



Neutrosophic soft compact spaces

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Abstract

In this paper, some cover types are defined on neutrosophic soft sets and the concept of compactness on neutrosophic soft topological spaces is presented using these covers. We also defined the concept of locally compactness on neutrosophic soft topological spaces and some properties are investigated.

Keywords Neutrosophic set · Neutrosophic soft set · Neutrosophic soft cover neutrosophic soft compactness · Neutrosophic soft locally compactness

Mathematics Subject Classification 54D10 · 54D15 · 54D45 · 54E45

1 Introduction

In recent years; new set structures are defined which are different from classical set structure which brings a new approach to many problems that cannot be solved in science, social, engineering, medical and similar sciences. The fuzzy set theory [20], intuitionistic set theory [1], soft set theory [12], rough set theory [15] and neutrosophic set theory [17, 18], which lead these set structures, have caused intense interest in all fields of science. Many different studies are presented in all sub-branches of mathematics belonging to these set structures [6–8, 16, 19]. The fuzzy soft set, intuitionistic fuzzy set, intuitionistic fuzzy soft set, neutrosophic soft set combinations of these defined set structures are presented in many important studies [2–5, 9–11, 13, 14].

The class of compact spaces, which is one of the main subjects of classical mathematics, contains closed, bounded subsets of Euclidean spaces and carries the most important proper-

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ties of these sets in their structure. Therefore, it is important to study the concept of compact spaces in these new set theories.

In this study, the concept of compactness on neutrosophic soft topological spaces is presented. Firstly, various neutrosophic soft cover concepts are defined. Later, using these notions, neutrosophic soft compact space and neutrosophic soft set are defined. We also proved necessary theorems. In addition, neutrosophic soft compactness concept and neutrosophic soft separation axioms have been associated. Finally, neutrosophic soft locally compactness concept is defined and some properties are investigated.

2 Preliminary

In this section, the basic definitions and theorems of neutrosophic soft sets are given by using [3, 9, 10, 13, 14] for a better understanding of future sections.

Definition 2.1. [9] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the set of all neutrosophic sets of X . A neutrosophic soft set $\tilde{F}_E$ over X is a set defined by a set valued function $\tilde{F}$ representing a mapping $\tilde{F} : E \rightarrow P(X)$ where $\tilde{F}$ is called approximate function of $\tilde{F}_E$. In other words, $\tilde{F}_E$ is a parameterized family of some elements of the set $P(X)$ and therefore it can be written as a set of ordered pairs,

$$\tilde{F}_E = \left\{ \left(e, \langle x, T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \rangle : x \in X \right) : e \in E \right\}$$

where $T_{\tilde{F}(e)}(x), I_{\tilde{F}(e)}(x), F_{\tilde{F}(e)}(x) \in [0, 1]$ called the truth-membership, indeterminacy-membership, falsity-membership function of $\tilde{F}(e)$, respectively. Since supremum of each T, I, F is 1 so the inequality $0 \leq T_{\tilde{F}(e)}(x) + I_{\tilde{F}(e)}(x) + F_{\tilde{F}(e)}(x) \leq 3$ is obvious.

Throughout this work, $NSS(X_E)$ refers to the class of all neutrosophic soft sets over X .

Definition 2.2. [3] Let $\tilde{F}_E \in NSS(X_E)$. The complement of $\tilde{F}_E$ is denoted by $\tilde{F}_E^c$ and is defined by:

$$\tilde{F}_E^c = \left\{ \left(e, \langle x, F_{\tilde{F}(e)}(x), 1 - I_{\tilde{F}(e)}(x), T_{\tilde{F}(e)}(x) \rangle : x \in X \right) : e \in E \right\}$$

Obvious that, $(\tilde{F}_E^c)^c = \tilde{F}_E$.

Definition 2.3. [10] Let $\tilde{F}_E, \tilde{G}_E \in NSS(X_E)$. Then $\tilde{F}_E$ is a subset of $\tilde{G}_E$, denoted by $\tilde{F}_E \subseteq \tilde{G}_E$. If for $\forall e \in E, \forall x \in X$;

1. $T_{\tilde{F}(e)}(x) \leq T_{\tilde{G}(e)}(x)$
2. $I_{\tilde{F}(e)}(x) \leq I_{\tilde{G}(e)}(x)$
3. $F_{\tilde{F}(e)}(x) \geq F_{\tilde{G}(e)}(x)$.

$\tilde{F}_E$ equals to $\tilde{G}_E$ if $\tilde{F}_E \subseteq \tilde{G}_E$ and $\tilde{G}_E \subseteq \tilde{F}_E$. It is denoted by $\tilde{F}_E = \tilde{G}_E$.

Definition 2.4. [13] Let $\tilde{F}_E, \tilde{G}_E \in NSS(X_E)$. Then, their union is denoted by $\tilde{F}_E \cup \tilde{G}_E = \tilde{H}_E$ and is defined by:

$$\tilde{H}_E = \left\{ \left(e, \langle x, T_{\tilde{H}(e)}(x), I_{\tilde{H}(e)}(x), F_{\tilde{H}(e)}(x) \rangle : x \in X \right) : e \in E \right\}$$

where, $T_{\tilde{H}(e)}(x) = \max\{T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)\}$, $I_{\tilde{H}(e)}(x) = \max\{I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x)\}$,
 $F_{\tilde{H}(e)}(x) = \min\{F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)\}$

Definition 2.5. [13] Let $\tilde{F}_E, \tilde{G}_E \in NSS(X_E)$. Then, their intersection is denoted by $\tilde{F}_E \cap \tilde{G}_E = \tilde{H}_E$ and is defined by:

$$\tilde{H}_E = \left\{ \left(e, \langle x, T_{\tilde{H}(e)}(x), I_{\tilde{H}(e)}(x), F_{\tilde{H}(e)}(x) \rangle : x \in X \right) : e \in E \right\}$$

where, $T_{\tilde{H}(e)}(x) = \min\{T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)\}$, $I_{\tilde{H}(e)}(x) = \min\{I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x)\}$,

$$F_{\tilde{H}(e)}(x) = \max\{F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)\}$$

Definition 2.6. [13] Let $\tilde{F}_E, \tilde{G}_E \in NSS(X_E)$. Then, $\tilde{F}_E$ difference $\tilde{G}_E$ operation on them is denoted by,

$$\tilde{F}_E \setminus \tilde{G}_E = \tilde{H}_E$$

and is defined by $\tilde{H}_E = \tilde{F}_E \cap \tilde{G}_E^c$ as follows:

$$\tilde{H}_E = \left\{ \left(e, \langle x, T_{\tilde{H}(e)}(x), I_{\tilde{H}(e)}(x), F_{\tilde{H}(e)}(x) \rangle : x \in X \right) : e \in E \right\}$$

where, $T_{\tilde{H}(e)}(x) = \min\{T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)\}$, $I_{\tilde{H}(e)}(x) = \min\{I_{\tilde{F}(e)}(x), 1 - I_{\tilde{G}(e)}(x)\}$,

$$F_{\tilde{H}(e)}(x) = \max\{F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)\}$$

Definition 2.7. [13] $\tilde{F}_E \in NSS(X_E)$. Then, $\tilde{F}_E$ is said to be,

1. a null neutrosophic soft set if $T_{\tilde{F}(e)}(x) = 0$, $I_{\tilde{F}(e)}(x) = 0$, $F_{\tilde{F}(e)}(x) = 1$; $\forall e \in E$, $\forall x \in X$. It is denoted by 0_{X_E} .
 2. an absolute neutrosophic soft set if $T_{\tilde{F}(e)}(x) = 1$, $I_{\tilde{F}(e)}(x) = 1$, $F_{\tilde{F}(e)}(x) = 0$; $\forall e \in E$, $\forall x \in X$. It is denoted by 1_{X_E} .
- Clearly, $0_{X_E}^c = 1_{X_E}$ and $1_{X_E}^c = 0_{X_E}$.

