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Dynamical behaviour of the foam drainage equation

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ABSTRACT

In this study, we examine the well-known foam drainage equation in the form of the nonlinear partial differential equation using a new extended direct algebraic method. This remarkable powerful method transforms the equation under study into an ordinary differential equation using a wave transformation which provides an exact form to the problem considered in the paper. Graphical representation for the acquired solution is presented through 3-D plots which variable parameters to show the efficiency and simplicity of the method used. From these plots, the methods prove to be a reliable and effective approach for solving nonlinear similar problems and can be accounted for in the near future for possible application to similar models.

Introduction

Nonlinear partial differential equations (NPDE) are considered as one of the most important forms of equations that have a lot of different applications in science and engineering. Most nonlinear real-life phenomena are simulated using NPDE which motivates the researchers to investigate more in their behavior and solutions over the past few years. These solutions can be found through different analytical and numerical approaches and will help in better understand their physical properties. For example, a Sine-Gordon expansion method has been applied for simulating a fractional form of the Regularized long wave (RLW) equation in [1]. New solutions for the fractional Drinfeld-Sokolov-Wilson system which have an application in the study of waves in shallow water systems are found in [2] using a novel analytical approach. Also, with an application in optics and semiconductors laser, a complex form of Ginzburg–Landau equation is investigated in [3]. Constructing soliton solutions for these types of equations are of great importance for better understanding their dynamic and physical behavior. The authors in [4] found new periodic wave solutions for the time-fractional integrable shallow water equation which is potentially a new application. Kolmogorov-Petrovskii-Piskunov equation has been studied in [5] using two ansätze methods with the aid of a Cole-Hopf transformation. In addition, the authors in [6] adapted the rational function method for solving the 3 + 1-dimensional Jimbo–Miwa equation combined with a Buckland transform. Ma in [7], analyzed the N-soliton solutions and Hirota N-soliton conditions for some 1 + 1-dimensional scalar equations including a class of generalized KdV equations and higher-order KdV equations. In the same manner, the same author in [8], investigated the N-soliton solution and Hirota N-soliton conditions and applied theses to three integrable equations named the (2 + 1)-dimensional KdV equation, the Kadomtsev–Petviashvili equation, the (2 + 1)-dimensional Hirota–Satsuma–Ito equation succeeding in finding a new solution to these equations. Wen et al. in [9] applied some innovative algorithm for finding an N-soliton solution to the under general dispersion relations for solving the B-type Kadomtsev–Petviashvili equation. Jinxi et al. in [10] investigated the solutions to the (2 + 1)-dimensional negative-order breaking soliton equation using the truncated Painlevé expansion deriving some explicit

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soliton–cnoidal wave solutions. Fractional order Schrödinger equation was found to possess some symmetry and anti-symmetry soliton solutions in [11] and the influence of the Lévy index on different solitons is examined. Xiaoyan et al. [12], investigated the similarities and differences between different planes of soliton solutions of the 3 + 1-dimensional coupled variable coefficient system with potential application in the evolution of polarization modes in nonlinear fibers. Some dynamical characteristics of the analytical soliton solution to the fractional space–time fractional Fokas-Lenells equation was given in [13]. Many other forms of NPDE were solved including space–time fractional equal wave and modified equal wave equations [14], space–time fractional NPDE models [15], higher-order nonlinear Schrödinger equation [16], (2 + 1)-dimensional Konopelchenko–Dubrovsky equation [17], nonlinear fractional biological models [18], (3 + 1)-dimensional no autonomous-coupled nonlinear Schrödinger equation [19], stochastic nonlinear Schrödinger equation [20] and 2 + 1 dimensional space–time fractional Schrödinger equation [21].Those were only a few examples of the possible models with applications and others can be found in [22–53] and references therein.

One of the important applications of NPDE can be found in the simulating of a formed foam. Foams are of great importance in the field of industry and many other applications and their properties are subject to extensive research [54]. Foam is defined as the object formed by holding the gas pockets into a liquid or solid with some well-defined structure. This phenomenon was first noticed back in the 19th century by the famous physicists' Joseph Plateau. In [55], Weaire et al. found some new findings regarding the structure of foams answering some of the main questions arising in this field but not answering all of the raised questions, and the research to continue their work. Foams can be found in many industrial products that can be used in everyday life like personal care products and cosmetics such as creams and lotions and also can be formed during cleaning clothes, dishes, and dispensing process [56]. They have some important applications in various fields such as the food and chemical, material science, and firefighting industries [57]. Less obviously, they appear in acoustic cladding, lightweight mechanical components, and impact-absorbing parts on cars, heat exchangers, and textured wallpapers (incorporated as foaming inks) and even have an analogy in cosmology. The formation of bubbles or cells can be formed in order or randomly such as the form of the bees' honeycomb. The are many applications of polymeric foams [58] and recently there has been a new form of foams called metallic foams which are mainly formed of metals such as aluminum [59]. These mentioned applications of foams in the industry may include the reduction of the impact of explosions and cleaning up the oil spills. In addition, polymeric foams are of use as a heat exchange in structural engineering and material science such as glass, ceramic, and metal foams [60]. Also, mineral processing uses foams to separate important products by floating. In terms of geophysical applications, foams find their way in describing volcanic eruptions [61]. Fig. 1 describes the three main topics of foams which are drainage, coarsening, and rheology.

As a non-linear partial differential equation (NPDE), the foam drainage (FD) equationsare applied to specify the improvement of the vertical foam density with the gravitational acceleration. Under the capillarity and gravity effect [62,63], the FD equation is a simple model of the liquid flow in the channels [64] and nodes between the bubbles, and this is why these studies are essential. Foam is also crucial not only for natural but also for industrial daily activities. Substantially, the foam has generated a great deal of interest in academic researches. The foam will satisfy and even be a crucial part of a process in processing industries. Hence, separation of the froth flotation can be an identifying example, especially about minerals and coal. Because of the significance of foams in several processes and applications, they are produced for distillation and absorption. Their properties are tied to intensive investigational efforts from performance enhancers and experimental researchers [65].

