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REGULAR PAPER

A new approach to neutrosophic soft rough sets

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Abstract
Rough set is a world-renowned innovation for dealing with ambiguous, incomplete, and imprecise situations. Soft set theory and neutrosophic set theory are other advanced mathematical techniques for dealing with ambiguous, partial, and inconsistent data. The aim of this paper is to broaden the scope of rough set theory, soft set theory, and neutrosophic set theory. The notion of neutrosophic soft rough sets have been re-introduced. On neutrosophic soft rough set, several definitions, properties, and examples have been established. We also develop the concept of neutrosophic soft rough topology, which is based on a novel neutrosophic soft rough set approach. We have defined open sets, closed sets, interior, and closure as characteristics of neutrosophic soft rough topology.

Keywords Rough sets · Neutrosophic soft rough sets · Neutrosophic soft rough topology

1 Introduction

In 1999, F. Smarandache presented the idea of neutrosophic set (NS) [1], which is an extension of classical sets, fuzzy set [2] , intuitionistic fuzzy set [3], and so on. Indeterminate data are handled by the neutrosophic set notion, however fuzzy set theory and intuitionistic fuzzy set theory fail when the relation is indeterminate. Truth membership function (T), indeterminacy membership function (I), and falsity membership function (F) characterize neutrosophic sets. This theory is particularly essential in many application areas since indeterminacy is explicitly specified and the truth, indeterminacy, and falsity membership functions are independent. Wang et al. [4] presented an extension of neutrosophic set called interval neutrosophic set (INS). In the actual world, the interval neutrosophic set (INS) can

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reflect ambiguous, inaccurate, partial, and inconsistent information. The notion of single valued neutrosophic set was presented by Wang et al. [5]. The single-valued neutrosophic set may represent truth-membership degree, indeterminacy-membership degree, and falsity-membership degree separately, and it deals with incomplete, indeterminate, and inconsistent information. Because of the imperfection of knowledge that humans get or witness from the outside world, all of the components indicated by the single-valued neutrosophic set are particularly fit for human reasoning.

The rough set theory, which was presented by Pawlak [6] in 1982 as a strategy for handling uncertainty and imperfection, may successfully assess incomplete information. Moreover, many new rough set models have been developed by many scholars [7–12] based on different aspects, namely the universe, relations, objects, and operators, such as rough fuzzy sets, fuzzy rough sets, generalized fuzzy rough sets, and intuitionistic fuzzy rough sets. Broumi et al. [7] defined and demonstrated the basic features of the rough neutrosophic set. In the literature, certain theoretical advances and applications have been presented [13–17].

Molodtsov presented a new mathematical tool called "soft set theory" [18] in 1999 for dealing with uncertainty, and how soft set theory is free of the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, and probability theory. This notion generalizes the soft set, fuzzy soft set [19], intuitionistic fuzzy soft set [20], neutrosophic soft set [21] and intuitionistic neutrosophic soft set [22]. Hybrid structures consisting of soft sets and other set structures continue to be studied intensively [23–33].

By combining rough sets and neutrosophic soft sets, Broumi et al. [34] created a hybrid structure known as rough neutrosophic soft sets (NSRSs). They also introduced interval valued neutrosophic soft rough sets by fusing interval valued neutrosophic soft sets and rough sets. By fusing single valued neutrosophic soft sets with rough sets, Yang et al. [35] presented single valued neutrosophic rough sets (SVNRSs), defined SVNRSs on two universes, and created an algorithm for a problem based on SVNRSs on two universes. Neutrosophic soft graphs have been introduced by Akram and Shahzadi [36]. Marei [37] in 2015 , S. Debnath [38] in 2019, Al-Quran et al. [14] in 2019, Riaz et al. [39] in 2020 and Das. et al. [40] in 2021 have redefined neutrosophic soft rough sets. However, the methods utilized for operations on a neutrosophic rough set are not all-encompassing and have certain drawbacks. As a result, we provide a novel concept for neutrosophic soft rough sets.

These are the study's primary contributions, in brief:

1. In previous approaches [14, 37–40], membership functions are either completely or partially ignored. In this study, membership functions are fully used in a functional state.
2. Neutrosophic soft rough set structure is reconstructed and the neutrosophic soft rough topology is constructed using this structure.
3. Not using membership functions greatly affects the difference of the structure from the classical rough soft structure. With this study, all these problems have been eliminated and the structure has been made more functional.
4. Basic concepts such as open set, closed set, interior and closure are studied on newly defined neutrosophic soft rough topological spaces.

The organization of this work is as follows: The literature material, motivation, and requirement to develop new concepts about the neutrosophic soft rough sets and its topology are presented in Sect. 1. A brief background of soft sets, neutrosophic sets, and rough sets is provided in Sect. 2. In Sect. 3, the neutrosophic soft rough set structure is redefined and important propositions and examples are presented. In Sect. 4, neutrosophic soft rough topological spaces are defined and important propositions and examples are presented. In Sect. 5, the proposed method using the cosine similarity measure was applied to decision making

problems. In Sect. 6, we compared the proposed neutrosophic soft rough approach with other approaches that combine rough set with neutrosophic soft sets [14, 39, 40]. In Sect. 7, information about the results of the study is presented.

2 Preliminaries

In this section, we present some basic definitions including soft set, rough set, neutrosophic soft sets and some operations on neutrosophic soft sets.

Definition 1 [18] Let Σ be an initial universe, Υ be a set of all parameters and $P(\Sigma)$ denotes the power set of Σ. A pair (N, Υ) is called a soft set over Σ, where N is a mapping given by $N : \Upsilon \to P(\Sigma)$.

In other words, the soft set is a parameterized family of subsets of the set Σ . For $\sigma \in \Upsilon$, $N(\sigma)$ may be considered as the set of σ-elements of the soft set (N, Υ), or as the set of σ-approximate elements of the soft set, i.e.,

$$(N, \Upsilon) = \{(\sigma, N(\sigma)) : \sigma \in \Upsilon, N : \Upsilon \to P(\Sigma)\}.$$

Definition 2 [6] Let Σ be the initial universe and Υ be a set of parameters. Then, lower, upper, and boundary approximations of Υ are defined as

$$\Re_{_}(\Upsilon) = \bigcup_{n \in \Sigma} \{\Re(n) : \Re(n) \subseteq \Upsilon\},$$
$$\Re^{-}(\Upsilon) = \bigcup_{n \in \Sigma} \{\Re(n) : \Re(n) \cap \Upsilon \neq \emptyset\},$$

and

$$B\Re(\Upsilon) = \Re^{-}(\Upsilon) \setminus \Re_{_}(\Upsilon),$$

respectively, where $\Re$ is an indiscernibility relation $\Re \subseteq \Sigma \times \Sigma$ which indicates our information about elements of Σ. The set Υ is said to be defined if $\Re^{-}(\Upsilon) = \Re_{_}(\Upsilon)$. If $\Re^{-}(\Upsilon) \neq \Re_{_}(\Upsilon)$, i.e., $B\Re(\Upsilon) \neq \varnothing$, the set Υ is rough set w.r.t $\Re$.

