



TPACK development model for pre-service mathematics teachers

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Abstract

Technological Pedagogical Content Knowledge (TPACK) describes a significant qualification for teachers and pre-service teachers to address the integration of contemporary technological advancements into lessons that adequately reflect the subject matter for a diverse set of students. Various methods and models have been identified to develop and evaluate the TPACK of teachers and pre-service teachers. This case study aimed to enrich and deepen the descriptors of the Niess (2013) model to better assist pre-service mathematics teacher educators as they redesign their teacher education programs for developing the TPACK needed in this digital age. Data were collected from semi-structured interview forms, lesson plans, micro teaching experiences, and interviews. The study group comprised seven volunteer mathematics pre-service teachers in the final term of their undergraduate education program. Analysis of the data generated a revised model to guide the TPACK development of pre-service mathematics teachers.

Keywords Pre-service mathematics teachers · Technological Pedagogical Content Knowledge · Model development · Mathematics education

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1 Introduction

Pedagogical Content Knowledge (PCK) describes teachers' knowledge as the knowledge that differentiates a content expert from a teacher whose responsibility is to teach the content in a manner that guides diverse students' understandings of the content. Basically, PCK is the special teacher knowledge that blends and integrates Content Knowledge (CK) and Pedagogical Knowledge (PK) into an understanding of how particular topics, problems or issues are organized, represented and adapted to the diverse interests, and abilities of learners, and presented for instruction (Shulman, 1986).

Integrating technology into teaching has become a critical expectation as teachers develop their technological knowledge with respect to specific content and for use in the teaching process (Niess, 2005, 2011). This type of knowledge, derived from integrating technological knowledge into the other types of knowledge that teachers need for teaching in the digital age, has been identified as Technological Pedagogical Content Knowledge (TPACK) (Koehler & Mishra, 2005a; Niess, 2005). As noted by Mishra and Koehler (2006), developing the TPACK of pre-service teachers must be one of the most important goals for teacher education this century. Recognizing this expectation, Niess (2005) used Grossmann's (1990) central components to more clearly describe pre-service mathematics teachers' knowledge for teaching with technology:

- An overarching conception about the purposes for incorporating technology in teaching mathematics;
- Knowledge of students' understandings, thinking, and learning of mathematics with technology;
- Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics;
- Knowledge of instructional strategies and representations for teaching and learning mathematics with technologies. (Niess et al., 2009, p. 8)

Later, she extended each of the components to provide more clarification (Niess, 2013):

- Overarching conceptions: What teachers know and believe about the nature of the subject matter, what is important to students' learning, and how technology supports learning forms the basis for teachers' instructional decisions.
- Students understandings, thinking and learning: Teachers rely on and benefit from students' knowledge and beliefs about understanding, thinking, and learning about technologies in particular content topics.
- Curriculum and curricular materials: Teachers discuss and apply various technologies for teaching specific subjects and how concepts and processes are organized throughout the curriculum in a technology-enhanced environment to understand that they are structured and evaluated.

- **Instructional strategies and representations:** When learning content with certain technologies, teachers adapt their instructions to guide students in learning about those technologies. They use specific representations with technologies to meet their instructional goals and the needs of students in their classrooms.

Subsequently, the Niess' research group (2007) observed in-service teachers as they learned to use spreadsheets and how they integrated this technology into their mathematics lessons over the course of nearly four years. Their analysis indicated that these teachers progressed through five levels of development as they learned to integrate a particular technology in learning and teaching mathematics with technologies. These levels were described by Niess et al. (2009) as:

1. *Recognizing* (knowledge), where teachers are able to use the technology and recognize the alignment of the technology with mathematics content yet do not integrate the technology in teaching and learning of mathematics.
2. *Accepting* (persuasion), where teachers form a favorable or unfavorable attitude toward teaching and learning mathematics with an appropriate technology.
3. *Adapting* (decision), where teachers engage in activities that lead to a choice to adopt or reject teaching and learning mathematics with an appropriate technology.
4. *Exploring* (implementation), where teachers actively integrate teaching and learning of mathematics with an appropriate technology.
5. *Advancing* (confirmation), where teachers evaluate the results of the decision to integrate teaching and learning mathematics with an appropriate technology (p. 9).

This developmental model included four central components with these five levels for determining the in-service mathematics teachers' TPACK. In, 2013, the Niess' research group enhanced the developmental model by adding descriptors that extended each component in each developmental level with respect to spreadsheet integration. However, the researchers argued that additional research was needed to more clearly describe mathematics pre-service teachers' actions and characteristics that transformed their TPACK knowledge for teaching mathematics with additional technologies other than spreadsheets. Specifically, they noted the novice nature of pre-service teachers who had little experience in designing and teaching lessons in real classroom environments as well as with little experience in working with various technologies for exploring mathematical ideas. Further examination of the literature revealed an absence of more results that extended an integrated homogeneous model that shaped the TPACK of pre-service teachers as provided by the Niess (2013) model. This void inspired the research for this paper with the intent of expanding the Niess (2013) model with components and levels to more accurately describe pre-service mathematics teachers' transformations in their TPACK. The research goal was to provide clarity for the TPACK components as well as the TPACK levels for pre-service mathematics teacher educators as they engaged in redesigning their programs for more appropriately preparing mathematics teachers with the TPACK knowledge base needed for teaching in this more digital age.

2 Theoretical framework and literature review

Technological Pedagogical Content Knowledge (TPACK) (Mishra & Koehler, 2006) proposes that teachers' knowledge is a specialized amalgam of technology, content, and pedagogy. The model describes how the subsets of Technological Knowledge (TK), Pedagogical Knowledge (PK), and Content Knowledge (CK) interact with each other, forming three additional subsets: Pedagogical Content Knowledge (PCK), Technological Pedagogical Knowledge (TPK), and Technological Content Knowledge (TCK) (Koehler et al., 2007; Mishra & Koehler, 2006). The center of the model highlights TPACK as the intersection of all the previously mentioned subsets.

Initially, these multiple subsets were surrounded by the word *Contexts* to describe the conditions in which teachers teach with technologies (such as diversity of students, school social networks, and parental concerns). Porras-Hernández and Salinas-Amescua (2013) described the importance of various contexts in which teachers teach with technology declaring that these contexts needed more clarity. They identified multiple context levels: the *Micro*, *Meso* and *Macro* levels. They further described the *micro level context* as the in-class conditions for learning, highlighting the “conditions that influence or determine teachers' practice” (p. 239) and, thus, influence teachers' TPACK. Given these ideas, in 2019 Mishra changed the word *Contexts* in the TPACK model to *ConteXtual Knowledge*, to portray teachers “as *intrapreneurs*—knowing how their organization functions, and how levers of power and influence can effect sustainable change” (Mishra, 2019, p. 76). These adjustments resulted in Fig. 1 for describing the current vision of the TPACK model.