Proposition 2.1. [13] Let $\tilde{F}_E, \tilde{G}_E \in NSS(X_E)$. Then,

1. $[\tilde{F}_E \cup \tilde{G}_E]^c = \tilde{F}_E^c \cap \tilde{G}_E^c$
2. $[\tilde{F}_E \cap \tilde{G}_E]^c = \tilde{F}_E^c \cup \tilde{G}_E^c$

Definition 2.8. [13] Let τ^{NSS} be a subset $NSS(X_E)$. Then τ^{NSS} is called a neutrosophic soft topology on X if,

1. $0_{X_E}, 1_{X_E}$ belongs to τ^{NSS}
2. $\tilde{F}_E, \tilde{G}_E \in \tau^{NSS}$ then $\tilde{F}_E \cap \tilde{G}_E \in \tau^{NSS}$
3. $(\tilde{F}_E)_i \in \tau^{NSS}$ $\forall i \in I$, then $\bigcup_{i \in I} (\tilde{F}_E)_i \in \tau^{NSS}$

The triplet $\left(X, \tau^{NSS}, E \right)$ is called a neutrosophic soft topological space. The members of τ^{NSS} are said to be neutrosophic soft open in X .

Definition 2.9. [13] Let $\left(X, \tau^{NSS}, E \right)$ be a neutrosophic soft topological space over X and $\tilde{F}_E \in NSS(X_E)$.

- (a) Then, $\tilde{F}_E$ is said to be neutrosophic soft closed set iff its complement is a neutrosophic soft open set.
- (b) Then, the neutrosophic soft interior of $\tilde{F}_E$, denoted $(\tilde{F}_E)^\circ$, is defined as the neutrosophic soft union of all neutrosophic soft open subsets of $\tilde{F}_E$.
 $(\tilde{F}_E)^\circ = \bigcup \{ \tilde{G}_E \in NSS(X_E) : \tilde{G}_E \text{ is a neutrosophic soft open set and } \tilde{G}_E \subset \tilde{F}_E \}$
 Clearly, $(\tilde{F}_E)^\circ$ is the biggest neutrosophic soft open set that is contained by $\tilde{F}_E$.
- (c) Then, the neutrosophic soft closure of $\tilde{F}_E$, denoted $\overline{(\tilde{F}_E)}$, is defined as the neutrosophic soft intersection of all neutrosophic soft closed supersets of $\tilde{F}_E$.
 $\overline{(\tilde{F}_E)} = \bigcap \{ \tilde{K}_E \in NSS(X_E) : \tilde{K}_E \text{ is a neutrosophic soft closed set and } \tilde{F}_E \subset \tilde{K}_E \}$
 Clearly, $\overline{(\tilde{F}_E)}$ is the smallest neutrosophic soft closed set that contains $\tilde{F}_E$.

Definition 2.10. [14] The neutrosophic soft set $x_{(\alpha, \beta, \gamma)}^e$ is called a neutrosophic soft point, for every $x \in X$, $0 < \alpha, \beta, \gamma \leq 1$, $e \in E$, and is defined as follows:

$$x_{(\alpha, \beta, \gamma)}^e(e')(y) = \begin{cases} (\alpha, \beta, \gamma) & \text{if } e' = e \text{ and } y = x, \\ (0, 0, 1) & \text{if } e' \neq e \text{ or } y \neq x \end{cases}$$

Definition 2.11. [14] Let $\tilde{F}_E$ be a neutrosophic soft set over X . We say that $x_{(\alpha, \beta, \gamma)}^e \in \tilde{F}_E$ read as belonging to the neutrosophic soft set $\tilde{F}_E$ whenever $\alpha \leq T_{\tilde{F}(e)}(x)$, $\beta \leq I_{\tilde{F}(e)}(x)$ and $\gamma \geq F_{\tilde{F}(e)}(x)$.

Definition 2.12. [14] Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $\tilde{F}_E \in NSS(X_E)$. Then, $\tilde{F}_E$ is called a neutrosophic soft neighborhood of the neutrosophic soft point $x_{(\alpha, \beta, \gamma)}^e \in \tilde{F}_E$, if there exists a neutrosophic soft open set $\tilde{G}_E$ such that $x_{(\alpha, \beta, \gamma)}^e \in \tilde{G}_E \subset \tilde{F}_E$.

Theorem 2.1. [14] Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space and $\tilde{F}_E \in NSS(X_E)$. Then, $\tilde{F}_E$ is a neutrosophic soft open set if and only if $\tilde{F}_E$ is a neutrosophic soft neighborhood of its neutrosophic soft points.

Definition 2.13. [14] Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $x_{(\alpha, \beta, \gamma)}^e$ and $y_{(\alpha', \beta', \gamma')}^{e'}$ are distinct neutrosophic soft points. If there exist neutrosophic soft open sets $\tilde{F}_E$ and $\tilde{G}_E$ such that,

$$\begin{aligned} x_{(\alpha, \beta, \gamma)}^e \in \tilde{F}_E, x_{(\alpha, \beta, \gamma)}^e \cap \tilde{G}_E &= 0_{X_E} \text{ and} \\ y_{(\alpha', \beta', \gamma')}^{e'} \in \tilde{G}_E, y_{(\alpha', \beta', \gamma')}^{e'} \cap \tilde{F}_E &= 0_{X_E} \end{aligned}$$

Then, $(X, \overset{NSS}{\tau}, E)$ is called a neutrosophic soft T_1 -space.

Definition 2.14. [14] Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X and $x_{(\alpha, \beta, \gamma)}^e$ and $y_{(\alpha', \beta', \gamma')}^{e'}$ are distinct neutrosophic soft points. If there exist neutrosophic soft open sets $\tilde{F}_E$ and $\tilde{G}_E$ such that,

$$x_{(\alpha, \beta, \gamma)}^e \in \tilde{F}_E, \quad y_{(\alpha', \beta', \gamma')}^{e'} \in \tilde{G}_E \quad \text{and} \quad \tilde{F}_E \cap \tilde{G}_E = 0_{X_E}$$

Then, $\left(X, \begin{smallmatrix} NSS \\ \tau \end{smallmatrix}, E\right)$ is called a neutrosophic soft T_2 -space (neutrosophic soft Hausdorff space).

Definition 2.15. [14] A neutrosophic soft topological space $\left(X, \begin{smallmatrix} NSS \\ \tau \end{smallmatrix}, E\right)$ over X is called a neutrosophic soft normal space if for every pair of disjoint neutrosophic soft closed sets $\tilde{F}_E^1, \tilde{F}_E^2$, there exists disjoint neutrosophic soft open sets $\tilde{G}_E^1, \tilde{G}_E^2$ such that $\tilde{F}_E^1 \subset \tilde{G}_E^1$ and $\tilde{F}_E^2 \subset \tilde{G}_E^2$.

$\left(X, \begin{smallmatrix} NSS \\ \tau \end{smallmatrix}, E\right)$ is said to be a neutrosophic soft T_4 -space if it is both a neutrosophic soft normal and neutrosophic soft T_1 -space.

Definition 2.16. [14] Let $\left(X, \begin{smallmatrix} NSS \\ \tau \end{smallmatrix}, E\right)$ be a neutrosophic soft topological space over X and $\tilde{F}_E$ be an arbitrary neutrosophic soft set. Then $\tau_{\tilde{F}_E} = \left\{(\tilde{F}_E \cap \tilde{H}_E) : \tilde{H}_E \in \tau\right\}$ is said to be neutrosophic soft topology on $\tilde{F}_E$ and $\left(\tilde{F}_E, \begin{smallmatrix} NSS \\ \tau_{\tilde{F}_E} \end{smallmatrix}, E\right)$ is called a neutrosophic soft topological subspace of $\left(X, \begin{smallmatrix} NSS \\ \tau \end{smallmatrix}, E\right)$.

3 Neutrosophic soft compact spaces

In this section, we give a definition of neutrosophic soft compact space on a neutrosophic soft set and its related properties.