Definition 3 [1] A neutrosophic set N on the universe of discourse Σ is defined as:

$$N = \{< n, \mu_N(n), \upsilon_N(n), \omega_N(n) >: n \in \Sigma\},$$

where $\mu, \upsilon, \omega : \Sigma \to \left]^{-}0, 1^{+}\right[$ and $^{-}0 \leq \mu_N(n) + \upsilon_N(n) + \omega_N(n) \leq 3^{+}$.

Definition 4 [41] Let Σ be an initial universe set and Υ be a set of parameters. Let $NS(\Sigma)$ denote the set of all neutrosophic sets of Σ. Then, a neutrosophic soft set $\left(\widetilde{N}, \Upsilon\right)$ over Σ is a set defined by a set valued function $\widetilde{N}$ representing a mapping $\widetilde{N} : \Upsilon \to NS(\Sigma)$ where $\widetilde{N}$ is called approximate function of the neutrosophic soft set $\left(\widetilde{N}, \Upsilon\right)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $NS(\Sigma)$ and therefore it can be written as a set of ordered pairs,

$$\left(\widetilde{N}, \Upsilon\right) = \{\left(\sigma, < n, \mu_{\widetilde{N}(\sigma)}(n), \upsilon_{\widetilde{N}(\sigma)}(n), \omega_{\widetilde{N}(\sigma)}(n) >: n \in \Sigma\right) : \sigma \in \Upsilon\}.$$

where $\mu_{\widetilde{N}(\sigma)}(n), \upsilon_{\widetilde{N}(\sigma)}(n), \omega_{\widetilde{N}(\sigma)}(n) \in [0, 1]$, respectively, called the truth-membership, indeterminacy-membership, falsity-membership function of $\widetilde{N}(\sigma)$. Since supremum of each μ,υ,ω is 1 so the inequality $0 \leq \mu_{\widetilde{N}(\sigma)}(n) + \upsilon_{\widetilde{N}(\sigma)}(n) + \omega_{\widetilde{N}(\sigma)}(n) \leq 3$ is obvious. We denote the set of all neutrosophic soft sets of Σ by $NSS(\Sigma, \Upsilon)$.

Definition 5 [42] Let $\left(\widetilde{N}, \Upsilon\right)$ be neutrosophic soft set over the universe set Σ. The complement of $\left(\widetilde{N}, \Upsilon\right)$ is denoted by $\left(\widetilde{N}, \Upsilon\right)^c$ and is defined by:

$$\left(\widetilde{N}, \Upsilon\right)^c = \{\left(\sigma, < n, w_{\widetilde{N}(\sigma)}(n), 1 - \upsilon_{\widetilde{N}(\sigma)}(n), \mu_{\widetilde{N}(\sigma)}(n) >: n \in \Sigma\right) : \sigma \in \Upsilon\}.$$

Obvious that, $\left(\left(\widetilde{N}, \Upsilon\right)^c\right)^c = \left(\widetilde{N}, \Upsilon\right)$.

Definition 6 [21]. Let $\left(\widetilde{N}_1, \Upsilon\right)$ and $\left(\widetilde{N}_2, \Upsilon\right)$ be two neutrosophic soft sets over the universe set Σ . $\left(\widetilde{N}_1, \Upsilon\right)$ is said to be neutrosophic soft subset of $\left(\widetilde{N}_2, \Upsilon\right)$ if $\mu_{\widetilde{N}_1(\sigma)}(n) \leq \mu_{\widetilde{N}_2(\sigma)}(n)$, $\upsilon_{\widetilde{N}_1(\sigma)}(n) \leq \upsilon_{\widetilde{N}_2(\sigma)}(n)$, $\omega_{\widetilde{N}_1(\sigma)}(n) \geq \omega_{\widetilde{N}_2(\sigma)}(n)$, for all $\sigma \in \Upsilon$, for all $n \in \Sigma$. It is denoted by $\left(\widetilde{N}_1, \Upsilon\right) \subseteq \left(\widetilde{N}_2, \Upsilon\right)$.

$\left(\widetilde{N}_1, \Upsilon\right)$ is said to be neutrosophic soft equal to $\left(\widetilde{N}_2, \Upsilon\right)$ if $\left(\widetilde{N}_1, \Upsilon\right)$ is neutrosophic soft subset of $\left(\widetilde{N}_2, \Upsilon\right)$ and $\left(\widetilde{N}_2, \Upsilon\right)$ is neutrosophic soft subset of $\left(\widetilde{N}_1, \Upsilon\right)$. It is denoted by $\left(\widetilde{N}_1, \Upsilon\right) = \left(\widetilde{N}_2, \Upsilon\right)$.

Definition 7 [43] Let $\left(\widetilde{N}_1, \Upsilon\right)$ and $\left(\widetilde{N}_2, \Upsilon\right)$ be two neutrosophic soft sets over the universe set Σ. Then, their union is denoted by $\left(\widetilde{N}_1, \Upsilon\right) \cup \left(\widetilde{N}_2, \Upsilon\right) = \left(\widetilde{N}_3, \Upsilon\right)$ and is defined by:

$$\left(\widetilde{N}_3, \Upsilon\right) = \{\left(\sigma, < n, \mu_{\widetilde{N}_3(\sigma)}(n), \upsilon_{\widetilde{N}_3(\sigma)}(n), \omega_{\widetilde{N}_3(\sigma)}(n) >: n \in \Sigma\right) : \sigma \in \Upsilon\}.$$

where

$$\begin{aligned}
\mu_{\widetilde{N}_3(\sigma)}(n) &= \max\left\{\mu_{\widetilde{N}_1(\sigma)}(n), \mu_{\widetilde{N}_2(\sigma)}(n)\right\}, \\
\upsilon_{\widetilde{N}_3(\sigma)}(n) &= \max\left\{\upsilon_{\widetilde{N}_1(\sigma)}(n), \upsilon_{\widetilde{N}_2(\sigma)}(n)\right\}, \\
\omega_{\widetilde{N}_3(\sigma)}(n) &= \min\left\{\omega_{\widetilde{N}_1(\sigma)}(n), \omega_{\widetilde{N}_2(\sigma)}(n)\right\}.
\end{aligned}$$