This TPACK model presents challenges about both its nature and how it is developed. Both the center subset and the entire model are called TPACK. In so doing, the model proposes two approaches for how teachers' knowledge is developed and assessed through teacher preparation programs. First, an *interdisciplinary vision* of TPACK recognizes the distinctness of the multiple subsets in the model and calls for specific preparation in each of the domains in order to develop this teacher knowledge for the digital age. Secondly, a different view considers teacher knowledge as a distinct whole, developed through the integration and evolution of the multiple subsets where TPACK results in a transformation of a teacher's knowledge, describing the educational and developmental processes where the multiple subsets are rearranged, merged, organized, integrated and assimilated in such a way that none are any longer individually discernible. This alternate view, a *transformation vision, a homogeneous model*, proposes TPACK as a distinct whole where teachers' knowledge is transformed for the digital age, often using the acronym of TPCK to differentiate the vision from TPACK to display the importance of the multiple subsets in the model. Thus, from this point in the paper, the model is referred to as TPCK/TPACK in order to recognize the worldwide understandings and views concerning teachers' knowledge for the twenty-first century (Angeli & Valanides, 2005, 2015; Koehler & Mishra, 2008; Niess, 2005, 2019).

Each of these views suggest pedagogical techniques that apply technologies constructively for teaching the content, resulting in the knowledge of what makes

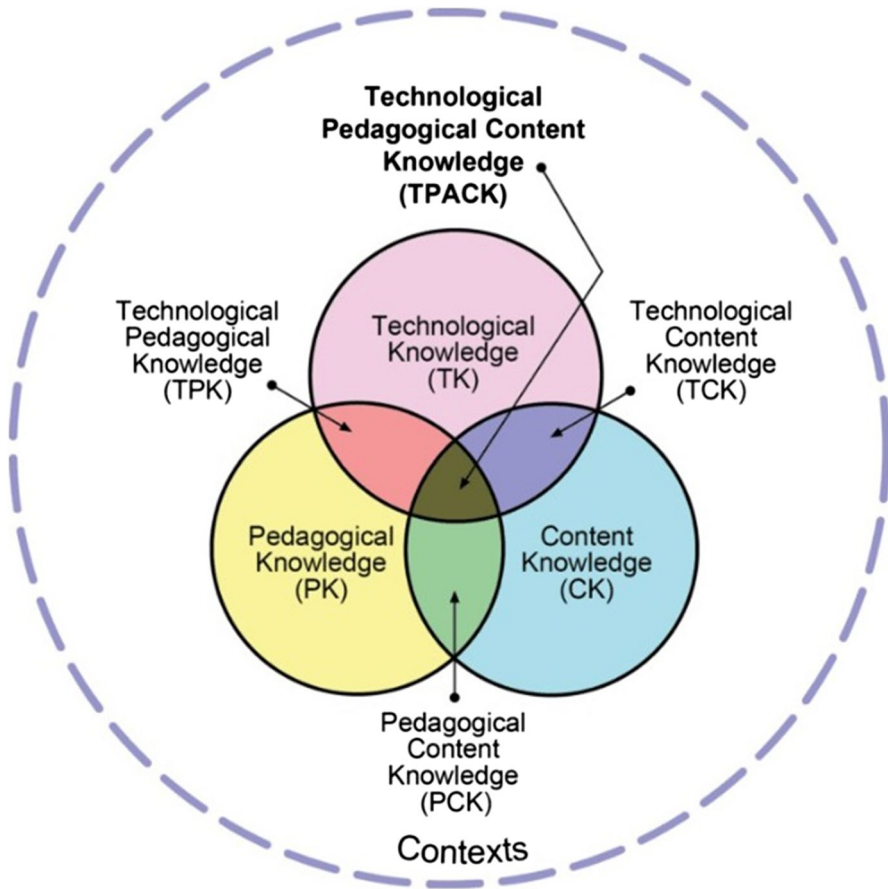


Fig. 1 Revised version of the TPACK image. © Mishra, 2018. Reproduced with permission

learning concepts difficult or easy and how technology can help in solving some of the problems that students face. Instruction is then based on the knowledge of students' prior knowledge and theories of epistemology, and how technologies can be used to build on their existing knowledge to develop new epistemologies or strengthen old ones (Koehler & Mishra, 2009).

2.1 TPACK development

The challenge for teacher educators is how TPCK/TPACK should be developed and assessed given these different views of the knowledge. Should researchers focus on TPCK/TPACK as a transformed knowledge that integrates the various knowledge subsets? Or, should they focus on the interdisciplinary view highlighting the various subsets and emphasize the development of TPCK/TPACK through the combination

of the various knowledge subsets? What has happened over the past 20 years of research and development with respect to the understanding of TPCK/TPACK?

2.1.1 Interdisciplinary TPACK development models

Multiple researchers have focused on TPCK/TPACK as an interdisciplinary construct, typically adding to or changing the interdisciplinary view. A sample of these interdisciplinary views includes the following:

- Saengbanchong et al. (2014) revised the TPACK-S model, adding a student knowledge component suggesting teachers need knowledge of students in terms of their learning readiness and needs. Their model engaged this Student Knowledge component with the other TPCK/TPACK subsets, proposing that teachers' TPCK/TPACK was the sum of 14 student-centered subsets.
- Lee and Kim (2014) proposed a multidisciplinary model for developing pre-service teachers' TPCK/TPACK that they called TPACK-IDDIRR (Introduce, Demonstrate, Develop, Implement, Reflect, and Revise). This instructional design model highlighted multidisciplinary technology integration courses grounded in the "Learning by Design" approach developed by Koehler and Mishra (2005b) to emphasize the "understanding of the complex set of interrelationships between artifacts, users, tools, and practices" (p. 94).
- Hsu et al. (2015) created the TPACK-Practice (TPACKCK-P) framework, to reflect how teachers apply TPCK/TPACK while teaching science in their classrooms. Their model highlights eight knowledge dimensions refined with 17 indicators to provide the specifics for the multiple knowledge dimensions.
- Huang (2018) developed a T-TPACK framework for mathematics teachers that included TTP (Technological Teaching Pedagogy), TPACK, and TTCK (Technological Teaching Content Knowledge).
- Arifin et al. (2020) developed a new model for building knowledge with special expertise in the field of vocational education and learning, also shifting pedagogical concepts towards andragogy. This model was referred to as "technology, andragogy, work, and content knowledge," or TAWOCK.
- Guggemos and Seufert (2021) built a new structure using both the TPCK/TPACK framework and the will, skill, and tool model. This structure consisted of the components of TK, TPK, TPACK, Technological Collaboration Knowledge (TCoK), and attitudes. In essence models such as these viewed TPACK more as an interdisciplinary construct considering the various subsets and their interactions as important constructs (such as TK, PK, CK, TPK, PCK, and TCK) for developing TPACK.