Definition 3.1. (a) Let I be a set $\tilde{v} = \left\{(\tilde{F}_E)_i : i \in I\right\}$ a family of subset of $NSS(X_E)$. If

$\tilde{F}_E \subset \bigcup_{i \in I} (\tilde{F}_E)_i$, then $\tilde{v}$ is called a cover of the neutrosophic soft subset 1_{X_E} . If

$\tilde{F}_E = 1_{X_E}$, then $\tilde{v}$ is called a neutrosophic soft cover of 1_{X_E} .

(b) If the number of elements of $\tilde{v}$ is finite or countable, then $\tilde{v}$ is called a finite or countable neutrosophic soft cover, respectively.

(c) Let $\tilde{v} = \left\{(\tilde{F}_E)_i : i \in I\right\}$ be a neutrosophic soft cover of 1_{X_E} . If, $\tilde{v}' = \left\{(\tilde{F}_E)_i : i \in I', I' \subset I\right\}$ is the neutrosophic soft subfamily of $\tilde{v}$ and it is also a cover of 1_{X_E} , then $\tilde{v}'$ is called a neutrosophic soft subcover of $\tilde{v}$.

(d) A the family $\tilde{v}$ of neutrosophic soft sets is called the centralized neutrosophic soft if every finite neutrosophic soft subfamily of $\tilde{v}$ has non-empty intersection.

Definition 3.2. (a) Let $\left(X, \begin{smallmatrix} NSS \\ \tau \end{smallmatrix}, E\right)$ be a neutrosophic soft topological spaces over X and $\tilde{v}$ be a neutrosophic soft cover of 1_{X_E} . If every element of the cover $\tilde{v}$ is a neutrosophic

- soft open (closed) in $\left(X, \overset{NSS}{\tau}, E\right)$, then $\tilde{v}$ is called a neutrosophic soft open (closed) cover.
- (b) If there is a neutrosophic soft neighborhood that intersects only the finite number of elements of the cover $\tilde{v}$ of each $x_{(\alpha, \beta, \gamma)}^e \in NSS(X_E)$, $\tilde{v}$ is called a neutrosophic soft locally finite cover.
- (c) If each $\left(\tilde{F}_E\right)_i$ of the cover $\tilde{v}$ intersects only a finite number of elements, then $\tilde{v}$ is called a neutrosophic soft star finite cover.
- (d) If each $x_{(\alpha, \beta, \gamma)}^e \in NSS(X_E)$ belongs to only a finite number of elements of the cover $\tilde{v}$, then $\tilde{v}$ is called neutrosophic soft point finite cover.

Remark 3.1. If the neutrosophic soft coverings are open in Definition 3.2, each neutrosophic soft star with finite cover is neutrosophic soft locally finite, and every neutrosophic soft locally finite cover is neutrosophic soft point finite.

Definition 3.3. Let $\left(X, \overset{NSS}{\tau}, E\right)$ be a neutrosophic soft topological space over X and $\tilde{F}_E \in NSS(X_E)$.

- (a) If every neutrosophic soft open cover of $\left(X, \overset{NSS}{\tau}, E\right)$ has a finite neutrosophic soft subcover, then $\left(X, \overset{NSS}{\tau}, E\right)$ is called neutrosophic soft compact space.
- (b) If $\left(\tilde{F}_E, \overset{NSS}{\tau}_{\tilde{F}_E}, E\right)$ is a neutrosophic soft compact space, then $\tilde{F}_E$ is called a neutrosophic soft compact set in $\left(X, \overset{NSS}{\tau}, E\right)$.

Example 3.1. Let $X = \{x_1, x_2, x_3\}$ be an initial universe set, $E = \{e_1, e_2\}$ be a set of parameters and $\overset{NSS}{\tau} = \{0_{(X, E)}, 1_{(X, E)}, (\tilde{F}_1, E), (\tilde{F}_2, E), (\tilde{F}_3, E)\}$ where $(\tilde{F}_1, E)$, $(\tilde{F}_2, E)$ and $(\tilde{F}_3, E)$ defined as following:

$$\begin{aligned} (\tilde{F}_1, E) &= \left\{ \begin{array}{l} e_1 = \{\langle x_1, (1, 0, 1) \rangle, \langle x_2, (0, 1, 1) \rangle, \langle x_3, (0, 0, 1) \rangle\} \\ e_2 = \{\langle x_1, (0, 0, 1) \rangle, \langle x_2, (1, 0, 1) \rangle, \langle x_3, (1, 1, 0) \rangle\} \end{array} \right\} \\ (\tilde{F}_2, E) &= \left\{ \begin{array}{l} e_1 = \{\langle x_1, (0, 1, 0) \rangle, \langle x_2, (1, 1, 0) \rangle, \langle x_3, (1, 1, 0) \rangle\} \\ e_2 = \{\langle x_1, (1, 1, 0) \rangle, \langle x_2, (1, 1, 0) \rangle, \langle x_3, (0, 1, 0) \rangle\} \end{array} \right\} \\ (\tilde{F}_3, E) &= \left\{ \begin{array}{l} e_1 = \{\langle x_1, (0, 0, 1) \rangle, \langle x_2, (0, 1, 1) \rangle, \langle x_3, (0, 0, 1) \rangle\} \\ e_2 = \{\langle x_1, (0, 0, 1) \rangle, \langle x_2, (1, 0, 1) \rangle, \langle x_3, (0, 1, 0) \rangle\} \end{array} \right\}. \end{aligned}$$

Here, $1_{(X, E)} = \bigcup_{i=1}^3 (\tilde{F}_i, E)$. Hence $\{(\tilde{F}_1, E), (\tilde{F}_2, E), (\tilde{F}_3, E)\}$ is an open cover of $\left(X, \overset{NSS}{\tau}, E\right)$. Also, $1_{(X, E)} = (\tilde{F}_1, E) \cup (\tilde{F}_2, E)$. Therefore, $\left(X, \overset{NSS}{\tau}, E\right)$ is neutrosophic soft compact space.

Theorem 3.1. Let $\left(X, \overset{NSS}{\tau}, E\right)$ be a neutrosophic soft topological space over X and $\tilde{F}_E \in NSS(X_E)$. $\tilde{F}_E$ is a neutrosophic soft compact set if and only if every neutrosophic soft open cover of $\tilde{F}_E$ has a finite neutrosophic soft subcover in $\left(X, \overset{NSS}{\tau}, E\right)$.

Proof. Let $\tilde{F}_E \in NSS(X_E)$ be a neutrosophic soft compact set and the family $\tilde{v} = \{(\tilde{F}_E)_i : i \in I\}$ is a neutrosophic soft open cover of $\tilde{F}_E$ in (X, τ^{NSS}, E) . Then,

$$\tilde{F}_E \subset \bigcup_{i \in I} (\tilde{F}_E)_i \Rightarrow \tilde{F}_E = \bigcup_{i \in I} (\tilde{F}_E \cap (\tilde{F}_E)_i)$$

can be written. Since each $i \in I$ is $(\tilde{F}_E \cap (\tilde{F}_E)_i) \in \tau_{\tilde{F}_E}^{NSS}$, neutrosophic soft family $\{(\tilde{F}_E \cap (\tilde{F}_E)_i)\}_{i \in I}$ is a neutrosophic soft open cover of $\tilde{F}_E$ and since $(\tilde{F}_E, \tau_{\tilde{F}_E}^{NSS}, E)$ is neutrosophic soft compact, for

$$\exists i_1, \dots, i_n \cdot \tilde{F}_E = \bigcup_{j=1}^n (\tilde{F}_E \cap (\tilde{F}_E)_{i_j}) \subset \bigcup_{j=1}^n (\tilde{F}_E)_{i_j}$$

is obtained, that is, family $\{(\tilde{F}_E)_{i_j}\}_{j=1, \dots, n}$ is a neutrosophic soft finite subcover of $\tilde{F}_E$.