The mathematical form of the FD equation is given as:

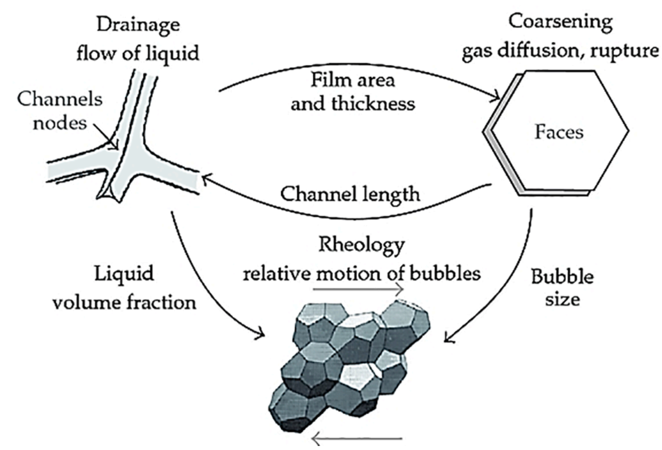


Fig. 1. Schematic diagram of the interdependence of drainage, coarsening, and rheology of foams [54].

$$\vartheta_t + 2\vartheta^2\vartheta_x - \vartheta_x^2 - \frac{1}{2}\vartheta_{xx}\vartheta = 0, \qquad -\infty < x < \infty, \quad 0 < t < \infty, \tag{1}$$

where both x and t are scaled position and duration coordinates, respectively. Here, we seek the traveling wave solutions for Eq. (1) using anew direct algebraic method.

In the past several years, many researchers studied the FD equation using different analytical and approaches. The authors in [66], studied the dynamics of coarsening foams subjects for describing their physical properties. Also, in [67,68] the authors reviewed the history and recent development of the FD equation. In 2019, Koursari et al. researched FD placed on a thin porous layer [69], and the authors in [70], described analytic results of the FD equation. The author in [71] found an exact solution for the problems of the FD equation by the new modified (G'/G) −expansion method. The authors in [72], did research on the FD equation for drainage dynamics in unsaturated permeable instruments. In addition, a reliable approach based on the homotopy analysis method has been performed for solving the FD equation in [73]. A closed-form soliton solution to the space–time fractional foam drainage was found in [74]. Also, conservation law and Lie symmetry analysis for the given problem is investigated in [75].

In this paper, we are concerned with solving the FD equation represented in Eq. (1) using a new extended direct algebraic method (NEDAM). This method was proved to be a reliable method for solving NPDE and was first introduced in [31] for finding resonant optical solitons with variable coefficients in Kerr and non-Kerr law media by Hadi et al. This method has been used ever since for solving different problems such as Tzitzica type evolution with application in nonlinear optics [76], Biswas-Arshed equation [77], Zoomeron equation [78] and many other relative models. Some novel aspects of the designed novel system are described as follows:

- A new direct algebraic methods is adapted for solving the foam drainage equation with application in related industry.
- The obtained solutions considered novel and attain some new physical behavior findings.
- Cases of solutions are introduced for various values of parameters.
- The results proves that the new adapted method is effective in providing accurate solutions with physics.

The organization of the paper is as follows: in Section "Methodology of the NEDAM", we describe the methodology of the new NEDAM. Section New Exact Solutions and their graphical representations" is devoted to finding new solutions for the FD equation. Some graphical representation for the obtained solutions with different parameters is illustrated in Section "Comparison and physical explanation". In Section

"Conclusions", a conclusion for the study is given.

Methodology of the NEDAM

In this section, we will illustrate the main steps for the NEDAM which were first introduced by Rezazadeh in [19]. First, we consider a general form of the NPDES in the following form

$$P(\vartheta, \vartheta_t, \vartheta_x, \vartheta_{tt}, \vartheta_{xx}, \ldots) = 0. \tag{2}$$

This equation can be reduced into a nonlinear ordinary differential equation (ODE) which will take the form

$$G(\Omega, \Omega', \Omega'', \ldots) = 0, \tag{3}$$

The obtained ODE can be obtained using the following wave transformation

$$\vartheta(x,t) = \Omega(\eta), \qquad \eta = x + vt, \tag{4}$$

where $v \neq 0$, and $\Omega' = \frac{d\Omega}{d\eta}$. Second, we assume that the solution of ODE in Eq. (3) can be defined by a polynomial in $\Theta(\eta)$ as in the form

$$\Omega(\eta) = \sum_{k=1}^{N} b_k \Theta^k(\eta), \quad b_N \neq 0, \tag{5}$$

where$b_k(0 \le k \le N)$are constant coefficients and $\Theta(\eta)$ provides the ODE in the form

$$\Theta'(\eta) = \mathrm{Ln}(A)\left(\alpha + \beta\Theta(\eta) + \sigma\Theta^2(\eta)\right), \quad A \neq 0, 1, \tag{6}$$

where α, β and σ are constants. Many approximate solutions can be found by assigning different values of parameters monitored in the auxiliary equations. We summarize the different cases as follows

Case 1. *When assuming $\beta^2 - 4\alpha\sigma < 0$ and $\sigma \neq 0$, the first five approximate solutions to the equation can be found in the following form*

$$\Theta_1 = -\frac{\beta}{2\sigma} + \frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2\sigma} tan_A\left(\frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2}\eta\right), \tag{7}$$

$$\Theta_2 = -\frac{\beta}{2\sigma} - \frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2\sigma} cot_A\left(\frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2}\eta\right), \tag{8}$$

$$\Theta_3 = -\frac{\beta}{2\sigma} + \frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2\sigma}\left(tan_A\left(\sqrt{-(\beta^2 - 4\alpha\sigma)}\,\eta\right) \pm \sqrt{pq}\, sec_A\left(\sqrt{-(\beta^2 - 4\alpha\sigma)}\,\eta\right)\right),$$

$$\Theta_4 = -\frac{\beta}{2\sigma} + \frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{2\sigma}\left(-cot_A\left(\sqrt{-(\beta^2 - 4\alpha\sigma)}\,\eta\right) \pm \sqrt{pq}\, csc_A\left(\sqrt{-(\beta^2 - 4\alpha\sigma)}\,\eta\right)\right), \tag{10}$$

$$\Theta_5 = -\frac{\beta}{2\sigma} + \frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{4\sigma}\left(tan_A\left(\frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{4}\eta\right) - cot_A\left(\frac{\sqrt{-(\beta^2 - 4\alpha\sigma)}}{4}\eta\right)\right) \tag{11}$$