2.1.2 Integrated and transforming TPACK development models

On the other hand, multiple researchers have focused on TPCK/TPACK as a transformed knowledge that integrates the various subsets in the model. The transformation was initially described by two scholars. Angeli and Valanides (2005) described TPCK/TPACK as an expanded view of Shulman's (1986) concept of

pedagogical content knowledge (PCK). Niess (2005) modified the central PCK components presented by Grossmann (1990), mentioning four components in determining the TPCK/TPACK development of pre-service teachers:

1. An overarching conception of teaching science/mathematics with technology;
2. Instructional strategies and representations for teaching with technologies;
3. Students' understanding, thinking, and learning about a subject with technology; and
4. Curricula and curriculum materials (Niess, 2005, p. 520–521).

Following this description, Niess' research group conducted developmental level studies where they observed teachers learning to use spreadsheets and how they integrated them into mathematics lessons over the course of nearly four years. According to their analysis of those observations, the teachers progressed through five levels of development as they learned to integrate specific technology in learning and teaching mathematics. These levels were *recognizing*, *accepting*, *adapting*, *exploring*, and *advancing* (Niess et al., 2007). Later, Niess et al. (2009) created a model consisting of four themes, 11 descriptors, and five levels in determining the TPCK/TPACK of mathematics teachers. With this model, the curriculum and assessment, learning, teaching, and assessment themes were examined by taking into account the identified levels.

- A teacher at the *recognizing* level is aware of the technology. However, technology knowledge, which is subject-specific, has not yet been transferred to the learning-teaching environment. Thus, the teacher does not integrate technology into his/her courses and does not use it in class.
- At the *accepting* level, a teacher knows of the technologies that can be used in his/her courses and has a general positive or negative attitude about using technology or not.
- At the *adapting* level, teachers have the means to use or reject technology in mathematics in the context of both learning and teaching.
- A teacher at the *exploring* level is in a position to learn and teach technology and mathematics in the classroom. At this stage, students also use technology.
- At the *advancing* level, the teacher prefers to use technology in lessons and evaluates the results.

Subsequently, Niess (2013) explained the model more clearly by connecting the four TPCK/TPACK components with each of the five levels to emphasize the differences in teachers' knowledge levels, where spreadsheets were integrated as learning tools in mathematics. Given these additions to the Niess model, Lyublinskaya and Tournaki (2013) used the model to examine pre-service teachers' lesson plans in response to a variety of strategies and techniques for using instructional technologies in teaching concepts. Over the period of, 2013–2022, Lyublinskaya's research group continued examining various learning trajectories in pre-service teachers' courses, as shown in Lyublinskaya and Du (2021)

and Lyublinskaya and Kaplon-Schilis (2022). Their work continues to assess the influence on the participants' TPCK/TPACK level development and how the redesigns in the courses are effective in transforming the participants' TPCK/TPACK.

By 2018, Tatar et al. (2018) added a *pre-recognizing* level to the model, recognizing that teachers who do not have knowledge of the technology are unable to use technology for any reason. In 2021, the research of Bueno et al. clarified the Niess developmental model providing a visual as shown in Fig. 2 to enhance the transformative nature of the integrated knowledge development. Their model emphasized that the transition from one level to another was not configured in a regular way or through a consistent growing pattern toward TPCK/TPACK as initially described by Niess et al. (2009). Instead the design demonstrates how the subsets of Technology, Pedagogy and Content begin to merge as the teacher's knowledge passes through different stages and where "the spiral movement suggests a crucial idea of passing through proximal zones repeatedly among the pathway, as a sewing process" (Bueno et al., 2021, p. 113).

Rakes et al. (2022) used the Niess developmental model from 2009 to examine the ways secondary pre-service mathematics teachers showed evidence of TPCK/TPACK growth, paying specific attention to the four components (Niess, 2005) for demonstrating the growth. More specifically, they considered the relationship between the TPCK/TPACK model and the Mathematics Classroom Observation

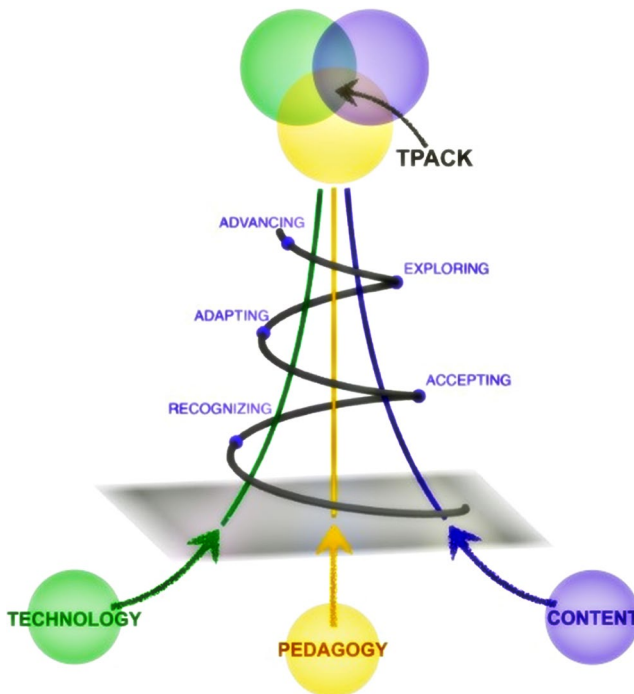


Fig. 2 Visual description of the TPCK/TPACK development created by Bueno et al., 2021 (available at: <https://www.geogebra.org/m/dxa2erhf>)

Protocol for Practices (MCOP²). The MCOP² results indicated that pre-service teachers increased their use of effective mathematics teaching practices. However, their growth in TPCK/TPACK was not significant. The authors alluded to the enhanced nature of TPCK/TPACK as a reason that the results did not match with that of MCOP², specifically through their recognition of the various TPCK/TPACK levels as taking time for teacher development.