Definition 8 [43] Let $\left(\widetilde{N}_1, \Upsilon\right)$ and $\left(\widetilde{N}_2, \Upsilon\right)$ be two neutrosophic soft sets over the universe set Σ. Then, their intersection is denoted by $\left(\widetilde{N}_1, \Upsilon\right) \cap \left(\widetilde{N}_2, \Upsilon\right) = \left(\widetilde{N}_3, \Upsilon\right)$ and is defined by:

$$\left(\widetilde{N}_3, \Upsilon\right) = \{\left(\sigma, < n, \mu_{\widetilde{N}_3(\sigma)}(n), \upsilon_{\widetilde{N}_3(\sigma)}(n), \omega_{\widetilde{N}_3(\sigma)}(n) >: n \in \Sigma\right) : \sigma \in \Upsilon\}.$$

where

$$\begin{aligned}
\mu_{\widetilde{N}_3(\sigma)}(n) &= \min\left\{\mu_{\widetilde{N}_1(\sigma)}(n), \mu_{\widetilde{N}_2(\sigma)}(n)\right\}, \\
\upsilon_{\widetilde{N}_3(\sigma)}(n) &= \min\left\{\upsilon_{\widetilde{N}_1(\sigma)}(n), \upsilon_{\widetilde{N}_2(\sigma)}(n)\right\}, \\
\omega_{\widetilde{N}_3(\sigma)}(n) &= \max\left\{\omega_{\widetilde{N}_1(\sigma)}(n), \omega_{\widetilde{N}_2(\sigma)}(n)\right\}.
\end{aligned}$$

Definition 9 [43] Let $\left(\widetilde{N}_1, \Upsilon\right)$ and $\left(\widetilde{N}_2, \Upsilon\right)$ be two neutrosophic soft sets over the universe set Σ. Then, "$\left(\widetilde{N}_1, \Upsilon\right)$ difference $\left(\widetilde{N}_2, \Upsilon\right)$" operation on them is denoted by $\left(\widetilde{N}_1, \Upsilon\right) \setminus \left(\widetilde{N}_2, \Upsilon\right) = \left(\widetilde{N}_3, \Upsilon\right)$ and is defined by $\left(\widetilde{N}_3, \Upsilon\right) = \left(\widetilde{N}_1, \Upsilon\right) \cap \left(\widetilde{N}_2, \Upsilon\right)^c$ as follows:

$$\left(\widetilde{N}_3, \Upsilon\right) = \{\left(\sigma, < n, \mu_{\widetilde{N}_3(\sigma)}(n), \upsilon_{\widetilde{N}_3(\sigma)}(n), \omega_{\widetilde{N}_3(\sigma)}(n) >: n \in \Sigma\right) : \sigma \in \Upsilon\}.$$

where

$$\mu_{\widetilde{N}_3(\sigma)}(n) = \min\left\{\mu_{\widetilde{N}_1(\sigma)}(n), \mu_{\widetilde{N}_2(\sigma)}(n)\right\},$$
$$\upsilon_{\widetilde{N}_3(\sigma)}(n) = \min\left\{\upsilon_{\widetilde{N}_1(\sigma)}(n), 1-\upsilon_{\widetilde{N}_2(\sigma)}(n)\right\},$$
$$\omega_{\widetilde{N}_3(\sigma)}(n) = \max\left\{\omega_{\widetilde{N}_1(\sigma)}(n), \omega_{\widetilde{N}_2(\sigma)}(n)\right\}.$$

Definition 10 [43]

1. A neutrosophic soft set $\left(\widetilde{N}, \Upsilon\right)$ over the universe set Σ is said to be null neutrosophic soft set if $\mu_{\widetilde{N}_3(\sigma)}(n) = 0$, $\upsilon_{\widetilde{N}_3(\sigma)}(n) = 0$, $\omega_{\widetilde{N}_3(\sigma)}(n) = 1$; for all $\sigma \in \Upsilon$, for all $n \in \Upsilon$. It is denoted by $0_{(\Sigma,\Upsilon)}$.
2. A neutrosophic soft set $\left(\widetilde{N}, \Upsilon\right)$ over the universe set Σ is said to be absolute neutrosophic soft set if $\mu_{\widetilde{N}_3(\sigma)}(n) = 1$, $\upsilon_{\widetilde{N}_3(\sigma)}(n) = 1$, $\omega_{\widetilde{N}_3(\sigma)}(n) = 0$; for all $\sigma \in \Upsilon$, for all $n \in \Upsilon$. It is denoted by $1_{(\Sigma,\Upsilon)}$.
   Clearly, $0^c_{(\Sigma,\Upsilon)} = 1_{(\Sigma,\Upsilon)}$ and $1^c_{(\Sigma,\Upsilon)} = 0_{(\Sigma,\Upsilon)}$.

Definition 11 [44] Let $\Sigma = \{n_1, n_2, \ldots, n_k\}$ be an initial universe set and $\Upsilon = \{\sigma_1, \sigma_2, \ldots, \sigma_m\}$ be a set of parameters. Let $\left(\widetilde{N}_1, \Upsilon\right)$ and $\left(\widetilde{N}_2, \Upsilon\right)$ be two neutrosophic soft sets over the universe set Σ. Then, cosine similarity measures between $\left(\widetilde{N}_1, \Upsilon\right)$ and $\left(\widetilde{N}_2, \Upsilon\right)$ defined as; $CS(\wp)$

$$= \frac{1}{km}\sum_{i=1}^{m}\sum_{j=1}^{k}\cos\left[\frac{\pi}{6}\left(\begin{array}{c}\left|\mu_{\widetilde{N}_1(\sigma_i)}(n_j) - \mu_{\widetilde{N}_2(\sigma_i)}(n_j)\right| + \left|\upsilon_{\widetilde{N}_1(\sigma_i)}(n_j) - \upsilon_{\widetilde{N}_2(\sigma_i)}(n_j)\right| + \\ \left|\omega_{\widetilde{N}_1(\sigma_i)}(n_j) - \omega_{\widetilde{N}_2(\sigma_i)}(n_j)\right|\end{array}\right)\right].$$