Case 2. *Assigning the values of $\beta^2 - 4\alpha\sigma > 0$ and $\sigma \neq 0$, the later five approximate solutions are*

$$\Theta_6(\eta) = -\frac{\beta}{2\sigma} - \frac{\sqrt{\beta^2 - 4\alpha\sigma}}{2\sigma} tanh_A\left(\frac{\sqrt{\beta^2 - 4\alpha\sigma}}{2}\eta\right), \tag{12}$$

$$\Theta_7(\eta) = -\frac{\beta}{2\sigma} - \frac{\sqrt{\beta^2 - 4\alpha\sigma}}{2\sigma} coth_A\left(\frac{\sqrt{\beta^2 - 4\alpha\sigma}}{2}\eta\right), \tag{13}$$

$$\Theta_8(\eta) = -\frac{\beta}{2\sigma} + \frac{\sqrt{\beta^2 - 4\alpha\sigma}}{2\sigma}\left(-tanh_A\left(\sqrt{\beta^2 - 4\alpha\sigma}\,\eta\right) \pm i\sqrt{pq}\,\mathrm{sech}_A\left(\sqrt{\beta^2 - 4\alpha\sigma}\,\eta\right)\right),$$

$$\Theta_9(\eta) = -\frac{\beta}{2\sigma} + \frac{\sqrt{\beta^2 - 4\alpha\sigma}}{2\sigma}\left(-coth_A\left(\sqrt{\beta^2 - 4\alpha\sigma}\,\eta\right) \pm \sqrt{pq}\,\mathrm{csch}_A\left(\sqrt{\beta^2 - 4\alpha\sigma}\,\eta\right)\right), \tag{16}$$

$$\Theta_{10}(\eta) = -\frac{\beta}{2\sigma} - \frac{\sqrt{\beta^2 - 4\alpha\sigma}}{4\sigma}\left(tanh_A\left(\frac{\sqrt{\beta^2 - 4\alpha\sigma}}{4}\eta\right) + coth_A\left(\frac{\sqrt{\beta^2 - 4\alpha\sigma}}{4}\eta\right)\right)$$

Case 3. *Assigning the values of $\alpha\sigma > 0$ and $\beta = 0$, the form of the solutions $\Theta_{11}(\eta)$ up to $\Theta_{15}(\eta)$ are as follows*

$$\Theta_{11}(\eta) = \sqrt{\frac{\alpha}{\sigma}} tan_A\left(\sqrt{\alpha\sigma}\,\eta\right), \tag{17}$$

$$\Theta_{12}(\eta) = -\sqrt{\frac{\alpha}{\sigma}} cot_A\left(\sqrt{\alpha\sigma}\,\eta\right), \tag{18}$$

$$\Theta_{13}(\eta) = \sqrt{\frac{\alpha}{\sigma}}\left(tan_A\left(2\sqrt{\alpha\sigma}\,\eta\right) \pm \sqrt{pq}\, sec_A\left(2\sqrt{\alpha\sigma}\,\eta\right)\right), \tag{19}$$

$$\Theta_{14}(\eta) = \sqrt{\frac{\alpha}{\sigma}}\left(-cot_A\left(2\sqrt{\alpha\sigma}\,\eta\right) \pm \sqrt{pq}\, csc_A\left(2\sqrt{\alpha\sigma}\,\eta\right)\right),$$

$$\Theta_{15}(\eta) = \frac{1}{2}\sqrt{\frac{\alpha}{\sigma}}\left(tan_A\left(\frac{\sqrt{\alpha\sigma}}{2}\eta\right) - cot_A\left(\frac{\sqrt{\alpha\sigma}}{2}\eta\right)\right) \tag{21}$$

Case 4. *Assigning the value of $\alpha\sigma < 0$ and $\beta = 0$, the next few solutions are in the form*

$$\Theta_{16}(\eta) = -\sqrt{-\frac{\alpha}{\sigma}} tanh_A\left(\sqrt{-\alpha\sigma}\,\eta\right), \tag{22}$$

$$\Theta_{17}(\eta) = -\sqrt{-\frac{\alpha}{\sigma}} coth_A\left(\sqrt{-\alpha\sigma}\,\eta\right), \tag{23}$$

$$\Theta_{18}(\eta) = \sqrt{-\frac{\alpha}{\sigma}}\left(-tanh_A\left(2\sqrt{-\alpha\sigma}\,\eta\right) \pm i\sqrt{pq}\,\mathrm{sech}_A\left(2\sqrt{-\alpha\sigma}\,\eta\right)\right), \tag{24}$$

$$\Theta_{19}(\eta) = \sqrt{-\frac{\alpha}{\sigma}}\left(-coth_A\left(2\sqrt{-\alpha\sigma}\,\eta\right) \pm \sqrt{pq}\,\mathrm{csch}_A\left(2\sqrt{-\alpha\sigma}\,\eta\right)\right), \tag{25}$$

$$\Theta_{20}(\eta) = -\frac{1}{2}\sqrt{-\frac{\alpha}{\sigma}}\left(tanh_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\eta\right) + coth_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\eta\right)\right) \tag{26}$$