In addition to these studies for enhancing the original Niess model, other studies focused on the development of TPCK/TPACK as a new and transformed and integrated knowledge:

- Koh and Divaharan (2011) presented a teaching process to advance the TPCK/TPACK of pre-service teachers during the teaching of information and communication technologies tools through three stages of fostering teachers' acceptance and technical proficiency, pedagogical modeling, and pedagogical application.
- Chai et al. (2014) created an Extended TPCK/TPACK framework by expanding the concept to include all types of educators. This expanded framework, based on ecological systems theory, provided a comprehensive assessment of different impact levels in information and communication integration.
- Lee and Kim (2017) shifted their previous views on TPCK/TPACK development from that of 2014. They presented a TPCK/TPACK-based instructional design model that “incorporates the distinctive, transformative, and integrative views of TPACK into a comprehensive actionable framework” (p. 1627). Their new model was threefold for helping pre-service teachers strengthen their TPCK/TPACK: Understand TPCK/TPACK, Experience TPCK/TPACK and Practice TPCK/TPACK.

3 The research study

Mishra and Koehler (2006) urged that developing the TPCK/TPACK of pre-service teachers should be one of the most important goals of teacher education. Yet, teacher education programs have varied considerably with respect to their coursework. This diversity leads to several questions: What is required to develop TPCK/TPACK? Where and how in the teacher preparation program is TPCK/TPACK developed? Is the pre-service teachers' TPCK/TPACK the same as that of teachers who have taught for several years? Some teacher education programs separate the teaching content coursework from the pedagogical courses. Some programs incorporate the technology-infused courses with the pedagogical courses while others have technology courses in computer science programs. What does seem to be consistent is that pre-service participants are provided with opportunities to practice, typically through micro teaching experiences and student teaching to provide opportunities for blending, reflecting on and solidifying the knowledge gained from the various courses.

The challenge for teacher educators is to assure that pre-service teachers complete a program that provides appropriate opportunities for developing the knowledge that adequately integrates their knowledge of technologies with their knowledge of the specific

content and with their pedagogical knowledge about teaching and learning. If TPCK/TPACK is a transformed knowledge where the multiple subsets are rearranged, merged, organized, integrated and assimilated in such a way that none are any longer individually discernible (as some researchers propose), what expectations must be included in the teacher education program? This question was the one that the researchers in this study chose to consider. In essence, this study was aimed at revising the Niess' (2013) model that included both levels and components; the challenge was to identify and describe more detailed descriptors for determining the TPCK/TPACK of pre-service mathematics teachers – one that considered the impact of how multiple technologies might be integrated in the development of the pre-service teachers' TPCK/TPACK.

The case study model from among the qualitative research methods was used to gather in-depth and detailed information from multiple current and limited sources. Case descriptions or case themes were accordingly developed and presented (Creswell, 2007).

3.1 Participants

Seven pre-service teachers studying in the mathematics teaching department of a state university in Turkey volunteered to participate in the study, all of whom were in the last term of their undergraduate education. The enrollment of individuals in their final term of undergraduate education was based on the assumption that pre-service teachers' TPCK/TPACK was confirmed after reaching a sufficient PCK level in their program. These participants were coded as S₁, S₂, S₃, S₄, S₅, S₆, and S₇.

3.2 Data collection tools and processes

The pre-service teachers were informed about the purposes of the study, answering their questions prior to the beginning of the research. A semi-structured interview form, prepared with the opinions of four experts based on the model proposed by Niess (2013), gathered important preliminary data. This form consisted of four parts representing the four TPCK/TPACK components (Niess, 2005). For each component, six statements described the possible levels, totaling 24 statements. Pre-service teachers were asked to mark the most fitting responses representing their personal knowledge and to explain the reasons for their choices. Statements reflecting the levels for each component were presented randomly to eliminate any perception of a certain order. A researcher supported the pre-service teachers while they completed their forms and gave explanations about any misunderstood expressions.

Semi-structured interviews were then conducted with each pre-service teacher individually to clarify their answers in the interview form. Thus, more information was acquired about any unclear points and the relevant answers were adjusted. These interviews were between 10 and 35 min in length. The pre-service teachers were then asked to prepare a detailed lesson plan on any mathematics or geometry subject of their choice and to perform micro teaching in line with those lesson plans. Micro teaching sessions were conducted after the collection of the plans, lasting between 25 and 65 min. Finally, to clarify the outcomes of the micro teaching sessions, individual semi-structured interviews were conducted

with the pre-service teachers by appointment after the micro teaching sessions. Semi-structured interview questions were created with the consensus of three experts. Micro teaching interviews lasted between 10 and 25 min. Permissions were obtained prior to audio-recording and video-recording the micro teachings. The interviews were conducted one-on-one between researchers and participants. Table 1 presents the workflow for the various tasks of this research.

3.3 Data analysis

Descriptive analysis was used to analyze the data. Audio-recordings were transcribed, summarized, and interpreted. Answers to the semi-structured interview forms, interviews, lesson plans, and micro teaching sessions were examined separately and coded within the framework of the four central components as follows:

- 1- An overarching conception about the purposes for incorporating technology in teaching subject matter topics,
- 2- Knowledge of students' understandings, thinking, and learning in subject matter topics with technology,
- 3- Knowledge of curriculum and curricular materials that integrate technology in learning and teaching subject matter topics, and
- 4- Knowledge of instructional strategies and representations for teaching and learning subject matter topics with technologies.

The six levels of pre-recognizing, recognizing, acceptance, adapting, exploring, and advancing were also considered. The opinions of three experts were obtained for the analysis. Where there was disagreement, it was discussed and consensus was reached.

4 Results

The research findings considered the six TPCK/TPACK levels. While there were no results from any participants for some components and levels, in some cases there were results from more than one participant for a single component or level.

Table 1 Workflow of the tasks involved in this research

Completed Tasks	Hours	Week
Providing information about the work	2 h	1 week
Application of the semi-structured interview form	2 h	1 week
Post-form interviews	3 h	1 week
Preparation of lesson plans	Preparation at home	2 week
Micro teaching	10 h	4 week
Post-micro teaching interviews	3 h	2 week
Total	20 h + preparation at home	11 week

4.1 Pre-recognizing

Since the participants knew about and practiced the use of technology in mathematics education, data for this particular level were not gathered in this study.

4.2 Recognizing

A teacher at the *recognizing* level is aware of the technology. However, technology knowledge, which is subject-specific, has not yet been transferred to the learning-teaching environment. Thus, the teacher at this level does not integrate technology into his/her courses and does not use it in class. At this level, no findings were obtained for the first or second components, that of teachers' overarching conceptions and their knowledge of students thinking and understanding. Yet, for the third component dealing with the curriculum, S_5 wrote the following on the semi-structured interview form:

Since the concepts of mathematics can be learned more easily with technology applications, more permanent learning can be achieved. With the visualization of mathematical concepts, learning is achieved beyond memorization. However, I'm not sure how to use the technology, because I don't know if every subject can be explained with technology.