Conversely, let family $\{(\tilde{G}_E)_i : i \in I\}$ be a neutrosophic soft open cover of $(\tilde{F}_E, \tau_{\tilde{F}_E}^{NSS}, E)$. Since each $i \in I$ is $(\tilde{G}_E)_i \in \tau_{\tilde{F}_E}^{NSS}$, there is $(\tilde{F}_E)_i \in \tau^{NSS}$ such that $(\tilde{G}_E)_i = \tilde{F}_E \cap (\tilde{F}_E)_i$. Thus, family $\{(\tilde{F}_E)_i : i \in I\}$ is a neutrosophic soft open covering of the $\tilde{F}_E$ in (X, τ^{NSS}, E) and

$$\begin{aligned} \exists (\tilde{F}_E)_{i_1}, \dots, (\tilde{F}_E)_{i_l} : \tilde{F}_E \subset \bigcup_{j=1}^n (\tilde{F}_E)_{i_j} &\Rightarrow \tilde{F}_E = \tilde{F}_E \cap \left(\bigcup_{j=1}^n (\tilde{F}_E)_{i_j} \right) \\ &= \bigcup_{j=1}^n \tilde{F}_E \cap (\tilde{F}_E)_{i_j} = \bigcup_{j=1}^n (\tilde{G}_E)_{i_j} \end{aligned}$$

is obtained from the condition of theorem.

It is sometimes quite difficult to prove that a given neutrosophic soft topological space is neutrosophic soft compact by appealing directly to the definition. The following theorems give several equivalent forms of neutrosophic soft compactness which are often easier to apply.

Theorem 3.2. Let (X, τ^{NSS}, E) be a neutrosophic soft topological space over X . 1_{X_E} is neutrosophic soft compact space if and only if every family of neutrosophic soft closed sets with empty intersection in (X, τ^{NSS}, E) has a finite subfamily with empty intersection.

Proof. Let (X, τ^{NSS}, E) be a neutrosophic soft compact space, and the intersection of the family $\tilde{v} = \{(\tilde{F}_E)_i : i \in I\}$ is the neutrosophic soft closed sets family which is empty. Then, the family,

$$\tilde{v} = \{(\tilde{G}_E)_i = 1_{X_E} \setminus (\tilde{F}_E)_i\}_{i \in I}$$

is the neutrosophic soft open sets family and we obtain,

$$\bigcup_{i \in I} (\tilde{G}_E)_i = \bigcup_{i \in I} (1_{X_E} \setminus (\tilde{F}_E)_i) = 1_{X_E} \setminus \left(\bigcap_{i \in I} (\tilde{F}_E)_i \right) = 1_{X_E} \setminus 0_{X_E} = 1_{X_E}$$

Thus, the family $\tilde{v} = \{(\tilde{G}_E)_i\}_{i \in I}$ is a neutrosophic soft open covering of 1_{X_E} . Since $(X, \overset{NSS}{\tau}, E)$ is a neutrosophic soft compact space,

$$\exists (\tilde{G}_E)_{i_1}, \dots, (\tilde{G}_E)_{i_n} : 1_{X_E} = \bigcup_{j=1}^n (\tilde{G}_E)_{i_j}$$

is obtained. Now the intersection of the finite subfamily $\left\{(\tilde{F}_E)_{i_j}\right\}_{j=1, n}$ of the neutrosophic soft family $\tilde{v} = \{(\tilde{F}_E)_i\}_{i \in I}$ and we obtain,

$$\bigcap_{j=1}^n (\tilde{F}_E)_{i_j} = \bigcap_{j=1}^n (1_{X_E} \setminus (\tilde{G}_E)_{i_j}) = 1_{X_E} \setminus \left(\bigcup_{j=1}^n (\tilde{G}_E)_{i_j} \right) = 1_{X_E} \setminus 1_{X_E} = 0_{X_E}$$

Conversely, let family $\tilde{v} = \{(\tilde{G}_E)_i\}_{i \in I}$ be a neutrosophic soft open covering of the 1_{X_E} . The intersection of the neutrosophic soft closed sets family $\tilde{v} = \{(\tilde{F}_E)_i = 1_{X_E} \setminus (\tilde{G}_E)_i\}_{i \in I}$ is empty. Really,

$$\bigcap_{i \in I} (\tilde{F}_E)_i = \bigcap_{i \in I} (1_{X_E} \setminus (\tilde{G}_E)_{i_j}) = 1_{X_E} \setminus \left(\bigcup_{i \in I} (\tilde{G}_E)_{i_j} \right) = 1_{X_E} \setminus 1_{X_E} = 0_{X_E}.$$

Then, from the condition of theorem

$$\exists (\tilde{F}_E)_{i_1}, \dots, (\tilde{F}_E)_{i_n} : \bigcap_{j=1}^n (\tilde{F}_E)_{i_j} = 0_{X_E}$$

can be written. Hence,

$$\bigcup_{j=1}^n (\tilde{G}_E)_{i_j} = \bigcup_{j=1}^n (1_{X_E} \setminus (\tilde{F}_E)_{i_j}) = 1_{X_E} \setminus \left(\bigcap_{j=1}^n (\tilde{F}_E)_{i_j} \right) = 1_{X_E} \setminus 0_{X_E} = 1_{X_E}.$$

Thus, the neutrosophic soft finite subcovering of neutrosophic soft $\tilde{v} = \{(\tilde{F}_E)_i\}_{i \in I}$ covering of 1_{X_E} was found. As a result, $(X, \overset{NSS}{\tau}, E)$ is neutrosophic soft compact space.

We recall from Definition 3.1. (d) that a neutrosophic soft class of subsets of a non-empty neutrosophic soft set is said to have the centralized neutrosophic soft if every finite neutrosophic soft subclass has non-empty intersection. This concept enables us to express Theorem 3.2 as follows.

Theorem 3.3. Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X .

$(X, \overset{NSS}{\tau}, E)$ is a neutrosophic soft compact space if and only if the neutrosophic soft intersection of all sets of every neutrosophic soft centered closed sets family is different from empty in 1_{X_E} .

Proof. Let $(X, \frac{NSS}{\tau}, E)$ be a neutrosophic soft compact space over X , and $\tilde{v} = \{(\tilde{F}_E)_i\}_{i \in I}$ be neutrosophic soft centered closed sets family. Suppose it is $\bigcap_{i \in I} (\tilde{F}_E)_i = 0_{X_E}$. Then, family $\tilde{v} = \{(\tilde{G}_E)_i = 1_{X_E} \setminus (\tilde{F}_E)_i\}_{i \in I}$ is an open covering of 1_{X_E} . Since $(X, \frac{NSS}{\tau}, E)$ is a neutrosophic soft compact space.

$$\exists (\tilde{G}_E)_{i_1}, \dots, (\tilde{G}_E)_{i_n} : 1_{X_E} = \bigcup_{j=1}^n (\tilde{G}_E)_{i_j}$$

written. In this case

$$\bigcap_{j=1}^n (\tilde{F}_E)_{i_j} = \bigcap_{j=1}^n (1_{X_E} \setminus (\tilde{G}_E)_{i_j}) = 1_{X_E} \setminus \left(\bigcup_{j=1}^n (\tilde{G}_E)_{i_j} \right) = 1_{X_E} \setminus 1_{X_E} = 0_{X_E}.$$

If it is, it contradicts with $\tilde{v}$ is a neutrosophic soft centered family.

In that case, the assumption is wrong, so, it is $\bigcap_{i \in I} (\tilde{F}_E)_i \neq 0_{X_E}$.

Conversely, the theorem is provided, but $(X, \frac{NSS}{\tau}, E)$ is not neutrosophic soft compact space. Then, there is no finite neutrosophic soft subcovering of $\tilde{v} = \{(\tilde{G}_E)_i\}_{i \in I}$ neutrosophic soft open covering of 1_{X_E} . Thus, $\bigcup_{j=1}^n (\tilde{G}_E)_{i_j} \neq 1_{X_E}$ is obtained for any $(\tilde{G}_E)_{i_1}, \dots, (\tilde{G}_E)_{i_n}$ finite neutrosophic soft subfamily of $\tilde{v}$ neutrosophic soft family. Consider the family of $\tilde{v} = \{(\tilde{F}_E)_i = 1_{X_E} \setminus (\tilde{G}_E)_i\}_{i \in I}$ neutrosophic soft closed sets. Here, because

$$\bigcap_{j=1}^n (\tilde{F}_E)_{i_j} = \bigcap_{j=1}^n (1_{X_E} \setminus (\tilde{G}_E)_{i_j}) = 1_{X_E} \setminus \left(\bigcup_{j=1}^n (\tilde{G}_E)_{i_j} \right) \neq 0_{X_E}.$$

is the $\tilde{v}$ family is a neutrosophic soft centered closed set family and the theorem of the condition $\bigcap_{i \in I} (\tilde{F}_E)_i \neq 0_{X_E}$. Then,

$$1_{X_E} = \bigcup_{i \in I} (\tilde{G}_E)_i = \bigcup_{i \in I} (1_{X_E} \setminus (\tilde{F}_E)_{i_j}) = 1_{X_E} \setminus \left(\bigcap_{i \in I} (\tilde{F}_E)_i \right) \neq 1_{X_E}.$$

So, the assumption is wrong. Thus, the proof is completed.