Case 5. *When $\beta = 0$ and $\sigma = \alpha$, the solutions from $\Theta_{21}(\eta)$ to $\Theta_{25}(\eta)$ are*

$$\Theta_{21}(\eta) = tan_A(\alpha\eta), \tag{27}$$

$$\Theta_{22}(\eta) = -cot_A(\alpha\eta), \tag{28}$$

$$\Theta_{23}(\eta) = tan_A(2\alpha\eta) \pm \sqrt{pq}sec_A(2\alpha\eta), \tag{29}$$

$$\Theta_{24}(\eta) = -cot_A(2\alpha\eta) \pm \sqrt{pq}csc_A(2\alpha\eta), \tag{30}$$

$$\Theta_{25}(\eta) = \frac{1}{2}\left(tan_A\left(\frac{\alpha}{2}\eta\right) - cot_A\left(\frac{\alpha}{2}\eta\right)\right) \tag{31}$$

Case 6. *For the values $\beta = 0$ and $\sigma = -\alpha$, the solutions from $\Theta_{26}(\eta)$ to $\Theta_{30}(\eta)$ are*

$$\Theta_{26}(\eta) = -tanh_A(\alpha\eta), \tag{32}$$

$$\Theta_{27}(\eta) = -coth_A(\alpha\eta), \tag{33}$$

$$\Theta_{28}(\eta) = -tanh_A(2\alpha\eta) \pm i\sqrt{pq}\mathrm{sech}_A(2\alpha\eta), \tag{34}$$

$$\Theta_{29}(\eta) = -coth_A(2\alpha\eta) \pm \sqrt{pq}\mathrm{csch}_A(2\alpha\eta), \tag{35}$$

$$\Theta_{30}(\eta) = -\frac{1}{2}\left(tanh_A\left(\frac{\alpha}{2}\eta\right) + coth_A\left(\frac{\alpha}{2}\eta\right)\right) \tag{36}$$

Case 7. *For the value $\beta^2 = 4\alpha\sigma$, the solution $\Theta_{31}(\eta)$is in the form*

$$\Theta_{31}(\eta) = \frac{-2\alpha((\mathrm{Ln}A)\beta\eta + 2)}{(\mathrm{Ln}A)\beta^2\eta}. \tag{37}$$

Case 8. *For the values of the parameters $\beta = k$, $\alpha = mk(m \neq 0)$ and $\sigma = 0$, the solution $\Theta_{32}(\eta)$ is in the form*

$$\Theta_{32}(\eta) = A^{k\eta} - m. \tag{38}$$

Case 9. *For the values of the parameters $\beta = \sigma = 0$, the solution $\Theta_{33}(\eta)$ is in the form*

$$\Theta_{33}(\eta) = (\mathrm{Ln}\ A)\alpha\eta. \tag{39}$$

Case 10. *For the values of the parameters $\beta = \alpha = 0$, the solution $\Theta_{34}(\eta)$ is in the form*

$$\Theta_{34}(\eta) = \frac{-1}{(\mathrm{Ln}\ A)\sigma\eta}. \tag{40}$$

Case 11. *For the values of the parameters $\alpha = 0$ and $\beta \neq 0$, the solution $\Theta_{35}(\eta)$ and $\Theta_{36}(\eta)$ is in the form*

$$\Theta_{35}(\eta) = -\frac{p\beta}{\sigma(cosh_A(\beta\eta) - sinh_A(\beta\eta) + p)}, \tag{41}$$

$$\Theta_{36}(\eta) = -\frac{\beta(sinh_A(\beta\eta) + cosh_A(\beta\eta))}{\sigma(sinh_A(\beta\eta) + cosh_A(\beta\eta) + q)}. \tag{42}$$

Case 12. *For the values of the parameters $\beta = k$, $\sigma = mk(m \neq 0)$ and $\alpha = 0$, the solution $\Theta_{37}(\eta)$ is in the form*

$$\Theta_{37}(\eta) = \frac{pA^{k\eta}}{p - mqA^{k\eta}}, \tag{43}$$

where the trigonometric values of the assigned parameters are in the form

$$sinh_A(\eta) = \frac{pA^\eta - qA^{-\eta}}{2}, cosh_A(\eta) = \frac{pA^\eta + qA^{-\eta}}{2},$$

$$tanh_A(\eta) = \frac{pA^\eta - qA^{-\eta}}{pA^\eta + qA^{-\eta}}, coth_A(\eta) = \frac{pA^\eta + qA^{-\eta}}{pA^\eta - qA^{-\eta}},$$

$$tan_A(\eta) = -i\frac{pA^{i\eta} - qA^{-i\eta}}{pA^{i\eta} + qA^{-i\eta}}, cot_A(\eta) = i\frac{pA^{i\eta} + qA^{-i\eta}}{pA^{i\eta} - qA^{-i\eta}}, \tag{44}$$

where $p, q > 0$ are arbitrary constants.

By balancing the highest order derivative with the highest power nonlinear term in Eq. (3), the determination of the N positive integer can be completed. Substitute Eq. (5), along with its requested derivatives into Eq. (3), and contrast the coefficients of powers $\Theta(\eta)$ in the final equation for obtaining the set of algebraic equations. We can get these unknowns $b_0, b_1, \ldots, b_N, c$ with the use of the symbolic computation system “Maple” in solving the over determined system of nonlinear algebraic equations. Next, we shall illustrate the steps for finding anew exact solution to Eq. (1).