On the other hand, S_3 stated the following in the interview conducted after administration of the form: "If a 12-h subject takes 20 h with the use of technology, then I can't make changes to my curriculum. If it rises to 20 h, it will take away from the time I would devote to other issues."

Regarding the fourth component, that of instructional strategies, in the semi-structured interview, S_3 said:

Even in the classroom, there are groups of students who are ignorant of technology. In order to have their participation, it is first of all necessary for someone with prior knowledge to teach them how to use technology and then, when it is learned, to carry out works via technology.

This expression was evaluated at the recognizing level, perhaps because of students' reluctance to use the technology as they had little understanding of the technology. In an interview conducted after administration of the form, S_6 expressed the challenges of teaching with technologies that were unfamiliar to students.

Technology is always used for gaming. That's why there is need for a teacher to guide the students. We need to teach them technology; we need to teach why we should use a tablet. I think it should also be taught how to use technology in mathematics. There may be a student who doesn't know it at all. I also think that it should not be a waste of time for that student. It can be taught in a different class; for instance, it could be a computer course. I think that, first of all, the attention of the

students should be attracted to technology in a different class. At first, it should be [via instructions], and then when the students know what to do, the instructions may become less frequent.

4.3 Accepting

The *accepting* level is one in which teachers know of the technologies that can be used in their courses and have a general positive or negative attitude about using technology or not. For the first component (their overarching conceptions), when the micro teaching and post-micro teaching interview data were examined for the accepting level, it was identified that technology could be used in limited settings of visualization, calculation, motivation, and the demonstration of things that are taught in the classroom. Four students voiced their positive opinions where they were willing to accept various uses for technologies in the classroom (S₁, S₃, S₅, S₆).

For the second component (that of students' understanding and thinking), S₅ recognized the value of technology use in the post-micro teaching interview: "I think it's useful because I visualize it with technology. Maybe it doesn't provide complete learning, but I think it's helpful. My use of technology motivated the students." There were also other similar statements:

Technology enabled the lectured group to form an idea while solving questions, and it also saved time. It would be better if I had solved even more examples with the application. I think technology supports and contributes to student understanding (S₇)

Similarly, S₆ indicated that "I think it helps students' learning."

For the third component (the curriculum), it was determined that from the micro teaching results the pre-service teachers wanted to integrate technology into their curriculum, but they had difficulties in doing so (S₃, S₅, S₆). In the interview that followed the micro teaching sessions, S₃ stated:

I was short of materials. I had problems with the application, I couldn't use the material as I had wanted. I had a hard time using and building the material. I didn't look at how the subject was presented in the curriculum, but I looked at the course of the subject; that is, I looked at the order. I benefited from the book in the curriculum. First, I looked at the available knowledge, and then what could be added on top of that. I examined the achievements section [of the curriculum].

On the form, S₄ stated: "Technology is an indispensable tool for mathematics, and technology should be integrated into lessons for continuous innovation. Still, although technology has such a wide range of uses, not every subject is compatible with technology support."

For the fourth component (instructional strategies), it was determined from the micro teaching data that technology is used in teacher-centered and teacher-directed

ways with strict instructions and for the purpose of summarizing what is being taught without going beyond the classroom instructions (S_3 , S_5 , S_6). In addition, it was determined that S_6 used technology in teaching but still had difficulties. In terms of the lesson plan, it was determined that the expressions “teaching by presentation,” “demonstration,” and “question and answer” were used in the context of this component, but detailed explanations were not provided (S_5). In each of these cases, it was determined that the student teachers had, in general, a mostly positive attitude and willingness to use technology in their instruction, but they often voiced some negative perceptions that were more likely focusing the use of the technologies in a teacher-entered instruction, rather than allowing student-centered instruction..

4.4 Adapting

At the *adapting* level, teachers have the means to use or reject technology in mathematics in the context of both learning and teaching. For the overarching conception component, in the post-form interview regarding the adapting level where teachers have the means to use or reject technology in mathematics in the context of both learning and teaching, S_3 said:

Without knowing a mathematical subject, it is not easy to apply it with the help of technology. The more you become aware of the subject, it becomes more understandable on the basis of technology. Technology can be instrumental in finding a different solution to the problem. It can be possible to see how mathematics can be used and explained with technology.

Meanwhile, S_6 said:

The teacher lectures, and we apply it. The student doesn't know what to do, how to use this tool, what to use it for, and I think the priority is that the student should have such knowledge. It should be teacher-centered, but the student should also be involved. First of all, I think mathematics should be directly taught... I don't think we should start with technology in the first place.

For students' understanding and conceptions, after the initial analysis of the micro teaching data, it was concluded that with the use of technology as a tool in learning mathematics, and as additional material in student learning, the changes of values and students' interpretations and the effects among the students on the levels of understanding and depth are doubtful (S_1 , S_3). In the interview that followed the micro teaching, S_3 stated: “First I lectured on the subject and gave the formulas. Later, I tried to discover formulas with technology.” On the form, S_1 said:

First, the subject must be known. Better learning is achieved by adding visuality to the solution [using technology], and by providing the ability to think from a different perspective. The individual also gains better learning by practicing personally. Technology contributes to the development of the individual's explorative side.

For the curriculum component, upon reviewing the micro teaching data, it was observed that S_1 used technological material mostly as a teaching tool. It was also observed that S_1 used technology as both an additional and a complementary tool, but more so as an additional tool. On the form, S_1 stated: “Teaching mathematics is considered to only consist of expressions. It doesn’t appeal to people because of the constant mathematical operations. But when combined with technology, the effect of visuality attracts interest to mathematics. Together, they are complementary.”

For the fourth component, the instructional strategies, S_1 used deduction (i.e., the presentation method) and conveyed information directly. The lectured group, on the other hand, filled out a worksheet by directly following the information and instructions. In S_3 ’s lesson plan, it was stated that he wanted to solve a problem with technology at the end of the lesson, but there was no information on the method for doing so. In response to the form, S_1 said:

Since individual levels of technology knowledge are not the same, it would be better to inform the lectured group and to make them proceed accordingly, and this will also avoid motivation problems among individuals. Being in line with the teacher is a better way for students to discover things themselves; in this way, by creating a sense of curiosity about discovering new things, the student will move forward with the help of the teacher.