Theorem 3.4. Every neutrosophic soft closed subset of a neutrosophic soft compact topological space is neutrosophic soft compact.

Proof. Let $(X, \frac{NSS}{\tau}, E)$ be a neutrosophic soft compact space, neutrosophic soft sets family $\tilde{v} = \{(\tilde{F}_E)_i\}_{i \in I}$ be an open cover of $\tilde{F}_E$.

In this case, $\tilde{F}_E \subset \bigcup_{i \in I} (\tilde{F}_E)_i$ and $\tilde{F}_E$ are neutrosophic soft closed, so the $1_{X_E} \setminus \tilde{F}_E$ set is open. On the other hand, it can also be written

$$\exists (\tilde{F}_E)_{i_1}, \dots, (\tilde{F}_E)_{i_n} : \left(\bigcup_{j=1}^n (\tilde{F}_E)_{i_j} \right) \cup (1_{X_E} \setminus (\tilde{F}_E)_{i_j}) = 1_{X_E}.$$

Here, $\tilde{F}_E \subset \bigcup_{j=1}^n (\tilde{F}_E)_{i_j}$ is obtained. Then, it is obtained that is neutrosophic soft compact set of $\tilde{F}_E$ from Theorem 3.1.

Corollary 3.1. $(\tilde{F}_E)_{i_1}, \dots, (\tilde{F}_E)_{i_n}$ neutrosophic soft sets, neutrosophic soft closed subsets be the family of $\left(X, \frac{NSS}{\tau}, E\right)$. $\tilde{F}_E \subset \bigcup_{j=1}^n (\tilde{F}_E)_i$ is a neutrosophic soft compact set if and only if for each $i = \overline{1, n}$, $(\tilde{F}_E)_i$ is a neutrosophic soft compact set.

In Theorem 3.4 we proved that every neutrosophic soft closed subset of a neutrosophic soft compact space is neutrosophic soft compact. We now use the preceding theorem to show that neutrosophic soft compact subset Hausdorff spaces are always neutrosophic soft closed.

Theorem 3.5. Every neutrosophic soft compact subset in a neutrosophic soft Hausdorff topological space is a neutrosophic soft closed.

Proof Let $\left(X, \frac{NSS}{\tau}, E\right)$ be a neutrosophic soft Hausdorff space, $\tilde{F}_E \subset NSS(X_E)$ neutrosophic soft be a compact set. To show that is closed of $\tilde{F}_E$, it is enough to show that is open of $\text{set}1_{X_E} \setminus \tilde{F}_E$. ${}^0x_{(\alpha, \beta, \gamma)}^e \in 1_{X_E} \setminus \tilde{F}_E$ is not equal to any $y_{(\alpha', \beta', \gamma')}^{e'} \in \tilde{F}_E$. Then, since $\left(X, \frac{NSS}{\tau}, E\right)$ is a neutrosophic soft Hausdorff space, for points ${}^0x_{(\alpha, \beta, \gamma)}^e \neq y_{(\alpha', \beta', \gamma')}^{e'} \in 1_{X_E}$,

$$\begin{aligned} & \exists \left(\tilde{U}_E\right)_{{}^0x_{(\alpha, \beta, \gamma)}^e, y_{(\alpha', \beta', \gamma')}^{e'}} , \left(\tilde{V}_E\right)_{y_{(\alpha', \beta', \gamma')}^{e'}, {}^0x_{(\alpha, \beta, \gamma)}^e} \\ & \in \frac{NSS}{\tau} : {}^0x_{(\alpha, \beta, \gamma)}^e \in \left(\tilde{U}_E\right)_{{}^0x_{(\alpha, \beta, \gamma)}^e, y_{(\alpha', \beta', \gamma')}^{e'}} , y_{(\alpha', \beta', \gamma')}^{e'} \in \left(\tilde{V}_E\right)_{y_{(\alpha', \beta', \gamma')}^{e'}, {}^0x_{(\alpha, \beta, \gamma)}^e} \\ & \text{ve } \left(\tilde{U}_E\right)_{{}^0x_{(\alpha, \beta, \gamma)}^e, y_{(\alpha', \beta', \gamma')}^{e'}} \cap \left(\tilde{V}_E\right)_{y_{(\alpha', \beta', \gamma')}^{e'}, {}^0x_{(\alpha, \beta, \gamma)}^e} = 1_{X_E} \end{aligned}$$

can be written. If the $y_{(\alpha', \beta', \gamma')}^{e'}$ walk in the all $\tilde{F}_E$ neutrosophic soft set, family $\left\{\left(\tilde{V}_E\right)_{y_{(\alpha', \beta', \gamma')}^{e'}, {}^0x_{(\alpha, \beta, \gamma)}^e}\right\}_{y_{(\alpha', \beta', \gamma')}^{e'} \in \tilde{F}_E}$ of $\tilde{F}_E$ is an open cover at 1_{X_E} . Since $\tilde{F}_E$ set is a neutrosophic soft compact set,

$$\exists \left(\tilde{V}_E\right)_{y_{(\alpha', \beta', \gamma')}^{e'}, {}^0x_{(\alpha, \beta, \gamma)}^e}, \dots, \left(\tilde{V}_E\right)_{y_{(\alpha', \beta', \gamma')}^{e_n}, {}^0x_{(\alpha, \beta, \gamma)}^e} : \tilde{F}_E \subset \bigcup_{i=1}^n \left(\tilde{V}_E\right)_{y_{(\alpha', \beta', \gamma')}^{e_i}, {}^0x_{(\alpha, \beta, \gamma)}^e}$$

can be written. Consider $\left(\tilde{U}_E\right)_{{}^0x_{(\alpha, \beta, \gamma)}^e, y_{(\alpha', \beta', \gamma')}^{e_i}}$ $i = \overline{1, n}$ neighborhoods that provide

$$\left(\tilde{U}_E\right)_{{}^0x_{(\alpha, \beta, \gamma)}^e, y_{(\alpha', \beta', \gamma')}^{e_i}} \cap \left(\tilde{V}_E\right)_{y_{(\alpha', \beta', \gamma')}^{e_i}, {}^0x_{(\alpha, \beta, \gamma)}^e} = {}^0x_{(\alpha, \beta, \gamma)}^e$$

condition, of the ${}^0x_{(\alpha,\beta,\gamma)}^e$ corresponding to the $(\tilde{V}_E)_{i_{y_{(\alpha',\beta',\gamma')}^e}, {}^0x_{(\alpha,\beta,\gamma)}^e}$ $i = \overline{1, n}$ sets. The set $(\tilde{U}_E)_{0_{x_{(\alpha,\beta,\gamma)}^e}} = \bigcap_{i=1}^n (\tilde{U}_E)_{0_{x_{(\alpha,\beta,\gamma)}^e}, i_{y_{(\alpha',\beta',\gamma')}^e}}$ is a neutrosophic soft open neighborhood of ${}^0x_{(\alpha,\beta,\gamma)}^e$ and