3 Neutrosophic soft rough sets

Definition 12 Let Σ be a non-empty universe set, Υ be a set of parameters, and $NS(\Sigma)$ be the set of all neutrosophic set over Σ. The family $\left(\widetilde{N}, \Upsilon\right)$ be a neutrosophic soft set over Σ, where $\widetilde{N}$ is a mapping given by $\widetilde{N} : \Upsilon \to NS(\Sigma)$. Then $\left(\Sigma, \widetilde{N}, \Upsilon\right)$ is called neutrosophic soft approximation space. The lower and upper neutrosophic soft approximation of $\zeta \subseteq NSS(\Sigma, \Upsilon)$ with respect to $\left(\Sigma, \widetilde{N}, \Upsilon\right)$ are denoted by $\underline{apr}_{NS}(\zeta)$ and $\overline{apr}_{NS}(\zeta)$, respectively, defined by

$$\underline{apr}_{NS}(\zeta) = \left\{\left(\sigma_i < \frac{n}{\underline{\mu}_{\sigma_i}(n), \underline{\upsilon}_{\sigma_i}(n), \underline{\omega}_{\sigma_i}(n)} >\right), \forall\sigma_i \in \Upsilon, \forall n \in \Sigma\right\}$$
$$\overline{apr}_{NS}(\zeta) = \left\{\left(\sigma_i < \frac{n}{\overline{\mu}_{\sigma_i}(n), \overline{\upsilon}_{\sigma_i}(n), \overline{\omega}_{\sigma_i}(n)} >\right), \forall\sigma_i \in \Upsilon, \forall n \in \Sigma\right\}$$

where,

$$\underline{\mu}_{\sigma_i}(n) = \left\{ \bigwedge \mu_{\sigma_i}(n) : \mu_{\sigma_i}(n) \in \zeta \cap \left(\widetilde{N}_i, \Upsilon\right); \forall \left(\widetilde{N}_i, \Upsilon\right) \subseteq \zeta, \forall \sigma_i \in \Upsilon, \forall n \in \Sigma \right\}$$

$$\underline{\upsilon}_{\sigma_i}(n) = \left\{ \bigwedge \upsilon_{\sigma_i}(n) : \upsilon_{\sigma_i}(n) \in \zeta \cap \left(\widetilde{N}_i, \Upsilon\right); \forall \left(\widetilde{N}_i, \Upsilon\right) \subseteq \zeta, \forall \sigma_i \in \Upsilon, \forall n \in \Sigma \right\}$$

$$\underline{\omega}_{\sigma_i}(n) = \left\{ \bigvee \omega_{\sigma_i}(n) : \omega_{\sigma_i}(n) \in \zeta \cap \left(\widetilde{N}_i, \Upsilon\right); \forall \left(\widetilde{N}_i, \Upsilon\right) \subseteq \zeta, \forall \sigma_i \in \Upsilon, \forall n \in \Sigma \right\}$$

$$\overline{\mu}_{\sigma_i}(n) = \left\{ \bigvee \mu_{\sigma_i}(n) : \mu_{\sigma_i}(n) \in \zeta \cup \left(\widetilde{N}_i, \Upsilon\right); \forall \left(\widetilde{N}_i, \Upsilon\right) \subseteq \zeta, \forall \sigma_i \in \Upsilon, \forall n \in \Sigma \right\}$$

$$\overline{\upsilon}_{\sigma_i}(n) = \left\{ \bigvee \upsilon_{\sigma_i}(n) : \upsilon_{\sigma_i}(n) \in \zeta \cup \left(\widetilde{N}_i, \Upsilon\right); \forall \left(\widetilde{N}_i, \Upsilon\right) \subseteq \zeta, \forall \sigma_i \in \Upsilon, \forall n \in \Sigma \right\}$$

$$\overline{\omega}_{\sigma_i}(n) = \left\{ \bigwedge \omega_{\sigma_i}(n) : \omega_{\sigma_i}(n) \in \zeta \cup \left(\widetilde{N}_i, \Upsilon\right); \forall \left(\widetilde{N}_i, \Upsilon\right) \subseteq \zeta, \forall \sigma_i \in \Upsilon, \forall n \in \Sigma \right\}$$

where "$\bigwedge$" and "$\bigvee$" mean "min" and "max" operators, respectively. It is easy see that $\underline{apr}_{NS}(\zeta)$ and $\overline{apr}_{NS}(\zeta)$ are two neutrosophic soft set on $NSS(\Sigma, \Upsilon)$. If $\underline{apr}_{NS}(\zeta) = \overline{apr}_{NS}(\zeta)$, then ζ is said to be neutrosophic soft definable set; otherwise, it is called a neutrosophic soft rough set.

Example 1 Let $\Sigma = \{n_1, n_2, n_3, n_4\}$ be a universe and $\Upsilon = \{\sigma_1, \sigma_2, \sigma_3\}$ be a parameter set. We consider following neutrosophic soft sets;

$$\left(\widetilde{N}_1, \Upsilon\right) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.3,0.5,0.7}, \frac{n_3}{0.6,0.5,0.4}, \frac{n_4}{0.7,0.3,0.2} >\right), \\ \left(\sigma_2 < \frac{n_2}{0.6,0.4,0.5}, \frac{n_3}{0.4,0.5,0.7}, \frac{n_4}{0.2,0.3,0.2} >\right) \end{array} \right\}$$

$$\left(\widetilde{N}_2, \Upsilon\right) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.6,0.5,0.5}, \frac{n_2}{0.3,0.4,0.3} >\right), \left(\sigma_2 < \frac{n_3}{0.8,0.6,0.2}, \frac{n_4}{0.7,0.3,0.2} >\right), \\ \left(\sigma_3 < \frac{n_2}{0.2,0.2,0.5}, \frac{n_4}{0.3,0.5,0.3} >\right) \end{array} \right\}$$

$$\left(\widetilde{N}_3, \Upsilon\right) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.8,0.7,0.3}, \frac{n_4}{0.6,0.2,0.3} >\right), \left(\sigma_2 < \frac{n_2}{0.7,0.4,0.3}, \frac{n_3}{0.8,0.7,0.1}, \frac{n_4}{0.7,0.5,0.1} >\right), \\ \left(\sigma_3 < \frac{n_2}{0.4,0.5,0.2}, \frac{n_3}{0.8,0.7,0.3}, \frac{n_4}{0.6,0.8,0.2} >\right) \end{array} \right\}$$

for the another neutrosophic soft set $\zeta \subset NSS(\Sigma, \Upsilon)$ with respect to $\left(\Sigma, \widetilde{N}, \Upsilon\right)$;

$$\zeta = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.7,0.6,0.4}, \frac{n_2}{0.8,0.6,0.2}, \frac{n_3}{0.8,0.6,0.2}, \frac{n_4}{0.9,0.5,0.1} >\right), \\ \left(\sigma_2 < \frac{n_2}{0.7,0.4,0.3}, \frac{n_3}{0.8,0.7,0.1}, \frac{n_4}{0.7,0.5,0.1} >\right), \\ \left(\sigma_3 < \frac{n_2}{0.4,0.5,0.2}, \frac{n_3}{0.8,0.7,0.3}, \frac{n_4}{0.6,0.8,0.2} >\right) \end{array} \right\}$$