4.5 Exploring

In the *exploring* level teachers actively integrate teaching and learning of mathematics with an appropriate technology. The students that demonstrated progress at this level were seen in the micro teaching data and considered their overarching conceptions for the first component, it was determined that S_2 and S_4 used technology to learn and teach, but mostly for learning purposes, and also in experimentation, application, and exploring. In the lesson plan of S_2 , there were statements about understanding the relationship with technology and reaching generalizations. On the form, S_4 stated:

Technology is not just a tool to be used to reinforce a subject that is taught. It is more common for technology to be used in mathematics for a subject to be discovered by the student, to go from part to whole with the inductive method, and finally to be used as a tool that the student can discover for himself. Technology is used both for exploration and to reinforce the subject being taught.

In the post-micro teaching interview, S_2 explained his views as follows:

I embodied it with the video. I showed the way to apply it in daily life. I used technology as a way to invent and for exploring. If I tried to have the student do it, we wouldn’t have had enough time and the students would have gotten bored. Therefore, I used ready-made materials for generalization purposes.

For the second component concerned with student understandings, it was observed from the micro teaching of S_2 and S_4 that, in developing a more solid understanding of mathematical ideas, their students discovered technology,

combined designs of technology with mathematical thinking and problem-solving, and enjoyed minor guidance and learning by discovery. S_1 expressed his opinion as follows on the administered form: “Since conceptual knowledge cannot be adequately expressed and reinforced in a group, solutions are only by means of memorizing. It is beneficial to consolidate conceptual knowledge with technology. Self-discovery helps strengthen conceptual knowledge.” Meanwhile, S_5 stated: “Technology is effective in learning by exploring mathematical concepts. It is necessary to ensure that discovery (the person doing it by himself with observation) is made with technology.”

Among the post-form interviews, the following statement of S_3 drew attention:

We know many concepts, but in a verbal way. Let me say this, there is not much in the image, but when we see it in GeoGebra, for example, information about the concept is formed. After the teacher gives information about how the situation occurs, the student also needs to show it. What I mean by exploring is actually exploring that knowledge... getting out of the monotony and revealing some other things... Something new that comes about with the inclusion of the technology as a tool in the content.

In the post-micro teaching interview, S_2 evaluated micro teaching as follows:

The student was at the center of the process, the student helped them learn. I think they learned the conceptual meaning. It became more permanent and meaningful as they combined technology with previous understandings. The student himself reached the knowledge and achieved practicality in terms of time.

For the third component, the curriculum, in the post-form interview, S_7 expressed his views by saying:

In the mathematics curriculum, it is written that information and communication technologies are used in some of the acquisitions. How can these be developed further, how can information and communication technologies be used? They can be used, but is there a need for guidance? For example, in the books it is said that the GeoGebra program is used. Could other programs also be used? Or, how much material is there in that program, for example, that software? How sufficient is the material, what are the shortcomings of the material, how can we improve it? In my opinion, a different perspective can be obtained by using several different materials for the same subject. I think one of those should be the teacher’s guidance. We should use technologies to solve problems. Since we have such an opportunity, we should do that.

In the post-micro teaching interview, S_2 expressed his view as follows:

I used the textbook, but the book was old. I took the applications from the state’s own book, I didn’t look at it in terms of duration, I created examples by looking at the achievements, I created my own material.

For the fourth instructional strategies component, it was observed that while S_2 and S_4 were integrating planning and implementation in learning and teaching, and technology in reflection, they focused on their students' understandings of mathematics and they carried out student-centered activities. Furthermore, it was also observed that they minimized the impact of the difficulties and used multiple teaching strategies (both deductive and inductive) via technology. S_2 and S_4 made use of teaching by means of innovation with technology in their lesson plans. On the form, S_5 stated:

Multiple teaching strategies should be used in the use of technology while teaching mathematical concepts because, while there is an understanding of the concept by going from the part to the whole in a geometry calculation, some subjects can be learned by going from the whole to the parts.

In the post-form interview, S_4 said:

Our aim is to teach mathematics, and to teach it by integrating technology while teaching mathematics, so that the students will understand... I think this way because it is aimed to understand a subject or learn by self-discovery, that is, by experiencing, by doing it. There are some subjects, for example, where the whole subject has to be given. However, maybe the student hasn't seen anything that could possibly form the basis of that subject before, and when that subject is explained, the deductive method might be more appropriate. So, I thought that it would be appropriate to use both methods.

4.6 Advancing

In the *advancing* level, the teacher prefers to use technology in lessons and evaluates the results. At this level, for the overarching conception component, S_2 said:

The concepts in the mathematics education process are generally abstract and difficult to understand. For this reason, the use of technology will provide students with a more understandable educational environment in terms of concretization and teaching of the concept. Active dynamic mathematical technologies create more solid and effective learning among students. Instead of understanding mathematics via static and abstract concepts, it is beneficial to make it more concrete and more dynamic by making use of technology.

Similarly, S_5 said:

Concepts can be made more understandable by explaining an abstract concept with the technology applications. 3D geometric shapes can be grasped better with technology applications. When the concepts that are difficult to understand are visualized and concretized with technology applications, it becomes possible to understand the mathematical concepts. The process is facilitated after the concepts are understood.

In the post-form interview, S_2 said:

...technology can be used in areas such as discovery and application. High-level process skills and applications and thinking and logical judgments beyond knowledge are required in these areas. There is a need to look from a holistic point of view; it is necessary to look at mathematics from a holistic view. In order to learn a mathematical concept, one needs to look at it from a broad perspective. So, I think that technology is advantageous at the point that it opens to that broad perspective of ours.

For the second component, the students' understandings, S_2 provided a good example:

One of the main purposes in mathematics education is to gain problem-solving skills for the student that will enable him or her to make life easier in real life. Based on this idea, it is necessary to develop high-level thinking skills and to carry out activities related to this during the education process. For this reason, the use of technologies in the process will both help develop the ability to relate and provide concrete learning by providing visualization. When the process works in this way, students will more meaningfully learn concepts and the relations between concepts in a more organized way. After this sense-making and organizing process, higher-order thinking and real-life problem-solving skills will develop to a larger degree.

In the post-survey interview, S_4 said:

It is useful to apply technology in these steps due to the facts that today most teaching is project-based, and the deductive inductive method is used, and it involves posing problems, exploring, and solving steps. Since these are the most used methods today, I thought it would be appropriate to use them, especially for having the students strengthen and reinforce deep understanding.

For the third component, the curriculum, S_2 wrote the following on the administered form:

Teaching programs should be planned in a way to provide a technology-supported education environment. In order to create effective and permanent learning in this process, sufficient time should be allocated. And in order to achieve these activities, teachers should be provided both guidance and developmental training. This process should not be static, and it should be planned as a constantly changing, self-developing, active, and dynamic process. It would be right to develop the teaching method or the technology emphasis in the program actively and dynamically, and also in a way that will meet the requirements and needs of the age. However, the pre-service teachers' beliefs in it is essential for the process to be successful.