$$\begin{aligned}\tilde{F}_E \cap (\tilde{U}_E)_{0_{x_{(\alpha,\beta,\gamma)}^e}} &= \tilde{F}_E \cap \left(\bigcap_{i=1}^n (\tilde{U}_E)_{0_{x_{(\alpha,\beta,\gamma)}^e}, i_{y_{(\alpha',\beta',\gamma')}^e}} \right) \\ &\subset \left(\bigcup_{i=1}^n (\tilde{V}_E)_{i_{y_{(\alpha',\beta',\gamma')}^e}, {}^0x_{(\alpha,\beta,\gamma)}^e} \right) \cap \left(\bigcap_{i=1}^n (\tilde{U}_E)_{0_{x_{(\alpha,\beta,\gamma)}^e}, i_{y_{(\alpha',\beta',\gamma')}^e}} \right) \\ &= \bigcup_{i=1}^n \left((\tilde{V}_E)_{i_{y_{(\alpha',\beta',\gamma')}^e}, {}^0x_{(\alpha,\beta,\gamma)}^e} \cap \bigcap_{i=1}^n (\tilde{U}_E)_{0_{x_{(\alpha,\beta,\gamma)}^e}, i_{y_{(\alpha',\beta',\gamma')}^e}} \right) \\ &\subset \bigcup_{i=1}^n \left((\tilde{V}_E)_{i_{y_{(\alpha',\beta',\gamma')}^e}, {}^0x_{(\alpha,\beta,\gamma)}^e} \cap (\tilde{U}_E)_{0_{x_{(\alpha,\beta,\gamma)}^e}, i_{y_{(\alpha',\beta',\gamma')}^e}} \right) \\ &= 0_{X_E}\end{aligned}$$

Here, ${}^0x_{(\alpha,\beta,\gamma)}^e \in (\tilde{U}_E)_{0_{x_{(\alpha,\beta,\gamma)}^e}} \subset 1_{X_E} \setminus \tilde{F}_E$ is obtained, so $1_{X_E} \setminus \tilde{F}_E$ neutrosophic soft set is obvious. In this case, $\tilde{F}_E$ is neutrosophic soft set closed.

Theorem 3.6. Every neutrosophic soft compact Hausdorff space is neutrosophic soft normal space.

Proof. Let (X, τ^{NSS}, E) be a neutrosophic soft compact Hausdorff space and $\tilde{F}_E^1, \tilde{F}_E^2$ and two discrete neutrosophic soft closed set. From Theorem 3.4., these sets are neutrosophic soft compact. For each $x_{(\alpha,\beta,\gamma)}^e \in \tilde{F}_E^1$ and $y_{(\alpha',\beta',\gamma')}^e \in \tilde{F}_E^2$ are $x_{(\alpha,\beta,\gamma)}^e \neq y_{(\alpha',\beta',\gamma')}^e$.

Since (X, τ^{NSS}, E) is a neutrosophic soft Hausdorff space,

$$\begin{aligned}\exists (\tilde{U}_E)_{x_{(\alpha,\beta,\gamma)}^e, y_{(\alpha',\beta',\gamma')}^e}, (\tilde{V}_E)_{y_{(\alpha',\beta',\gamma')}^e, x_{(\alpha,\beta,\gamma)}^e} \\ \in \tau^{NSS} : x_{(\alpha,\beta,\gamma)}^e \in (\tilde{U}_E)_{x_{(\alpha,\beta,\gamma)}^e, y_{(\alpha',\beta',\gamma')}^e}, y_{(\alpha',\beta',\gamma')}^e \in (\tilde{V}_E)_{y_{(\alpha',\beta',\gamma')}^e, x_{(\alpha,\beta,\gamma)}^e} \\ \text{ve } (\tilde{U}_E)_{x_{(\alpha,\beta,\gamma)}^e, y_{(\alpha',\beta',\gamma')}^e} \cap (\tilde{V}_E)_{y_{(\alpha',\beta',\gamma')}^e, x_{(\alpha,\beta,\gamma)}^e} = 0_{X_E}\end{aligned}$$

can be written for $x_{(\alpha,\beta,\gamma)}^e \neq y_{(\alpha',\beta',\gamma')}^e \in 1_{X_E}$. Let's keep ${}^0x_{(\alpha,\beta,\gamma)}^e \in \tilde{F}_E^1$ constant.

Then, for each $y_{(\alpha',\beta',\gamma')}^e \in \tilde{F}_E^2$ and ${}^0x_{(\alpha,\beta,\gamma)}^e \in \tilde{F}_E^1$

$$\begin{aligned}\exists (\tilde{U}_E)_{0_{x_{(\alpha,\beta,\gamma)}^e}, y_{(\alpha',\beta',\gamma')}^e}, (\tilde{V}_E)_{y_{(\alpha',\beta',\gamma')}^e, {}^0x_{(\alpha,\beta,\gamma)}^e} \\ \in \tau^{NSS} : 0_{x_{(\alpha,\beta,\gamma)}^e} \in (\tilde{U}_E)_{0_{x_{(\alpha,\beta,\gamma)}^e}, y_{(\alpha',\beta',\gamma')}^e}, y_{(\alpha',\beta',\gamma')}^e \in (\tilde{V}_E)_{y_{(\alpha',\beta',\gamma')}^e, {}^0x_{(\alpha,\beta,\gamma)}^e} \\ \text{ve } (\tilde{U}_E)_{0_{x_{(\alpha,\beta,\gamma)}^e}, y_{(\alpha',\beta',\gamma')}^e} \cap (\tilde{V}_E)_{y_{(\alpha',\beta',\gamma')}^e, {}^0x_{(\alpha,\beta,\gamma)}^e} = 0_{X_E}\end{aligned}$$

is obtained. It is clear that family $\left\{ \left(\tilde{V}_E \right)_{y_{(\alpha', \beta', \gamma')}^{e'}, 0_{x_{(\alpha, \beta, \gamma)}^e}} \right\}_{y_{(\alpha', \beta', \gamma')}^{e'} \in (\tilde{F}_E^2)}$ of $\tilde{F}_E^2$ is a neutrosophic soft open cover in 1_{X_E} . Since $\tilde{F}_E^2$ is neutrosophic soft compact,

$$\exists \left(\tilde{V}_E \right)_{y_{(\alpha', \beta', \gamma')}^{e'}, 0_{x_{(\alpha, \beta, \gamma)}^e}}, \dots, \left(\tilde{V}_E \right)_{n_{y_{(\alpha', \beta', \gamma')}^{e'}, 0_{x_{(\alpha, \beta, \gamma)}^e}}} : \tilde{F}_E^2 \subset \bigcup_{i=1}^n \left(\tilde{V}_E \right)_{i_{y_{(\alpha', \beta', \gamma')}^{e'}, 0_{x_{(\alpha, \beta, \gamma)}^e}}}$$

is writable.

Consider the $\left(\tilde{U}_E \right)_{0_{x_{(\alpha, \beta, \gamma)}^e}, i_{y_{(\alpha', \beta', \gamma')}^{e'}}}$ $i = \overline{1, n}$ neighborhoods that provide the $\left(\tilde{U}_E \right)_{0_{x_{(\alpha, \beta, \gamma)}^e}, i_{y_{(\alpha', \beta', \gamma')}^{e'}}} \cap \left(\tilde{V}_E \right)_{i_{y_{(\alpha', \beta', \gamma')}^{e'}, 0_{x_{(\alpha, \beta, \gamma)}^e}}} = 0_{X_E}$ condition of the $0_{x_{(\alpha, \beta, \gamma)}^e}$ corresponding to the $\left(\tilde{V}_E \right)_{i_{y_{(\alpha', \beta', \gamma')}^{e'}, 0_{x_{(\alpha, \beta, \gamma)}^e}}}$ $i = \overline{1, n}$ sets. Thus, $\left(\tilde{U}_E \right)_{0_{x_{(\alpha, \beta, \gamma)}^e}, (\tilde{F}_E^2)} = \bigcap_{i=1}^n \left(\tilde{U}_E \right)_{0_{x_{(\alpha, \beta, \gamma)}^e}, i_{y_{(\alpha', \beta', \gamma')}^{e'}}$ and $\left(\tilde{V}_E \right)_{\tilde{F}_E^2, 0_{x_{(\alpha, \beta, \gamma)}^e}} = \bigcup_{i=1}^n \left(\tilde{V}_E \right)_{i_{y_{(\alpha', \beta', \gamma')}^{e'}, 0_{x_{(\alpha, \beta, \gamma)}^e}}}$ sets are neutrosophic soft open neighborhoods of $0_{x_{(\alpha, \beta, \gamma)}^e}$ and $\tilde{F}_E^2$, respectively and

