,61	0,38	0,63	0,52	0,4	0,75	0,86	0,37	0,56	0,59	0,91	0,6	0,82	0,53	0,34	0,56	0,31	0,61	0,91	0,57	1	1	1	0	1	1	0,7	0,23	1	0,89	1	0,39	1	1	1
30	0,55	0,55	0,36	0,12	0,18	0,37	0,21	0,39	0,27	0,13	0,54	0,66	0,14	0,31	0,18	0,49	0,26	0,32	0,23	0,05	0,32	0,18	0,63	0,57	0,57	1	1	0,55	0,47	0	0	0,13	0,13	0,27	0,27	0,15	0,15	0,67	0,56	0,56
31	0,55	0,55	0,56	0,32	0,38	0,58	0,31	0,59	0,48	0,34	0,74	0,86	0,3	0,52	0,51	0,89	0,26	0,8	0,48	0,26	0,52	0,24	0,63	0,57	0,57	1	1	0,55	0,55	1	0	0,15	0,13	0,9	0,27	0,41	0,15	0,67	0,86	0,56
32	0,55	0,55	0,5	0,29	0,4	0,51	0,28	0,53	0,4	0,29	0,67	0,79	0,3	0,45	0,72	0,78	1	1	0,58	0,58	0,45	0,29	0,63	1	1	1	1	1	1	1	1	0	0	1	1	0,85	0,09	1	1	1
33	0,55	0,55	0,66	0,42	0,48	0,68	0,41	0,69	0,58	0,44	0,83	0,95	0,4	0,63	0,72	1	1	1	0,6	0,58	0,62	0,33	0,63	1	1	1	1	1	1	1	1	1	0	1	1	1	0,64	1	1	1
34	0,63	0,63	0,41	0,27	0,27	0,43	0,27	0,45	0,3	0,28	0,61	0,75	0,28	0,36	0,62	0,62	0,86	0,86	0,34	0,34	0,36	0,26	0,72	0,8	0,8	1	1	1	0,79	1	1	0,26	0,26	0	0	0,03	0,03	1	1	1
35	0,63	0,63	0,64	0,37	0,43	0,66	0,36	0,68	0,55	0,39	0,84	0,97	0,35	0,6	0,62	0,89	0,86	1	0,56	0,34	0,59	0,27	0,72	0,8	0,8	1	1	1	1	1	1	0,26	0,26	1	0	0,41	0,03	1	1	1
36	0,65	0,65	0,43	0,34	0,47	0,47	0,32	0,47	0,33	0,33	0,64	0,78	0,34	0,38	1	1	1	1	0,71	0,71	0,38	0,34	0,75	1	1	1	1	1	0,89	1	1	1	0,51	1	1	0	0	1	1	1
37	0,65	0,65	0,67	0,38	0,47	0,68	0,37	0,7	0,57	0,41	0,86	1	0,36	0,63	1	1	1	1	0,71	0,71	0,61	0,34	0,75	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
38	0,39	0,39	0,44	0,24	0,28	0,45	0,24	0,46	0,36	0,25	0,59	0,69	0,25	0,4	0,53	0,6	0,5	0,5	0,34	0,28	0,4	0,23	0,55	0,26	0,24	1	1	0,4	0,4	1	1	0,16	0,16	0,51	0,24	0,32	0,17	0	1	1
39	0,34	0,34	0,29	0,24	0,24	0,3	0,24	0,32	0,25	0,25	0,45	0,55	0,25	0,25	0,55	0,55	0,53	0,53	0,29	0,29	0,25	0,24	0,56	0,29	0,29	0,61	1	0,35	0,22	1	1	0,13	0,13	0,22	0,22	0,15	0,15	0,16	0	0
40	0,34	0,34	0,42	0,24	0,26	0,43	0,24	0,44	0,34	0,25	0,57	0,67	0,25	0,37	0,55	0,57	0,53	0,53	0,31	0,29	0,38	0,24	0,56	0,29	0,29	1	1	0,35	0,35	1	1	0,13	0,13	0,43	0,22	0,23	0,15	0,41	1	0

(f) The modified total superiority matrix

E	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0	0	0	0	0	0	0	1	1	1	1	1	0	1	1	1	1	1	0	1	1	1	1	1	1
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	0	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	0	1	1	1	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	0
6	1	1	1	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	1	0	1	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	1	1	1	1	0	0	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0
10	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	1	0	0	1	0	0	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	0
11	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0
12	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0
13	0	0	0	0	1	1	0	1	0	0	1	1	0	0	1	1	1	0	1	1	0	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	0	1	1	1
14	0	0	0	1	1	1	0	0	1	1	1	1	0	0	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	0	1	1	1
15	1	0	0	0	1	1	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0	1	1	1	1	1	0	0	1	1	1	1	1	1	0	0	1	1	1
16	1	1	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	1	1	1	1	1	0	0	1	1	1	1	1	1	0	0	1	1	1
17	1	1	1	0	0	1	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	1
18	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1	1	1
19	1	1	1	1	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	0
20	1	0	1	1	1	1	0	1	0	0	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	0
21	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
22	1	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	1	1	1	0	1	0	0	0	1	0	0	1	0	0
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	1	1	0	0	1	0	0
29	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	1	0	1	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
37	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	1	1	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0