New exact solutions and their graphical representations

In this section, we shall introduce some new exact solutions to Eq. (1). We will practice the NEDAM to form a traveling wave solution for the FD equation. First, we use the transformation represented in Eq. (4), Eq. (1) can be transformed into

$$v\Omega' - \frac{1}{2}\Omega\Omega'' + 2\Omega^2\Omega' - (\Omega')^2 = 0, \tag{45}$$

where the prime denotes differentiation for η. Next, by balancing the highest order derivative and nonlinear terms appearing in $\Omega\Omega''$ and $\Omega^2\Omega' \Rightarrow (2N+2 = 3N+1) \Rightarrow (N = 1)$, thus we can write the following equation

$$\Omega(\eta) = b_0 + b_1\Theta(\eta). \tag{46}$$

We reach the results for Eq. (7) by substituting into Eq. (6) as its derivatives and their coefficient of different powers $\Theta(\eta)$ after equalization to zero. Thus, we reach the following system of algebraic equations

$$\Theta^0: \ -\frac{1}{2}(\mathrm{Ln}\ A)b_1\alpha\left(-2v + (\mathrm{Ln}\ A)b_0\beta + 2(\mathrm{Ln}\ A)b_1\alpha - 4b_0^2\right) = 0,$$

$$\Theta^1: \ -\frac{1}{2}(\mathrm{Ln}\ A)b_1\left(2(\mathrm{Ln}\ A)\alpha b_0\sigma + 5(\mathrm{Ln}\ A)\alpha b_1\beta - 8\alpha b_0 b_1 - 2\beta v + (\mathrm{Ln}\ A)b_0\beta^2 - 4\beta b_0^2\right) = 0,$$

$$\Theta^2: \ -\frac{1}{2}(\mathrm{Ln}\ A)b_1\left(-4\alpha b_1^2 + 6(\mathrm{Ln}\ A)\alpha b_1\sigma + 3(\mathrm{Ln}\ A)\beta b_0\sigma + 3(\mathrm{Ln}\ A)b_1\beta^2 - 8\beta b_0 b_1 - 2\sigma v - 4\sigma b_0^2\right) = 0,$$

$$\Theta^3: \ -\frac{1}{2}(\text{Ln }A)b_1\left(-4\beta b_1^2+7(\text{Ln }A)\beta b_1\sigma+2(\text{Ln }A)b_0\sigma^2-8\sigma b_0b_1\right)=0,$$

$$\Theta^4: \ -2(\text{Ln }A)b_1^2\sigma\left(-b_1+(\text{Ln }A)\sigma\right)=0. \tag{47}$$

To reach the values of the coefficients, we solve the above system algebraic equations with Maple. Then by solving the above system, we reach the following

$$b_0=\frac{1}{2}(\text{Ln }A)\beta,\quad b_1=(\text{Ln }A)\sigma,\quad v=\left(\text{Ln}^2A\right)\left(-\frac{1}{4}\beta^2+\sigma\alpha\right) \tag{48}$$

The solutions of Eq. (1) corresponding to Eq. (8), (7) and Eq. (4) can be summarized in the following cases

Case 12. *When assifing the values of $\beta^2-4\alpha\sigma<0$ and $\sigma\neq0$, the following approximta solutions of $\vartheta(x,t)$ are as follows*

$$\vartheta_1(x,t)=\frac{(\text{Ln }A)\sqrt{-\Omega}}{2}tan_A\left(\frac{\sqrt{-\Omega}}{2}\eta\right), \tag{49}$$

$$\vartheta_2(x,t)=\frac{(\text{Ln }A)\sqrt{-\Omega}}{2\sigma}cot_A\left(\frac{\sqrt{-\Omega}}{2}\eta\right), \tag{50}$$

$$\vartheta_3(x,t)=\frac{(\text{Ln }A)\sqrt{-\Omega}}{2}\left(tan_A\left(\sqrt{-\Omega}\eta\right)\pm\sqrt{pq}\,sec_A\left(\sqrt{-\Omega}\eta\right)\right), \tag{51}$$

$$\vartheta_4(x,t)=\frac{(\text{Ln }A)\sqrt{-\Omega}}{2}\left(-cot_A\left(\sqrt{-\Omega}\eta\right)\pm\sqrt{pq}\,csc_A\left(\sqrt{-\Omega}\eta\right)\right), \tag{52}$$

$$\vartheta_5(x,t)=\frac{(\text{Ln }A)\sqrt{-\Omega}}{4}\left(tan_A\left(\frac{\sqrt{-\Omega}}{4}\eta\right)-cot_A\left(\frac{\sqrt{-\Omega}}{4}\eta\right)\right), \tag{53}$$

where $\Omega=\beta^2-4\alpha\sigma$ and $\eta=x-\frac{1}{4}(Ln^2A)\Omega t$.

Case 13. *When assigins the values of $\beta^2-4\alpha\sigma>0$ and $\sigma\neq0$, the solutions $\vartheta_6(x,t)$ to $\vartheta_{10}(x,t)$ are*

$$\vartheta_6(x,t)=\frac{(\text{Ln }A)\sqrt{\Omega}}{2}tanh_A\left(\frac{\sqrt{\Omega}}{2}\eta\right), \tag{54}$$

$$\vartheta_7(x,t)=-\frac{(\text{Ln }A)\sqrt{\Omega}}{2}coth_A\left(\frac{\sqrt{\Omega}}{2}\eta\right), \tag{55}$$

$$\vartheta_8(x,t)=\frac{(\text{Ln }A)\sqrt{\Omega}}{2}\left(-tanh_A\left(\sqrt{\Omega}\eta\right)\pm i\sqrt{pq}\,\text{sech}_A\left(\sqrt{\Omega}\eta\right)\right), \tag{56}$$

$$\vartheta_9(x,t)=\frac{(\text{Ln }A)\sqrt{\Omega}}{2}\left(-coth_A\left(\sqrt{\Omega}\eta\right)\pm\sqrt{pq}\,\text{csch}_A\left(\sqrt{\Omega}\eta\right)\right), \tag{57}$$

$$\vartheta_{10}(x,t)=-\frac{(\text{Ln }A)\sqrt{\Omega}}{4}\left(tanh_A\left(\frac{\sqrt{\Omega}}{4}\eta\right)+coth_A\left(\frac{\sqrt{\Omega}}{4}\eta\right)\right), \tag{58}$$

Where $\Omega=\beta^2-4\alpha\sigma$ and $\eta=x-\frac{1}{4}(\text{Ln}^2A)\Omega t$.