$$\begin{aligned} & \left(\tilde{U}_E \right)_{0_{x_{(\alpha, \beta, \gamma)}^e}, \tilde{F}_E^2} \cap \left(\tilde{V}_E \right)_{\tilde{F}_E^2, 0_{x_{(\alpha, \beta, \gamma)}^e}} \\ &= \left(\bigcap_{i=1}^n \left(\tilde{U}_E \right)_{0_{x_{(\alpha, \beta, \gamma)}^e}, i_{y_{(\alpha', \beta', \gamma')}^{e'}}} \right) \cap \left(\bigcup_{i=1}^n \left(\tilde{V}_E \right)_{i_{y_{(\alpha', \beta', \gamma')}^{e'}, 0_{x_{(\alpha, \beta, \gamma)}^e}}} \right) \\ &= \bigcup_{i=1}^n \left(\left(\bigcap_{i=1}^n \left(\tilde{U}_E \right)_{0_{x_{(\alpha, \beta, \gamma)}^e}, i_{y_{(\alpha', \beta', \gamma')}^{e'}}} \right) \cap \left(\tilde{V}_E \right)_{i_{y_{(\alpha', \beta', \gamma')}^{e'}, 0_{x_{(\alpha, \beta, \gamma)}^e}}} \right) \\ &\subset \bigcup_{i=1}^n \left(\left(\tilde{U}_E \right)_{0_{x_{(\alpha, \beta, \gamma)}^e}, i_{y_{(\alpha', \beta', \gamma')}^{e'}}} \cap \left(\tilde{V}_E \right)_{i_{y_{(\alpha', \beta', \gamma')}^{e'}, 0_{x_{(\alpha, \beta, \gamma)}^e}}} \right) \\ &= 0_{X_E}. \end{aligned}$$

Thus, each $x_{(\alpha, \beta, \gamma)}^e$ and $\tilde{F}_E^2$ have neighborhoods so that for each $x_{(\alpha, \beta, \gamma)}^e \in \tilde{F}_E^1$ and $\tilde{F}_E^2$, $\left(\tilde{U}_E \right)_{x_{(\alpha, \beta, \gamma)}^e, \tilde{F}_E^2} \cap \left(\tilde{V}_E \right)_{\tilde{F}_E^2, x_{(\alpha, \beta, \gamma)}^e} = 0_{X_E}$ is provided. From there, $\left\{ \left(\tilde{U}_E \right)_{x_{(\alpha, \beta, \gamma)}^e, \tilde{F}_E^2} \right\}_{x_{(\alpha, \beta, \gamma)}^e \in (\tilde{F}_E^1)}$ family of $\tilde{F}_E^1$ has a neutrosophic soft open cover in 1_{X_E} .

Since $\tilde{F}_E^1$ is neutrosophic soft compact,

$$\begin{aligned} & \exists \left(\tilde{U}_E \right)_{1_{x_{(\alpha, \beta, \gamma)}^e}, \tilde{F}_E^2}, \dots, \left(\tilde{U}_E \right)_{k_{x_{(\alpha, \beta, \gamma)}^e}, \tilde{F}_E^2} : \tilde{F}_E^1 \subset \bigcup_{i=1}^k \left(\tilde{U}_E \right)_{i_{x_{(\alpha, \beta, \gamma)}^e}, \tilde{F}_E^2} \\ & \subset \left(\tilde{V}_E \right)_{\tilde{F}_E^2, i_{x_{(\alpha, \beta, \gamma)}^e}}, \quad i = \overline{1, k} \\ & \text{ve } \left(\tilde{U}_E \right)_{i_{x_{(\alpha, \beta, \gamma)}^e}, \tilde{F}_E^2} \cap \left(\tilde{V}_E \right)_{\tilde{F}_E^2, i_{x_{(\alpha, \beta, \gamma)}^e}} = 0_{X_E}. \end{aligned}$$

Thus, the open sets $(\tilde{U}_E)_{\tilde{F}_E^1} = \bigcup_{i=1}^k (\tilde{U}_E)_{i_{x_{(\alpha,\beta,\gamma)}^e}, (\tilde{F}_E^2)}$ and $(\tilde{V}_E)_{\tilde{F}_E^2} = \bigcap_{i=1}^k (\tilde{V}_E)_{\tilde{F}_E^2, i_{x_{(\alpha,\beta,\gamma)}^e}}$ contain sets $\tilde{F}_E^1$ and $\tilde{F}_E^2$, respectively. And

$$\begin{aligned} (\tilde{U}_E)_{\tilde{F}_E^1} \cap (\tilde{V}_E)_{\tilde{F}_E^2} &= \left(\bigcup_{i=1}^k (\tilde{U}_E)_{i_{x_{(\alpha,\beta,\gamma)}^e}, \tilde{F}_E^2} \right) \cap \left(\bigcap_{i=1}^k (\tilde{V}_E)_{\tilde{F}_E^2, i_{x_{(\alpha,\beta,\gamma)}^e}} \right) \\ &= \bigcup_{i=1}^k \left((\tilde{U}_E)_{i_{x_{(\alpha,\beta,\gamma)}^e}, \tilde{F}_E^2} \cap \left(\bigcap_{i=1}^k (\tilde{V}_E)_{\tilde{F}_E^2, i_{x_{(\alpha,\beta,\gamma)}^e}} \right) \right) \\ &\subset \bigcup_{i=1}^n \left((\tilde{U}_E)_{i_{x_{(\alpha,\beta,\gamma)}^e}, \tilde{F}_E^2} \cap (\tilde{V}_E)_{\tilde{F}_E^2, i_{x_{(\alpha,\beta,\gamma)}^e}} \right) \\ &= 0_{X_E} \end{aligned}$$

is provided. Thus, the proof is completed.

4 Neutrosophic soft locally compact spaces

Now we can give a definition of neutrosophic soft locally compactness that is weaker than neutrosophic soft compactness.

Definition 4.1. Let $(X, \frac{NSS}{\tau}, E)$ be a neutrosophic soft topological space over X .

$(X, \frac{NSS}{\tau}, E)$ is called a neutrosophic soft locally compact (locally countable compact) space if for each $x_{(\alpha,\beta,\gamma)}^e \in NSS(X_E)$, there exists a neutrosophic soft neighborhood $\tilde{U}_E \in \frac{NSS}{\tau}$ of $x_{(\alpha,\beta,\gamma)}^e$ with neutrosophic soft closure $\overline{\tilde{U}_E}$ neutrosophic soft compact.

When $(X, \frac{NSS}{\tau}, E)$ is also neutrosophic soft Hausdorff, the property of neutrosophic soft local compactness becomes much stronger. Let's state this as a theorem.

Theorem 4.1. If $(X, \frac{NSS}{\tau}, E)$ is a neutrosophic soft locally compact and neutrosophic soft Hausdorff space, $\tilde{F}_E \in NSS(X_E)$ neutrosophic soft compact set and $\tilde{F}_E \subset \tilde{V}_E$, $\tilde{V}_E \in \frac{NSS}{\tau}$, then there exists $\tilde{U}_E \in \frac{NSS}{\tau}$ such that the closure $\overline{\tilde{U}_E}$ is neutrosophic soft compact and $\tilde{F}_E \subset \tilde{U}_E \subset \overline{\tilde{U}_E} \subset \tilde{V}_E$.