The lower and upper neutrosophic soft approximations of ζ are calculated as

$$\underline{apr}_{NS}(\zeta) = \left\{ \left(\sigma_1 < \frac{n_1}{0.3, 0.5, 0.7} >\right), \left(\sigma_2 < \frac{n_3}{0.4, 0.5, 0.7}, \frac{n_4}{0.2, 0.3, 0.2} >\right) \right\}$$

$$\overline{apr}_{NS}(\zeta) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.8,0.7,0.3}, \frac{n_2}{0.8,0.6,0.2}, \frac{n_3}{0.8,0.6,0.2}, \frac{n_4}{0.9,0.5,0.1} >\right), \\ \left(\sigma_2 < \frac{n_2}{0.7,0.4,0.3}, \frac{n_3}{0.8,0.7,0.1}, \frac{n_4}{0.6,0.8,0.1} >\right), \\ \left(\sigma_3 < \frac{n_2}{0.4,0.5,0.2}, \frac{n_3}{0.8,0.7,0.3}, \frac{n_4}{0.6,0.8,0.1} >\right) \end{array} \right\}$$

Remark 1 For any $\zeta \subseteq NSS(\Sigma, \Upsilon)$, the sets $POS_{NS}(\zeta) = \underline{apr}_{NS}(\zeta)$, $NEG_{NS}(\zeta) = \overline{apr}_{NS}(\zeta)$ ve $Bnd_{NS}(\zeta) = \overline{apr}_{NS}(\zeta) \setminus \underline{apr}_{NS}(\zeta)$ are called neutrosophic soft rough positive, neutrosophic soft rough negative, and neutrosophic soft rough boundary regions of a considered set ζ, respectively.

Example 2 We consider Example 1 . Then, we can write the neutrosophic soft rough positive, negative, and boundary regions as follows:

$$POS_{NS}(\zeta) = \underline{apr}_{NS}(\zeta) = \left\{\left(\sigma_1 < \frac{n_1}{0.3,0.5,0.7} >\right), \left(\sigma_2 < \frac{n_3}{0.4,0.5,0.7}, \frac{n_4}{0.2,0.3,0.2} >\right)\right\}$$

$$NEG_{NS}(\zeta) = \overline{apr}_{NS}(\zeta) = \left\{\begin{array}{c}\left(\sigma_1 < \frac{n_1}{0.8,0.7,0.3}, \frac{n_2}{0.8,0.6,0.2}, \frac{n_3}{0.8,0.6,0.2}, \frac{n_4}{0.9,0.5,0.1} >\right), \\ \left(\sigma_2 < \frac{n_2}{0.7,0.4,0.3}, \frac{n_3}{0.8,0.7,0.1}, \frac{n_4}{0.6,0.8,0.1} >\right), \\ \left(\sigma_3 < \frac{n_2}{0.4,0.5,0.2}, \frac{n_3}{0.8,0.7,0.3}, \frac{n_4}{0.6,0.8,0.1} >\right)\end{array}\right\}$$

$$Bnd_{NS}(\zeta) = \overline{apr}_{NS}(\zeta) \setminus \underline{apr}_{NS}(\zeta)$$

$$= \left\{\begin{array}{c}\left(\sigma_1 < \frac{n_1}{0.7,0.5,0.3}, \frac{n_2}{0.8,0.6,0.2}, \frac{n_3}{0.8,0.6,0.2}, \frac{n_4}{0.9,0.5,0.1} >\right), \\ \left(\sigma_2 < \frac{n_2}{0.7,0.4,0.3}, \frac{n_3}{0.7,0.5,0.4}, \frac{n_4}{0.2,0.5,0.2} >\right), \\ \left(\sigma_3 < \frac{n_2}{0.4,0.5,0.2}, \frac{n_3}{0.8,0.7,0.3}, \frac{n_4}{0.6,0.8,0.1} >\right)\end{array}\right\}$$

Obviously, since it is clear $\underline{apr}_{NS}(\zeta) \neq \overline{apr}_{NS}(\zeta)$. So ζ is neutrosophic soft rough set in the approximation space $\left(\Sigma, \widetilde{N}, \Upsilon\right)$.

Proposition 1 *Let $\left(\widetilde{N}, \Upsilon\right)$ be a neutrosophic soft set over Σ. $\left(\Sigma, \widetilde{N}, \Upsilon\right)$ be a neutrosophic soft approximation space and $\zeta, \Psi \in NSS(\Sigma, \Upsilon)$, then we have,*

1. $\underline{apr}_{NS}(\zeta) \subseteq \zeta \subseteq \overline{apr}_{NS}(\zeta)$,
2. $\underline{apr}_{NS}\left(0_{(\Sigma,\Upsilon)}\right) = 0_{(\Sigma,\Upsilon)}$, $\overline{apr}_{NS}\left(1_{(\Sigma,\Upsilon)}\right) = 1_{(\Sigma,\Upsilon)}$
3. *If* $\zeta \subseteq \Psi$, *then* $\underline{apr}_{NS}(\zeta) \subseteq \underline{apr}_{NS}(\Psi)$
4. *If* $\zeta \subseteq \Psi$, *then* $\overline{apr}_{NS}(\zeta) \subseteq \overline{apr}_{NS}(\Psi)$
5. $\underline{apr}_{NS}(\zeta \cap \Psi) \subseteq \underline{apr}_{NS}(\zeta) \cap \underline{apr}_{NS}(\Psi)$
6. $\overline{apr}_{NS}(\zeta \cap \Psi) \subseteq \overline{apr}_{NS}(\zeta) \cap \overline{apr}_{NS}(\Psi)$
7. $\underline{apr}_{NS}(\zeta \cup \Psi) \subseteq \underline{apr}_{NS}(\zeta) \cup \underline{apr}_{NS}(\Psi)$
8. $\overline{apr}_{NS}(\zeta \cup \Psi) \subseteq \overline{apr}_{NS}(\zeta) \cup \overline{apr}_{NS}(\Psi)$