Case 14. *When assigning the values of $\alpha\sigma>0$ and $\beta=0$, the solutions $\vartheta_{11}(x,t)$ to $\vartheta_{15}(x,t)$ are*

$$\vartheta_{11}(x,t)=(\text{Ln }A)\sqrt{\alpha\sigma}tan_A\left(\sqrt{\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right), \tag{60}$$

$$\vartheta_{12}(x,t)=-(\text{Ln }A)\sqrt{\alpha\sigma}cot_A\left(\sqrt{\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right), \tag{61}$$

$$\vartheta_{13}(x,t)=(\text{Ln }A)\sqrt{\alpha\sigma}\left(tan_A\left(2\sqrt{\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)\pm\sqrt{pq}\,sec_A\left(2\sqrt{\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)\right), \tag{62}$$

$$\vartheta_{14}(x,t)=(\text{Ln }A)\sqrt{\alpha\sigma}\left(-cot_A\left(2\sqrt{\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)\pm\sqrt{pq}\,csc_A\left(2\sqrt{\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)\right), \tag{63}$$

$$\vartheta_{15}(x,t)=\frac{(\text{Ln }A)}{2}\sqrt{\alpha\sigma}\left(tan_A\left(\frac{\sqrt{\alpha\sigma}}{2}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)-cot_A\left(\frac{\sqrt{\alpha\sigma}}{2}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)\right) \tag{64}$$

Case 15. *When assigning the values of $\alpha\sigma<0$ and $\beta=0$, the solutions $\vartheta_{16}(x,t)$ to $\vartheta_{20}(x,t)$ are*

$$\vartheta_{16}(x,t)=-(\text{Ln }A)\sqrt{-\alpha\sigma}tanh_A\left(\sqrt{-\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right), \tag{65}$$

$$\vartheta_{17}(x,t)=-(\text{Ln }A)\sqrt{-\alpha\sigma}coth_A\left(\sqrt{-\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right), \tag{66}$$

$$\vartheta_{18}(x,t)=(\text{Ln }A)\sqrt{-\alpha\sigma}\left(-tanh_A\left(2\sqrt{-\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)\pm i\sqrt{pq}\,\text{sech}_A\left(2\sqrt{-\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)\right), \tag{67}$$

$$\vartheta_{19}(x,t)=(\text{Ln }A)\sqrt{-\alpha\sigma}\left(-coth_A\left(2\sqrt{-\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)\pm\sqrt{pq}\,\text{csch}_A\left(2\sqrt{-\alpha\sigma}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)\right), \tag{68}$$

$$\vartheta_{20}(x,t)=-\frac{1}{2}(\text{Ln }A)\sqrt{-\alpha\sigma}\left(tanh_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)+coth_A\left(\frac{\sqrt{-\alpha\sigma}}{2}\left(x+\left(\text{Ln}^2A\right)\alpha\sigma t\right)\right)\right) \tag{69}$$

Case 16. *When assigning the values of $\beta=0$ and $\sigma=\alpha$, the solutions $\vartheta_{21}(x,t)$ to $\vartheta_{25}(x,t)$ are*

$$\vartheta_{21}(x,t)=(\text{Ln }A)\alpha tan_A\left(\alpha\left(x+\left(\text{Ln}^2A\right)\alpha^2t\right)\right), \tag{70}$$

$$\vartheta_{22}(x,t)=-(\text{Ln }A)\alpha cot_A\left(\alpha\left(x+\left(\text{Ln}^2A\right)\alpha^2t\right)\right), \tag{71}$$

$$\vartheta_{23}=(\text{Ln }A)\alpha\left(tan_A\left(2\alpha\left(x+\left(\text{Ln}^2A\right)\alpha^2t\right)\right)\pm\sqrt{pq}\,sec_A\left(2\alpha\left(x+\left(\text{Ln}^2A\right)\alpha^2t\right)\right)\right), \tag{72}$$

$$\vartheta_{24}=(\text{Ln }A)\alpha\left(-cot_A\left(2\alpha\left(x+\left(\text{Ln}^2A\right)\alpha^2t\right)\right)\pm\sqrt{pq}\,csc_A\left(2\alpha\left(x+\left(\text{Ln}^2A\right)\alpha^2t\right)\right)\right), \tag{73}$$

$$\vartheta_{25}=\frac{(\text{Ln }A)\alpha}{2}\left(tan_A\left(\frac{\alpha}{2}\left(x+\left(\text{Ln}^2A\right)\alpha^2t\right)\right)-cot_A\left(\frac{\alpha}{2}\left(x+\left(\text{Ln}^2A\right)\alpha^2t\right)\right)\right) \tag{74}$$

Case 17. *When assigning the values of $\beta=0$ and $\sigma=-\alpha$, the solutions $\vartheta_{26}(x,t)$ to $\vartheta_{30}(x,t)$ are*

$$\vartheta_{26}(x,t)=(\text{Ln }A)\alpha tanh_A\left(\alpha\left(x-\left(\text{Ln}^2A\right)\alpha^2t\right)\right), \tag{75}$$

$$\vartheta_{27}(x,t)=-(\text{Ln }A)\alpha coth_A(\alpha\xi), \tag{76}$$

$$\vartheta_{28} = -(\text{Ln } A)\alpha\left(-tanh_A\left(2\alpha\left(x - \left(\text{Ln}^2 A\right)\alpha^2 t\right)\right)\right.$$
$$\left.\pm i\sqrt{pq}\,\text{sech}_A\left(2\alpha\left(x - \left(\text{Ln}^2 A\right)\alpha^2 t\right)\right)\right), \quad (77)$$

$$\vartheta_{29} = -(\text{Ln } A)\alpha\left(-coth_A\left(2\alpha\left(x - \left(\text{Ln}^2 A\right)\alpha^2 t\right)\right)\right.$$
$$\left.\pm\sqrt{pq}\,\text{csch}_A\left(2\alpha\left(x - \left(\text{Ln}^2 A\right)\alpha^2 t\right)\right)\right), \quad (78)$$

$$\vartheta_{30} = \frac{(\text{Ln } A)}{2}\alpha\left(tanh_A\left(\frac{\alpha}{2}\left(x - \left(\text{Ln}^2 A\right)\alpha^2 t\right)\right) + coth_A\left(\frac{\alpha}{2}\left(x - \left(\text{Ln}^2 A\right)\alpha^2 t\right)\right)\right) \quad (79)$$