Proof. Since $(X, \frac{NSS}{\tau}, E)$ is a neutrosophic soft Hausdorff space, there exists a $(\tilde{V}_E)_{x_{(\alpha,\beta,\gamma)}^e}$ neighborhood for each $x_{(\alpha,\beta,\gamma)}^e \in \tilde{F}_E \subset \tilde{V}_E$ such that,

$$x_{(\alpha,\beta,\gamma)}^e \in (\tilde{V}_E)_{x_{(\alpha,\beta,\gamma)}^e} \subset \overline{(\tilde{V}_E)_{x_{(\alpha,\beta,\gamma)}^e}} \subset \tilde{V}_E.$$

Since $(X, \frac{NSS}{\tau}, E)$ is a neutrosophic soft lokal compact, each $x_{(\alpha,\beta,\gamma)}^e \in (\tilde{F}_E)$ has a $(\tilde{W}_E)_{x_{(\alpha,\beta,\gamma)}^e}$ compact neighborhood.

Therefore, for neighborhood $(\tilde{U}_E)_{x_{(\alpha,\beta,\gamma)}^e} = (\tilde{V}_E)_{x_{(\alpha,\beta,\gamma)}^e} \cap (\tilde{W}_E)_{x_{(\alpha,\beta,\gamma)}^e}$ of $x_{(\alpha,\beta,\gamma)}^e$, $(\tilde{U}_E)_{x_{(\alpha,\beta,\gamma)}^e}$ is compact since it is $(\tilde{U}_E)_{x_{(\alpha,\beta,\gamma)}^e} \subset \overline{(\tilde{W}_E)_{x_{(\alpha,\beta,\gamma)}^e}}$. Since neutrosophic soft set $\tilde{F}_E$ is compact, $\left\{ (\tilde{U}_E)_{x_{(\alpha,\beta,\gamma)}^e} \right\}_{x_{(\alpha,\beta,\gamma)}^e \in (\tilde{F}_E)}$ neutrosophic soft covering of $\tilde{F}_E$ has a finite neutrosophic soft subcover.

$$\text{Here, for open set } \tilde{U}_E = \bigcup_{i=1}^n (\tilde{U}_E)_{i_{x_{(\alpha,\beta,\gamma)}^e}} \\ \tilde{F}_E \subset \tilde{U}_E, \overline{\tilde{U}_E} = \bigcup_{i=1}^n \overline{(\tilde{U}_E)_{i_{x_{(\alpha,\beta,\gamma)}^e}}} \subset \bigcup_{i=1}^n \overline{(\tilde{V}_E)_{i_{x_{(\alpha,\beta,\gamma)}^e}}} \subset \tilde{V}_E$$

conditions are provided.

Theorem 4.2. If $(X, \overset{NSS}{\tau}, E)$ is a neutrosophic soft local compact, neutrosophic soft Hausdorff space and $\tilde{F}_E \in NSS(X_E)$ neutrosophic soft closed (open) set, then $(\tilde{F}_E, \overset{NSS}{\tau}_{\tilde{F}_E}, E)$ is neutrosophic soft locally compact space.

Proof. Let $(X, \overset{NSS}{\tau}, E)$ be a neutrosophic soft locally compact Hausdorff space and $\tilde{F}_E \in NSS(X_E)$ be a neutrosophic soft closed set. Consider compact neighborhood $\overline{\tilde{U}_E}$ of each $x_{(\alpha,\beta,\gamma)}^e \in \tilde{F}_E$ in $(X, \overset{NSS}{\tau}, E)$. For open neighborhood of $x_{(\alpha,\beta,\gamma)}^e$ $\tilde{U}_E \cap \tilde{F}_E$ in $(\tilde{F}_E, \overset{NSS}{\tau}_{\tilde{F}_E}, E)$, $\overline{\tilde{U}_E \cap \tilde{F}_E}$ is compact because it is $\overline{\tilde{U}_E \cap \tilde{F}_E} = \overline{\tilde{U}_E} \cap \tilde{F}_E \subset \overline{\tilde{U}_E}$, that is, $(\tilde{F}_E, \overset{NSS}{\tau}_{\tilde{F}_E}, E)$ is a neutrosophic soft locally compact space.

When we make the proof for neutrosophic soft closed set, $(X, \overset{NSS}{\tau}, E)$ does not need to be a neutrosophic soft Hausdorff space.

Now, let $\tilde{F}_E \in NSS(X_E)$ be a neutrosophic soft open set. Since the single-point set in the neutrosophic soft Hausdorff space is neutrosophic soft closed, $\tilde{U}_E \in \overset{NSS}{\tau}$ is present so that

$$\{x_{(\alpha,\beta,\gamma)}^e\} \subset \tilde{U}_E \subset \overline{\tilde{U}_E} \subset \tilde{F}_E, \tilde{U}_E \text{ compact}$$

is provided for each $x_{(\alpha,\beta,\gamma)}^e \in \tilde{F}_E$ from Theorem 4.1.

From there, $\tilde{U}_E \cap \tilde{F}_E$ in $(\tilde{F}_E, \overset{NSS}{\tau}_{\tilde{F}_E}, E)$ is a neighborhood of $x_{(\alpha,\beta,\gamma)}^e$ and

$$(\overline{\tilde{U}_E \cap \tilde{F}_E}) = \overline{\tilde{U}_E} \cap \tilde{F}_E \subset \overline{\tilde{U}_E}$$

is compact, that is, $(\tilde{F}_E, \overset{NSS}{\tau}_{\tilde{F}_E}, E)$ is a neutrosophic soft locally compact space.

Theorem 4.3. If $(X, \overset{NSS}{\tau}, E)$ is a neutrosophic soft Hausdorff space and $\tilde{M}_E \in NSS(X_E)$ neutrosophic soft locally compact subspace, then $\tilde{M}_E$ is a neutrosophic soft open set in $\overline{\tilde{M}_E}$

and $\tilde{M}_E$ can be written as $\tilde{M}_E = \tilde{F}_E \cap \tilde{V}_E$, where $\tilde{F}_E$ and $\tilde{V}_E$ are neutrosophic soft closed and neutrosophic soft open set in $(X, \overset{NSS}{\tau}, E)$, respectively.

Proof. Since $(\tilde{M}_E, \overset{NSS}{\tau}_{\tilde{M}_E}, E)$ is neutrosophic soft locally compact, a neighbourhood $\tilde{U}_E \in \overset{NSS}{\tau}_{\tilde{M}_E}$ of $x_{(\alpha, \beta, \gamma)}^e \in \tilde{M}_E$ can be found such that $\overline{\tilde{U}_E} \cap \tilde{M}_E$ is neutrosophic soft compact in $\tilde{M}_E$. $(X, \overset{NSS}{\tau}, E)$ is a neutrosophic soft Hausdorff space. Therefore neutrosophic soft set $\overline{\tilde{U}_E} \cap \tilde{M}_E$ is closed in $(X, \overset{NSS}{\tau}, E)$. For $\tilde{U}_E \subset \overline{\tilde{U}_E} \cap \tilde{M}_E$, we have $\overline{\tilde{U}_E} \subset \overline{\tilde{U}_E} \cap \tilde{M}_E \subset \tilde{M}_E$. We obtain $\tilde{U}_E = \tilde{M}_E \cap \tilde{W}_E$, $\tilde{W}_E \in \overset{NSS}{\tau}$ for $\tilde{U}_E \in \overset{NSS}{\tau}_{\tilde{M}_E}$. So, we consider that neutrosophic soft set $\tilde{M}_E$ is neutrosophic soft dense set in $\overline{\tilde{M}_E}$, then,

$$x_{(\alpha, \beta, \gamma)}^e \in \tilde{W}_E \subset \overline{\tilde{W}_E} = \overline{\tilde{M}_E \cap \tilde{W}_E} = \overline{\tilde{U}_E} \subset \tilde{M}_E$$

can be written. That is, $\tilde{M}_E$ is neutrosophic soft open set. Hence

$$\overline{\tilde{M}_E} = \overline{\tilde{M}_E} \cap \tilde{M}_E = \tilde{F}_E \cap \tilde{V}_E$$

is obtained.

5 Conclusion

In the present paper, we introduce the concept of neutrosophic soft compactness and investigate the basic properties. We also define neutrosophic soft locally compact spaces. In addition, the relationship between neutrosophic soft compactness and neutrosophic soft separation axioms has been investigated. We hope that the results of this study may help in the investigation of neutrosophic soft set theory.

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