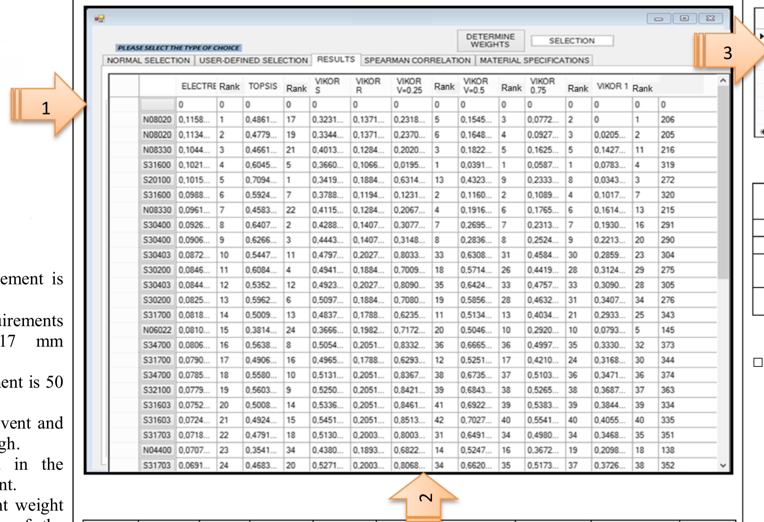
Appendix C

Example-1: Nitrogen tube

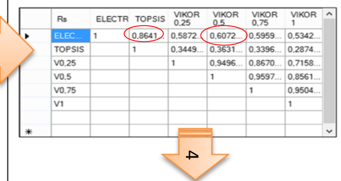
Requirements

a) Working pressure requirement is 150 psi.
b) Length and diameter requirements are 156.5 mm and 217 mm respectively.
c)Working volume requirement is 50 liters.
d) Resistance to organic solvent and water corrosion should be high.
e) Painting will be used in the construction of the component.
f) It is desirable to use light weight material for easy handling of the component.

PVSEL Results



Criteria	Strength MPa	Cost USD/kg	Young Module GPa	Fracture toughness MPa	Hardness HV	Thermal conductivity W/mK	Thermal expansion µstrain/^{0}C	Application Temperature ^{0}C	Fatigue strength MPa
Weight	8	8	5	8	4	N/A	N/A	N/A	6



Method	Material		
	N08020	S20100	S31600
ELECTRE	1	5	4
TOPSIS	17	1	5
VIKOR V=0.50	3	9	1
Total	21■	15■	10□

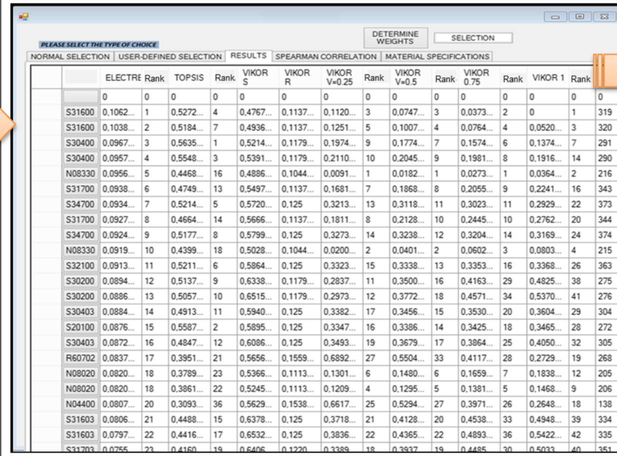
□ S31600 is selected for this example

Example -2: Membrane tank

Requirements

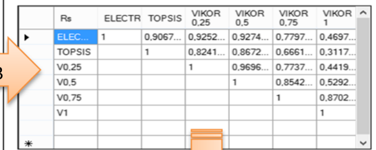
a) Working pressure requirement is 10 psi.
b) Length and diameter requirements are 156.5 mm and 217 mm respectively.
c)Working volume requirement is 24 liters.
d) Working temperature is 120 ºC .
e) Electro-static painting will be used in the construction of the component.
f) It is desirable to use light weight material for easy handling of the component.
g)Welding will be used in the construction of the component.