In the post-form interview, S_6 stated:

Technology education should be given beforehand, and new courses should be created for the mathematics curriculum. If mathematics lessons will be given

along with technology support, at first technology education must be given, and then technology intertwined with mathematics must be taught... If my curriculum requires amendment, I will amend it. I don't think strictly sticking to the curriculum is a good idea. When it is supported with technology, it will be materialized; I think it will be better this way. I think the content change could be in this direction.

In the fourth component, data could not be obtained in this study, so the expressions in the Niess (2013) model were used to describe this component.

5 Discussion

The understanding of TPCK/TPACK has advanced over several years through the Niess' research efforts in the search for clarifying the transformation of teachers' knowledge for teaching in the digital age. Throughout 2005–2018, the efforts described components, levels and themes but the key is the working group used for the identifications as summarized in Table 2.

As noted in Table 2, only components were considered in, 2005; only levels in 2007; themes, descriptors, and levels in 2009; and components and levels in, 2013. When we examined this information on a more in-depth level, work was carried out with pre-service teachers in, 2005 and in the other years with in-service teachers. From this point of view, considering that in the research conducted by Niess (2005) with pre-service teachers, there were only components, but no levels, and in the 2007, 2009, and 2013 research conducted with in-service teachers, pre-service teachers were not included, the following question arose: Can the four-component, five-level model of Niess (2013) be adapted for pre-service mathematics teachers?

Through this study, the model of Niess (2013) was revised for pre-service mathematics teacher with four components and six levels. Table 3 presents the revised model to more accurately describe pre-service mathematics teachers' TPCK/TPACK development.

Table 3 presents the research study's analysis of the descriptions for the six TPCK/TPACK levels organized by the four TPCK/TPACK components that were identified by Niess (2005). While the results provide some of the comments that were received, this table used all the pre-service teacher comments from the multiple interviews, lesson plans and micro teaching work with the pre-service teachers. As noted in the table, statements were not possible for the *pre-recognizing*

Table 2 TPCK/TPACK components, levels, themes, and study groups by years

Years	Components	Levels	Themes	Working Group
2005	x	-	-	Pre-service Teacher
2007	-	x	-	In-service Teacher
2009	-	x	x	In-service Teacher
2013	x	x	-	In-service Teacher
2018	x	x	-	In-service Teacher

Table 3 TPCK/TPACK development model for pre-service mathematics teachers

Levels	Identifiers of “Overarching Conception” component
Pre-recognizing	<ul style="list-style-type: none"> • It is not known for what purpose technologies can be used in mathematics education
Recognizing	<ul style="list-style-type: none"> • Technologies are used to make a difference only in the teaching of complex mathematical concepts and operations • It is thought that technology is not very necessary, and efficiency does not decrease without of technology • Use of technology is not embraced unless it is compulsory
Accepting	<ul style="list-style-type: none"> • Technology is used for educational purposes in the areas of visualization, motivation, calculation, supporting examples, demonstrating subjects taught in the exact same way with technology, making learning fun, concretizing, reinforcing, making a difference, increasing memorability, and drawing attention
Adapting	<ul style="list-style-type: none"> • Technology in learning and teaching is used to discover, try out, and apply new approaches to the subject after learning the subject of mathematics in the first place • Technology is used for both learning and teaching purposes, but mostly for teaching purposes
Exploring	<ul style="list-style-type: none"> • Technology is used for both learning and teaching purposes, but mostly for learning purposes • Technologies are used as incentives and support in learning and teaching mathematics (mostly in learning), exploring, understanding relationships, and reaching generalizations
Advancing	<ul style="list-style-type: none"> • Technologies are used by students to explore, experiment, and practice the mathematical concepts and operations so that they can be understood for learning purposes
	Identifiers of “Learning, Thinking, and Understanding of the Students” component
Pre-recognizing	<ul style="list-style-type: none"> • It is not known how technologies can be used for learning purposes in mathematics
Recognizing	<ul style="list-style-type: none"> • Technologies that can be used in mathematics lessons are known. However, such technologies are not considered as necessary in learning mathematics since they do not improve mathematics learning, mathematical thinking, or reasoning skills among the students. Therefore, they are not used for thinking, understanding, and learning among the students regarding technology
Accepting	<ul style="list-style-type: none"> • The lectured group uses technology by imitation and does exactly what is said • The lectured group focuses on the problems related to technology use rather than the content • Technology may not provide complete learning, but it helps learning • The lectured group is thought to be motivated by technology • Technology will provide advantages to the lectured group over time • There is concern that the lectured group’s attention may be focused on the technology rather than the subject
Adapting	<ul style="list-style-type: none"> • When the lectured group uses technology after learning about mathematics (after former understanding), their understanding of mathematical ideas improves • It is doubtful that the use of technology, a tool for learning mathematics, has an impact on the deep understanding of the group • Technology is used mostly as an addition and sometimes as a complementary tool in the learning of the lectured group • By adding visuality to solutions with technology, the ability to think from a different perspective is provided. Thus, the lectured group’s understanding of mathematical ideas develops

Table 3 (continued)

Levels	Identifiers of “Overarching Conception” component
Exploring	<ul style="list-style-type: none"> • The lectured group explores with technology in order to develop a deeper understanding of mathematical ideas. They combine technology designs with mathematical thinking and problem-solving • The lectured group, working with technology, needs little guidance • There is participation of the lectured group while working with technology, and the lectured group is at the center of the process • In order to strengthen the conceptual knowledge of the lectured group, the students are enabled to explore with technology • The subject is learned intertwined with technology
Advancing	<ul style="list-style-type: none"> • Technology is used as a learning tool to develop a deep understanding within the lectured group through high-level thinking activities such as project-based, problem-solving, and decision-making activities • Technology creates deeper, more permanent, and more effective learning in the group
	Identifiers of “Curriculum and Curriculum Materials” component
Pre-Recognizing	<ul style="list-style-type: none"> • It is not known how technologies can be integrated into the subjects of the mathematics curriculum
Recognizing	<ul style="list-style-type: none"> • It is thought that without technology, the mathematics curriculum is focused on memorizing rules and operations, and that mathematics can go beyond memorization with technology • The teacher is unsure about how to use technology in the curriculum • It is thought that the content of mathematics will change when technologies are used, and this idea is resisted • It is thought that deviations from and changes to the curriculum should not be made • Curriculum including technology is examined only superficially • It is thought that tangible material can be used instead of technology
Accepting	<ul style="list-style-type: none"> • It is difficult to conduct applications with technologies in many different subjects in the mathematics curriculum • Technology motivates the lectured group in terms of specific mathematics subjects • The technology is used by the person teaching the subject • The lecturer has difficulties with technological materials and applications and cannot use technology as originally desired • The curriculum is reviewed; however, it is difficult to integrate technology into the curriculum
Adapting	<ul style="list-style-type: none"> • Technological materials are used as both teaching and learning (mostly teaching) tools • Technologies are mostly additional and sometimes supplementary tools in the mathematics curriculum
Exploring	<ul style="list-style-type: none"> • Technological materials are used as both teaching and learning (mostly learning) tools • Worksheets are used for learning with technology • Technologies have a unifying role as problem-solving tools in the current curriculum to improve the learning of the lectured group • New ideas are sought in the curriculum regarding the use of technologies • Different materials are developed with technology; thus, different perspectives are obtained • The curriculum in which technology is integrated is examined in detail
Advancing	<ul style="list-style-type: none"> • When technology is integrated, curriculum reorganization and constant changes are made and new courses are created in order to benefit from effective and efficient learning • Technology is used constantly, even if it leads to changes in content
	Identifiers of “Instructional Strategies and Presentations” component