Case 18. *When $\beta = \alpha = 0$, the solution $\vartheta_{31}(x,t)$ is in the form*

$$\vartheta_{31}(x,t) = \frac{-1}{x - \frac{\left(\text{Ln}^2 A\right)}{4}\beta^2 t}, \quad (80)$$

Case 19. *When $\alpha = 0$ and $\beta \neq 0$, the solutions $\vartheta_{32}(x,t)$ and $\vartheta_{33}(x,t)$ are in the form*

$$\vartheta_{32} = (\text{Ln}A)\beta\left[\frac{1}{2} - \frac{p}{cosh_A\left(\beta\left(x - \frac{\left(\text{Ln}^2 A\right)}{4}\beta^2 t\right)\right) - sinh_A\left(\beta\left(x - \frac{\left(\text{Ln}^2 A\right)}{4}\beta^2 t\right)\right) + p}\right], \quad (81)$$

$$\vartheta_{33} = (\text{Ln}A)\beta\left[\frac{1}{2} - \frac{sinh_A\left(\beta\left(x - \frac{\left(\text{Ln}^2 A\right)}{4}\beta^2 t\right)\right) + cosh_A\left(\beta\left(x - \frac{\left(\text{Ln}^2 A\right)}{4}\beta^2 t\right)\right)}{sinh_A\left(\beta\left(x - \frac{\left(\text{Ln}^2 A\right)}{4}\beta^2 t\right)\right) + cosh_A\left(\beta\left(x - \frac{\left(\text{Ln}^2 A\right)}{4}\beta^2 t\right)\right) + q}\right] \quad (82)$$

Case 20. *When $\beta = k$, $\sigma = mk(m \neq 0)$ and $\alpha = 0$, the solution $\vartheta_{34}(x,t)$ is in the form*

$$\vartheta_{34}(x,t) = (\text{Ln } A)k\left[\frac{1}{2} + \frac{mpA^{k\left(x - \frac{\left(\text{Ln}^2 A\right)}{4}k^2 t\right)}}{p - mqA^{k\left(x - \frac{\left(\text{Ln}^2 A\right)}{4}k^2 t\right)}}\right] \quad (83)$$

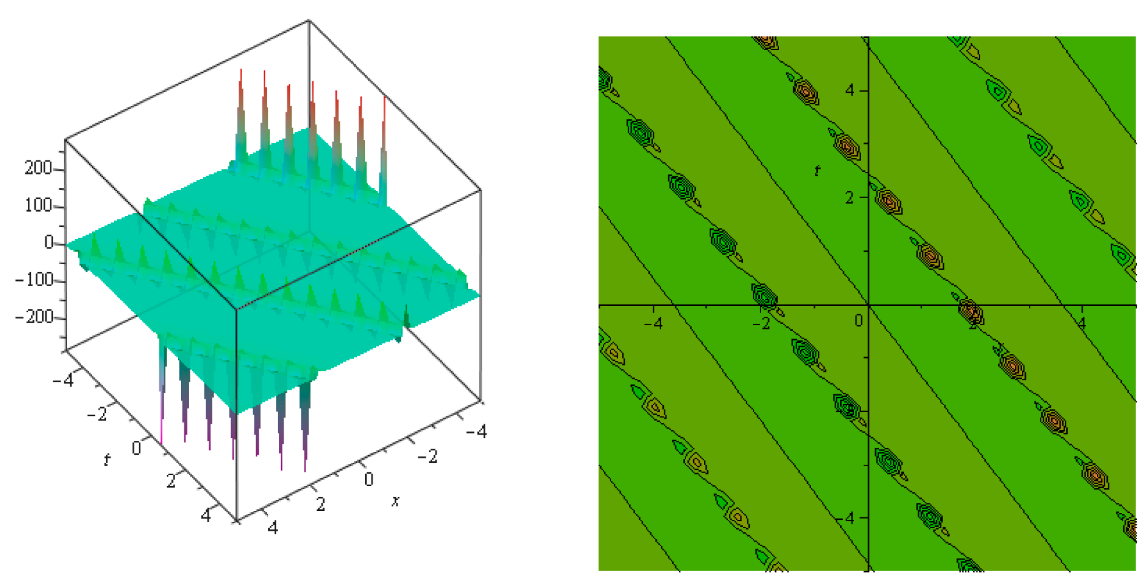
The above-mentioned cases gave a new form of solutions of the FD equation represented in Eq. (1). These solutions are new and have never been acquired before using this method. In the next section, we will illustrate the graphical representation of these solutions.

Comparison and physical explanation

Figs. 2 and 3 depict the 3D and contour plots of the $\vartheta_j(x,t)$, j = 1, 10, 34. Fig. 2 represents the FD equation solution given by Eq. (49). We observe in Fig. 1(a) and (b) that the $\vartheta_1(x,t)$ is a singular periodic wave. Fig. 3 depicts the wave solution given by Eq. (58). We observe, in Fig. 2 (a) and (b) that the $\vartheta_{10}(x,t)$ is a solitary wave. Fig. 4 illustrates the solitary wave solution given by eq. (83). We observe in Fig. 3(a) and (b) that the $\vartheta_{34}(x,t)$ is a singular solitary wave. In Figs. 2 and 3, we plot the solutions of Eqs. (49) and (58), by taking different values of parameters, β, α, σ, A, p, q respectively. We have shown the solution of Eq. (83) with parameters $m = 4$, $k = 3$, $A = 3$, $p = q$ in Fig. 4.