Proof

1. From Definition 12, we can easily see that $\underline{apr}_{NS}(\zeta) \subseteq \zeta$. Also, from the definition of neutrosophic soft upper approximation, $\forall \left(\widetilde{N}_i, \Upsilon\right) \cap \zeta \neq 0_{(\Sigma,\Upsilon)}, \overline{\mu}, \overline{\upsilon}, \overline{\omega} \in \zeta \cup \left(\widetilde{N}_i, \Upsilon\right)$. Hence, $\zeta \subseteq \overline{apr}_{NS}(\zeta)$. Thus, $\underline{apr}_{NS}(\zeta) \subseteq \zeta \subseteq \overline{apr}_{NS}(\zeta)$.
2. From Definition 12, the proof of 2 is straightforward.
3. Let $\zeta \subseteq \Psi$ and $\left(\widetilde{N}_i, \Upsilon\right) \subseteq \zeta$, $i = (1, 2)$. Then, $\underline{apr}_{NS}(\zeta) = \zeta \cap \left(\bigcap_{i=1}^{2} \left(\widetilde{N}_i, \Upsilon\right)\right)$. In addition, we have $\left(\widetilde{N}_i, \Upsilon\right) \subseteq \zeta$ then $\left(\widetilde{N}_i, \Upsilon\right) \subseteq \Psi$. Hence $\underline{apr}_{NS}(\Psi) = \Psi \cap \left(\bigcap_{i=1}^{2} \left(\widetilde{N}_i, \Upsilon\right)\right)$. This implies $\underline{apr}_{NS}(\zeta) \subseteq \underline{apr}_{NS}(\Psi)$.
4. Let $\zeta \subseteq \Psi$ and $\left(\widetilde{N}_i, \Upsilon\right) \cap \zeta \neq 0_{(\Sigma,\Upsilon)}$, $i = (1, 2)$. Then $\overline{apr}_{NS}(\zeta) = \zeta \cup \left(\bigcup_{i=1}^{2} \left(\widetilde{N}_i, \Upsilon\right)\right)$. For $\zeta \subseteq \Psi$, then $\left(\widetilde{N}_i, \Upsilon\right) \cap \Psi \neq 0_{(\Sigma,\Upsilon)}$ and $\overline{apr}_{NS}(\Psi) = \Psi \cup \left(\bigcup_{i=1}^{2} \left(\widetilde{N}_i, \Upsilon\right)\right)$ .This implies $\overline{apr}_{NS}(\zeta) \subseteq \overline{apr}_{NS}(\Psi)$.
5. Let $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \underline{apr}_{NS}(\zeta \cap \Psi)$. There exist $\left(\widetilde{N}, \Upsilon\right)$ such that $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{N}, \Upsilon\right) \subseteq \underline{apr}_{NS}(\zeta \cap \Psi), n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{N}, \Upsilon\right) \subseteq \zeta$ and $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{N}, \Upsilon\right) \subseteq \Psi$. Therefore $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in$

$\underline{apr}_{NS}(\zeta)$ and $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \underline{apr}_{NS}(\Psi)$, implying $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \underline{apr}_{NS}(\zeta) \cap \underline{apr}_{NS}(\Psi)$. Thus $\underline{apr}_{NS}(\zeta \cap \Psi) \subseteq \underline{apr}_{NS}(\zeta) \cap \underline{apr}_{NS}(\Psi)$.

6. Let $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \overline{apr}_{NS}(\zeta \cap \Psi)$. There exist $\left(\widetilde{N}, \Upsilon\right)$ such that $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{N}, \Upsilon\right) \cap (\zeta \cap \Psi) \neq 0_{(\Sigma,\Upsilon)}$, $\left(\widetilde{N}, \Upsilon\right) \cap (\zeta) \neq 0_{(\Sigma,\Upsilon)}$ and $\left(\widetilde{N}, \Upsilon\right) \cap (\Psi) \neq 0_{(\Sigma,\Upsilon)}$. Therefore $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \overline{apr}_{NS}(\zeta)$ and $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \overline{apr}_{NS}(\Psi)$, implying $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \overline{apr}_{NS}(\zeta) \cap \overline{apr}_{NS}(\Psi)$.
7. Let $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \notin \underline{apr}_{NS}(\zeta \cup \Psi)$. Then, there exist $\left(\widetilde{N}, \Upsilon\right)$ such that $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{N}, \Upsilon\right) \subsetneq (\zeta \cup \Psi)$. It follows that $\left(\widetilde{N}, \Upsilon\right) \subsetneq \zeta$ and $\left(\widetilde{N}, \Upsilon\right) \subsetneq \Psi$. Therefore $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \notin \underline{apr}_{NS}(\zeta)$ and $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \notin \underline{apr}_{NS}(\Psi)$, implying $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \notin \underline{apr}_{NS}(\zeta) \cup \underline{apr}_{NS}(\Psi)$. Thus $\underline{apr}_{NS}(\zeta \cup \Psi) \subseteq \underline{apr}_{NS}(\zeta) \cup \underline{apr}_{NS}(\Psi)$.
8. Let $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \overline{apr}_{NS}(\zeta \cup \Psi)$. There exist $\left(\widetilde{N}, \Upsilon\right)$ such that $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{N}, \Upsilon\right) \cap (\zeta \cup \Psi) \neq 0_{(\Sigma,\Upsilon)}$. It follows that $\left(\widetilde{N}, \Upsilon\right) \cap (\zeta) \neq 0_{(\Sigma,\Upsilon)}$ or $\left(\widetilde{N}, \Upsilon\right) \cap (\Psi) \neq 0_{(\Sigma,\Upsilon)}$. Therefore, $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \overline{apr}_{NS}(\zeta)$ or $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \overline{apr}_{NS}(\Psi)$. Hence $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \overline{apr}_{NS}(\zeta) \cup \overline{apr}_{NS}(\Psi)$. Thus, $\overline{apr}_{NS}(\zeta \cup \Psi) \subseteq \overline{apr}_{NS}(\zeta) \cup \overline{apr}_{NS}(\Psi)$.