Table 3 (continued)

Levels	Identifiers of “Overarching Conception” component
Pre- Recognizing	<ul style="list-style-type: none"> • It is not known how technologies can be used for teaching purposes in mathematics lessons
Recognizing	<ul style="list-style-type: none"> • It is thought that technology and mathematics should be taught separately • It is thought that more complex and difficult concepts can be introduced by supporting the initial learning with technology activities • Technology is known but is not used for teaching
Accepting	<ul style="list-style-type: none"> • When technology is combined with content, some difficulties and problems with classroom management and classroom assessment may arise. Therefore, technology is used by the person who teaches it in a limited way for the purpose of summarizing the subject and without going beyond strict instructions • Technology is teacher-centered and teacher-led • The lecturer uses technology in lectures but has difficulty
Adapting	<ul style="list-style-type: none"> • The lecturer primarily needs to give direct information with the presentation method (deductive) in technology activities • With technology, exploration, practice, and experimentation occur in teaching
Exploring	<ul style="list-style-type: none"> • Student-centered activities are carried out by focusing on the group’s understanding of mathematics while integrating technology in planning, implementation, and also reflection in learning and teaching • The impact of difficulties is minimized • Multiple teaching strategies (both deductive and inductive) are used with technology. The students in the lectured group discover and reinforce themselves • In teaching with technology, the person who explains is in the position of a guide. He or she activates the group with questions rather than transferring information
Advancing	<ul style="list-style-type: none"> • The main method is mathematical exploration with technology • Lessons are taught under the control of the lectured group (student-centered), and there are inquiries under the control of the lectured group

level since all of the participants had experiences with using technologies with mathematics. All other levels were represented by the students.

At the *recognizing* level, the statements primarily see technologies as not integral to mathematics instruction. Only at the instructional strategies and presentation component is technology considered as supporting the initial learning of more complex and difficult concepts. Otherwise, technology is not considered as being needed in learning mathematics since technologies “do not improve mathematics learning, mathematical thinking, or reasoning skills among the students.” Furthermore, there was a concern about technologies improving mathematical thinking and reasoning.

At the *accepting* level, the visualization and motivation capabilities of technologies are seen as having potential usefulness in learning mathematics. But the concern was that students’ attention would be on the technology rather than the mathematics. And, at this level the statements present a concern for classroom management which forces teacher-centered and teacher-led instruction.

At the *adapting* level there is a recognition of visualization capabilities but continues to see it used in as a teacher-centered activity where the technology is

more of an additional tool, supplementary to the instruction. The technological challenge appears as requiring exploration, practice and experimentation – which must happen when teaching.

At the *exploring* level, the notions appear to shift to student learning with the technologies. The overarching conceptions see technologies as useful incentives that support learning, exploring, understanding relationships and reaching generalizations. At this level worksheets are useful for learning with technology and there is a recognition of the lack of curricular materials regarding the use of technologies for learning mathematics, particularly with student-centered activities. These statements seem to move the incorporation of technologies to student-centered work where students work in groups and share ideas through explorations with the technologies.

At the *advancing* level the conceptions are that technologies used by students allow them to explore, experiment and practice the mathematics concepts and operation so that they can be understood. This work builds a deeper understanding, an understanding that is more effective. There is a recognition that if technology is used consistently it might lead to changes in the content. The mathematics instruction is seen as mathematical exploration with technology that involved high-level thinking activities such as project-based, problem-solving and decision-making activities.

6 Conclusion

Technological developments have deeply affected education and therefore changed teaching competencies. A good teacher is expected to integrate the CK, PK and TK into education. In this direction, the concept of TPCK/TPACK comes to the fore. It is expected that pre-service teachers who are on their way to becoming teachers, will also have activated the establishment of their TPCK/TPACK through their pre-service teacher preparation program. An important recognition in this study was that the theoretical vision of TPCK/TPACK was assumed to be an integrated knowledge type that resulted through the amalgam of technology, pedagogy and content in a homogeneous way.

When examining studies that considered TPCK/TPACK as an integrated, homogeneous transformative model, researchers primarily have conducted their research into TPCK/TPACK development with in-service teachers, rather than with novice pre-service teachers. Keeping in mind the nature of TPCK/TPACK envisioned as a homogenous transformed knowledge, it is important to think of the evolving nature of TPCK/TPACK knowledge, a knowledge that continues to grow and develop as changes happen in content, pedagogy and technology. The indicators for in-service teachers are certainly a result of the evolution of their TPCK/TPACK as they have had more advanced experiences in classroom environments. The TPCK/TPACK of in-service teachers represents a more matured knowledge beyond that of the pre-service teacher preparation programs. Thus, the TPCK/TPACK indicators resulting from studies with in-service teachers are likely different from similar research with pre-service teachers.

The model described in this study provides teacher educators with the opportunity to examine the development of pre-service teachers according to the TPCK/

TPACK components on a level basis considering the less advanced teaching knowledge and experiences of pre-service teachers. Focusing on their TPCK/TPACK levels with respect to each of the components enables teacher educators and researchers to view the nature of TPCK/TPACK at the novice level. Thus, the revised model in this study provides indicators to guide additional researchers to enhance the TPCK/TPACK development of pre-service teachers more clearly, toward recommending pre-service teaching experiences for enhancing their evolving TPCK/TPACK.

Data availability The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations

Conflict of interest There is no conflict of interest regarding this publication.

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