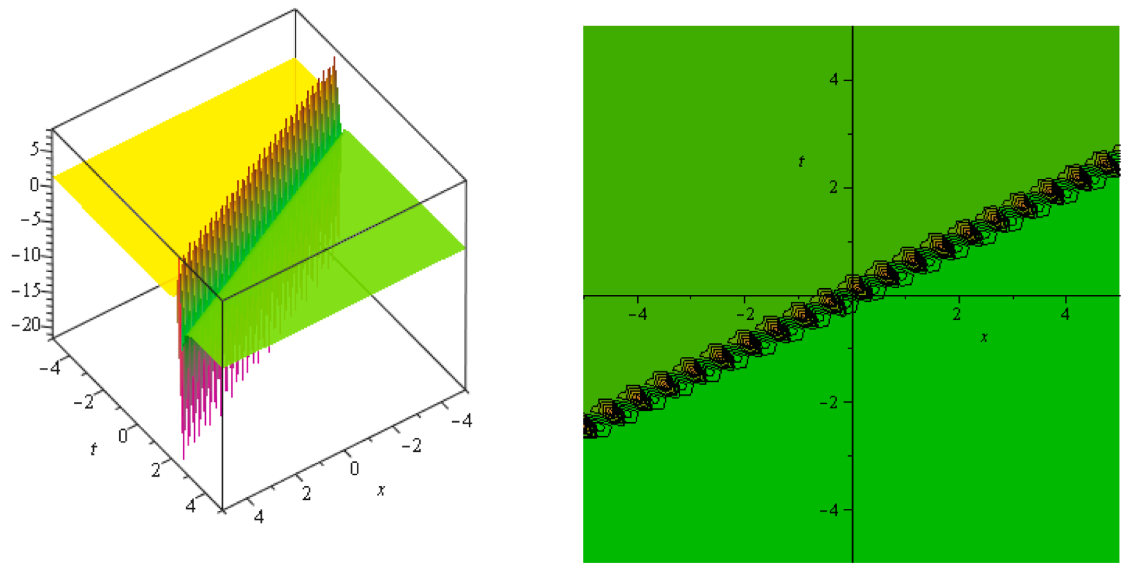
Conclusions

In this research, some novel solutions of the foam drainage (FD) equation are achieved with the aid of a new method called the new extended direct algebraic method (NEDAM). The method is based on transforming the nonlinear partial differential equation into an ordinary type through some wave transformation and this equation is then converted into a system of algebraic equations. By solving this system, the coefficients are found and several cases for the solutions of the FD equation are illustrated. We have shown and proved that NEDAM provides an effective and suitable method to find the exact solution and some physical behaviors of these solutions are plotted and illustrated in figures under the choice of suitable parameters. The symbolic computational calculations for the results here obtained were performed with the aid of Maple.



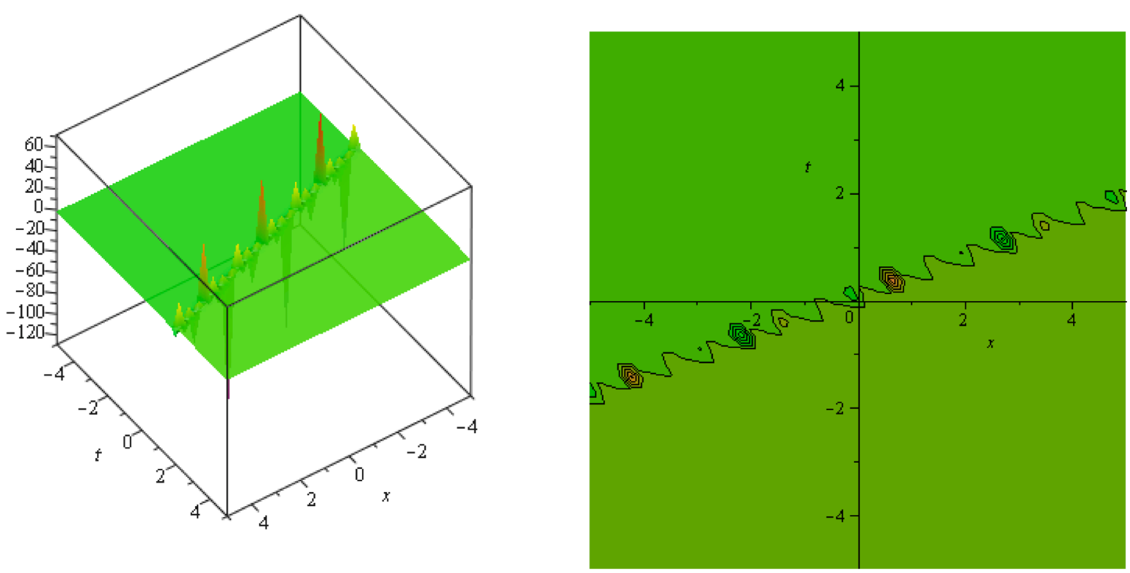
(a) 3D plot **(b)** contour plot

Fig. 2. Contour and 3-D plots of the solution to Eq. (49) with parameters $\beta = 3$, $\alpha = 3$, $\sigma = 1$, $A = e$, $p = 1$, $q = 1$.



(a) 3D plot **(b)** contour plot

Fig. 3. Contour and 3-D plots of the solution to Eq. (58) with parameters $\beta = 4$, $\alpha = 1$, $\sigma = 2$, $A = e$, $p = 0.95$, $q = 0.9$.



(a) 3D plot **(b)** contour plot

Fig. 4. Contour and 3-D plots of the solution to Eq. (83) with parameters $m = 4$, $k = 3$, $A = 3$, $p = q$.

CRediT authorship contribution statement

Wen-Hui Zhu: Investigation. **Arash Pashrashid:** Conceptualization, Supervision, Writing - review & editing, Validation. **Waleed Adel:** Software, Data curation, Formal analysis. **Hatira Gunerhan:** Data curation, Software, Investigation. **KottakkaranSooppy Nisar:** Investigation. **C. Ahamed Saleel:** Funding acquisition. **Mustafa Inc:** Formal analysis. **Hadi Rezazadeh:** Formal analysis.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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