□

Proposition 2 *Let $\left(\widetilde{N}, \Upsilon\right)$ be a neutrosophic soft set on a universe Σ and let $\zeta, \Psi \subseteq NSS(\Sigma, \Upsilon)$. Then, the following properties hold,*

1. $\underline{apr}_{NS}\left(\underline{apr}_{NS}(\zeta)\right) = \underline{apr}_{NS}(\zeta)$
2. $\overline{apr}_{NS}\left(\overline{apr}_{NS}(\zeta)\right) \supseteq \overline{apr}_{NS}(\zeta)$
3. $\overline{apr}_{NS}(\zeta) \subseteq \overline{apr}_{NS}\left(\underline{apr}_{NS}(\zeta)\right)$
4. $\overline{apr}_{NS}\left(\underline{apr}_{NS}(\zeta)\right) \supseteq \underline{apr}_{NS}(\zeta)$
5. $\underline{apr}_{NS}(\zeta^c) \supseteq \left[\overline{apr}_{NS}(\zeta)\right]^c$

Proof 1. Let $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \underline{apr}_{NS}(\zeta)$. Then, we have $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \left(\widetilde{N}, \Upsilon\right) \subseteq \underline{apr}_{NS}(\zeta)$. So $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \underline{apr}_{NS}\left(\underline{apr}_{NS}(\zeta)\right)$. Therefore $\underline{apr}_{NS}(\zeta) \subseteq \underline{apr}_{NS}\left(\underline{apr}_{NS}(\zeta)\right)$. From Proposition 1 $\underline{apr}_{NS}(\zeta) \subseteq \zeta$ using 3 of Proposition 1 we obtain $\underline{apr}_{NS}\left(\underline{apr}_{NS}(\zeta)\right) \subseteq \underline{apr}_{NS}(\zeta)$. Hence, $\underline{apr}_{NS}\left(\underline{apr}_{NS}(\zeta)\right) = \underline{apr}_{NS}(\zeta)$.

2. Let $H = \overline{apr}_{NS}(\zeta)$. Using property 1 of Proposition 1, we got $H \subseteq \overline{apr}_{NS}(\zeta)$. Hence $\overline{apr}_{NS}\left(\overline{apr}_{NS}(\zeta)\right) \supseteq \overline{apr}_{NS}(\zeta)$.
3. Let $H = \overline{apr}_{NS}(\zeta)$. Using property 1 of Proposition 1, we got $\underline{apr}_{NS}(\zeta) \subseteq H$. Hence $\underline{apr}_{NS}\left(\overline{apr}_{NS}(\zeta)\right) \subseteq \overline{apr}_{NS}(\zeta)$.
4. Let $H = \underline{apr}_{NS}(\zeta)$. Using property 1 of Proposition 1, we got $\overline{apr}_{NS}(\zeta) \supseteq H$. Hence $\underline{apr}_{NS}(\zeta) \subseteq \overline{apr}_{NS}\left(\underline{apr}_{NS}(\zeta)\right)$.
5. Let $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \notin \underline{apr}_{NS}(\zeta^c)$ and $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in (N, \Upsilon)$. We have $\left(\widetilde{N}, \Upsilon\right) \subsetneq \zeta^c$ and $\left(\widetilde{N}, \Upsilon\right) \cap \zeta^c = 0_{(\Sigma,\Upsilon)}$. Thus, $\left(\widetilde{N}, \Upsilon\right) \cap \zeta \neq 0_{(\Sigma,\Upsilon)}$, where $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \in \overline{apr}_{NS}(\zeta)$ but $n^{\sigma_i}_{(\alpha,\beta,\gamma)} \notin \left[\overline{apr}_{NS}(\zeta)\right]^c$. Therefore, $\underline{apr}_{NS}(\zeta^c) \supseteq \left[\overline{apr}_{NS}(\zeta)\right]^c$.

□

4 Neutrosophic soft rough topological spaces

In this section, we redefine and study neutrosophic soft rough topology and its some important properties using the new neutrosophic soft set concept.

Definition 13 Let Σ be a universe and $\left(\Sigma, \widetilde{N}, \Upsilon\right)$ be a neutrosophic soft approximation space. Then, neutrosophic soft rough topology is defined as;

$$\widetilde{\tau}_{NR}(\zeta) = \left\{0_{(\Sigma,\Upsilon)}, 1_{(\Sigma,\Upsilon)}, \underline{apr}_{NS}(\zeta), \overline{apr}_{NS}(\zeta), Bnd_{NS}(\zeta)\right\}$$

where $\zeta \subseteq NSS(\Sigma, \Upsilon)$ and $\widetilde{\tau}_{NR}(\zeta)$ satisfies the following conditions

1. $0_{(\Sigma,\Upsilon)}$ and $1_{(\Sigma,\Upsilon)}$ belong to $\widetilde{\tau}_{NR}(\zeta)$.
2. Union of members of any number of $\widetilde{\tau}_{NR}(\zeta)$ belongs to $\tau_{NS}(\zeta)$.
3. Intersection of members of finite number of $\widetilde{\tau}_{NR}(\zeta)$ belongs to $\tau_{NS}(\zeta)$.

the topology defined by $\widetilde{\tau}_{NR}(\zeta)$ on Σ is called neutrosophic soft rough topology on Σ and $(\Sigma, \widetilde{\tau}_{NR}(\zeta), \Upsilon)$ is called to be neutrosophic soft rough topological spaces.

Example 3 Let $\Sigma = \{n_1, n_2, n_3, n_4\}$ be a universe and $\Upsilon = \{\sigma_1, \sigma_2, \sigma_3\}$ be a parameter set. We consider following neutrosophic soft sets;

$$\left(\widetilde{N}_1, \Upsilon\right) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.4,0.7,0.8}, \frac{n_3}{0.6,0.4,0.5} >\right), \\ \left(\sigma_2 < \frac{n_1}{0.5,0.7,0.4}, \frac{n_3}{0.3,0.2,0.5}, \frac{n_4}{0.3,0.5,0.5} >\right) \end{array} \right\}$$

$$\left(\widetilde{N}_2, \Upsilon\right) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.7,0.9,0.3}, \frac{n_2}{0.7,0.6,0.4}, \frac{n_4}{0.4,0.6,0.8} >\right), \\ \left(\sigma_3 < \frac{n_1}{0.7,0.7,0.1}, \frac{n_3}{0.4,0.8,0.1} >\right) \end{array} \right\}$$

$$\left(\widetilde{N}_3, \Upsilon\right) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_2}{0.8,0.4,0.1}, \frac{n_3}{0.5,0.6,0.4} >\right), \left(\sigma_2 < \frac{n_2}{0.7,0.4,0.5}, \frac{n_4}{0.4,0.3,0.3} >\right), \\ \left(\sigma_3 < \frac{n_1}{0.5,0.6,0.7}, \frac{n_3}{0.4,0.2,0.1}, \frac{n_4}{0.3,0.6,0.3} >\right) \end{array} \right\}$$

for the another neutrosophic soft set $\zeta \subset NSS(\Sigma, \Upsilon)$ with respect to $\left(\Sigma, \widetilde{N}, \Upsilon\right)$;

$$\zeta = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.7,0.8,0.2}, \frac{n_2}{0.9,0.5,0.1}, \frac{n_3}{0.6,0.7,0.3}, \frac{n_4}{0.5,0.7,0.2} >\right), \\ \left(\sigma_2 < \frac{n_1}{0.6,0.8,0.1}, \frac{n_2}{0.8,0.5,0.3}, \frac{n_3}{0.5,0.3,0.1}, \frac{n_4}{0.4,0.6,0.1} >\right), \\ \left(\sigma_3 < \frac{n_1}{0.7,0.6,0.5}, \frac{n_3}{0.5,0.2,0.1}, \frac{n_4}{0.3,0.7,0.2} >\right) \end{array} \right\}$$