PVSEL Results



	ELECTRE	Rank	TOPSIS	Rank	VIKOR S	VIKOR R	VIKOR V=0.25	Rank	VIKOR V=0.5	Rank	VIKOR 0.75	Rank	VIKOR 1	Rank	
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S31600	0,1062...	1	0,5272...	4	0,4767...	0,1137...	0,1120...	3	0,0747...	3	0,0373...	2	0	1	319
S31600	0,1038...	2	0,5184...	7	0,4936...	0,1137...	0,1251...	5	0,1007...	4	0,0764...	4	0,0520...	3	320
S30400	0,0967...	3	0,5635...	1	0,5214...	0,1179...	0,1974...	9	0,1774...	7	0,1574...	6	0,1374...	7	291
S30400	0,0957...	4	0,5548...	3	0,5391...	0,1179...	0,2110...	10	0,2045...	9	0,1981...	8	0,1916...	14	290
N08330	0,0956...	5	0,4468...	16	0,4886...	0,1044...	0,0091...	1	0,0182...	1	0,0273...	1	0,0364...	2	216
S31700	0,0938...	6	0,4749...	13	0,5497...	0,1137...	0,1681...	7	0,1868...	8	0,2055...	9	0,2241...	16	343
S34700	0,0934...	7	0,5214...	5	0,5720...	0,125	0,3213...	13	0,3118...	11	0,3023...	11	0,2929...	22	373
S31700	0,0927...	8	0,4664...	14	0,5666...	0,1137...	0,1811...	8	0,2128...	10	0,2445...	10	0,2762...	20	344
S34700	0,0924...	9	0,5177...	8	0,5799...	0,125	0,3273...	14	0,3238...	12	0,3204...	14	0,3169...	24	374
N08330	0,0919...	10	0,4399...	18	0,5028...	0,1044...	0,0200...	2	0,0401...	2	0,0602...	3	0,0803...	4	215
S32100	0,0913...	11	0,5211...	6	0,5864...	0,125	0,3323...	15	0,3338...	13	0,3353...	16	0,3368...	26	363
S30200	0,0894...	12	0,5137...	9	0,6338...	0,1179...	0,2837...	11	0,3500...	16	0,4163...	29	0,4825...	38	275
S30200	0,0886...	13	0,5057...	10	0,6515...	0,1179...	0,2973...	12	0,3772...	18	0,4571...	34	0,5370...	41	276
S30403	0,0884...	14	0,4913...	11	0,5940...	0,125	0,3382...	17	0,3456...	15	0,3530...	20	0,3604...	29	304
S20100	0,0876...	15	0,5587...	2	0,5895...	0,125	0,3347...	16	0,3386...	14	0,3425...	18	0,3465...	28	272
S30403	0,0872...	16	0,4847...	12	0,6086...	0,125	0,3493...	19	0,3679...	17	0,3864...	25	0,4050...	32	305
R60702	0,0837...	17	0,3951...	21	0,5656...	0,1559...	0,6892...	27	0,5504...	33	0,4117...	28	0,2729...	19	268
N08020	0,0820...	18	0,3789...	23	0,5366...	0,1113...	0,1301...	6	0,1480...	6	0,1659...	7	0,1838...	12	205
N08020	0,0820...	18	0,3861...	22	0,5245...	0,1113...	0,1209...	4	0,1295...	5	0,1381...	5	0,1468...	9	206
N04400	0,0807...	20	0,3093...	36	0,5629...	0,1538...	0,6617...	25	0,5294...	27	0,3971...	26	0,2648...	18	138
S31603	0,0806...	21	0,4488...	15	0,6378...	0,125	0,3718...	21	0,4128...	20	0,4538...	33	0,4948...	39	334
S31603	0,0797...	22	0,4416...	17	0,6532...	0,125	0,3836...	22	0,4365...	22	0,4893...	36	0,5422...	42	335
S31703	0,0755	23	0,4160	19	0,6406	0,1220	0,3389	18	0,3937	19	0,4485	30	0,5033	40	351

Criteria	Strength MPa	Cost USD / kg	Young Module GPa	Fracture toughness MPa	Hardness HV	Thermal conductivity W/mK	Thermal expansion µstrain/^{0}C	Application Temperature ^{0}C	Fatigue strength MPa
Weight	5	8	5	6	4	3	6	6	5



	Rs	ELECTR	TOPSIS	VIKOR 0,25	VIKOR 0,5	VIKOR 0,75	VIKOR 1
▸	ELEC...	1	0,9067...	0,9252...	0,9274...	0,7797...	0,4697...
	TOPSIS		1	0,8241...	0,8672...	0,6661...	0,3117...
	V0,25			1	0,9696...	0,7737...	0,4419...
	V0,5				1	0,8542...	0,5292...
	V0,75					1	0,8702...
	V1						1
*							

Method	Material		
	S31600	S30400	N08330
ELECTRE	1	3	5
TOPSIS	4	1	16
VIKOR V=0.50	3	7	1
Total	8 □	11■	22■

□ S31600 is selected for this example

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