The lower and upper neutrosophic soft approximation and neutrosophic soft boundary region of ζ are calculated as,

$$\underline{apr}_{NS}(\zeta) = \left\{ \left(\sigma_1 < \frac{n_3}{0.5, 0.4, 0.5} > \right), \left(\sigma_2 < \frac{n_4}{0.3, 0.3, 0.5} > \right) \right\}$$

$$\overline{apr}_{NS}(\zeta) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.7,0.9,0.2}, \frac{n_2}{0.9,0.6,0.1}, \frac{n_3}{0.6,0.7,0.3}, \frac{n_4}{0.5,0.7,0.2} > \right), \\ \left(\sigma_2 < \frac{n_1}{0.6,0.8,0.1}, \frac{n_2}{0.8,0.5,0.3}, \frac{n_3}{0.5,0.3,0.1}, \frac{n_4}{0.4,0.6,0.1} > \right), \\ \left(\sigma_3 < \frac{n_1}{0.7,0.7,0.1}, \frac{n_3}{0.5,0.8,0.1}, \frac{n_4}{0.3,0.7,0.2} > \right) \end{array} \right\}$$

$$Bnd_{NS}(\zeta) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.7,0.9,0.2}, \frac{n_2}{0.9,0.6,0.1}, \frac{n_3}{0.5,0.6,0.5}, \frac{n_4}{0.5,0.7,0.2} > \right), \\ \left(\sigma_2 < \frac{n_1}{0.6,0.8,0.1}, \frac{n_2}{0.8,0.5,0.3}, \frac{n_3}{0.5,0.3,0.1}, \frac{n_4}{0.4,0.6,0.3} > \right), \\ \left(\sigma_3 < \frac{n_1}{0.7,0.7,0.1}, \frac{n_3}{0.5,0.8,0.1}, \frac{n_4}{0.3,0.7,0.2} > \right) \end{array} \right\}$$

then

$$\widetilde{\tau}_{NR}(\zeta) = \left\{ 0_{(\Sigma,\Upsilon)}, 1_{(\Sigma,\Upsilon)}, \underline{apr}_{NS}(\zeta), \overline{apr}_{NS}(\zeta), Bnd_{NS}(\zeta) \right\}$$

is a neutrosophic soft rough topology on Σ.

Definition 14 Let $(\Sigma, \widetilde{\tau}_{NR}(\zeta), \Upsilon)$ be a neutrosophic soft rough topological space. Then, numbers of $\widetilde{\tau}_{NR}(\zeta)$ are called neutrosophic soft rough open sets. A neutrosophic soft rough set is said to be a neutrosophic soft closed set if its complement belongs to $\widetilde{\tau}_{NR}(\zeta)$.

Proposition 3 *Consider* $(\Sigma, \widetilde{\tau}_{NR}(\zeta), \Upsilon)$ *as neutrosophic soft rough topological spaces over* Σ. *Then,*

1. $0_{(\Sigma,\Upsilon)}$ *and* $1_{(\Sigma,\Upsilon)}$ *are neutrosophic soft rough closed sets.*
2. *The intersection of any number of neutrosophic soft rough closed sets is a neutrosophic soft rough closed set over* Σ.
3. *The finite union of neutrosophic soft rough closed sets is a neutrosophic soft rough closed set over* Σ.

Proof Straightforward. □

Definition 15 Let $(\Sigma, \widetilde{\tau}_{NR}(\zeta), \Upsilon)$ be a neutrosophic soft rough topological space and $\Omega \in NSS(\Sigma, \Upsilon)$. Then,

$$\widetilde{\tau}_{NR_\Omega} = \left\{ \Omega \cap \left(\widetilde{N}_i, \Upsilon\right) : \left(\widetilde{N}_i, \Upsilon\right) \in \widetilde{\tau}_{NR}(\zeta) \text{ for } i \in I \right\}$$

is called a neutrosophic soft rough subspace topology on Ω and $\left(\Sigma_\Omega, \widetilde{\tau}_{NR_\Omega}, \Upsilon\right)$ is called a neutrosophic soft rough subspace of $(\Sigma, \widetilde{\tau}_{NR}(\zeta), \Upsilon)$.

Definition 16 Let $(\Sigma, \widetilde{\tau}_{NR}(\zeta), \Upsilon)$ be a neutrosophic soft rough topological space over Σ and $\Omega \in NSS(\Sigma, \Upsilon)$. Then, the neutrosophic soft rough interior of Ω is neutrosophic soft union of all neutrosophic soft rough open subsets of Ω and we denote it as $Int_{NSR}(\Omega)$.

Clearly, $Int_{NSR}(\Omega)$ is the largest neutrosophic soft rough open set contained by Ω.

Example 4 Let consider Example 3. Suppose that any $\Omega \in NSS(\Sigma, \Upsilon)$ be defined as follows:

$$\Omega = \left\{ \begin{array}{l} \left(\sigma_1 < \frac{n_1}{0.8,0.9,0.1}, \frac{n_2}{0.9,0.7,0.1}, \frac{n_3}{0.6,0.6,0.4}, \frac{n_4}{0.6,0.8,0.1} >\right), \\ \left(\sigma_2 < \frac{n_1}{0.6,0.9,0.1}, \frac{n_2}{0.9,0.6,0.2}, \frac{n_3}{0.6,0.3,0.1}, \frac{n_4}{0.5,0.6,0.2} >\right), \\ \left(\sigma_3 < \frac{n_1}{0.8,0.7,0.1}, \frac{n_2}{0.5,0.6,0.3}, \frac{n_3}{0.6,0.8,0.1}, \frac{n_4}{0.4,0.8,0.2} >\right) \end{array} \right\}$$

Then, $0_{(\Sigma,\Upsilon)}, 1_{(\Sigma,\Upsilon)}, \underline{apr}_{NS}(\zeta), \overline{apr}_{NS}(\zeta), Bnd_{NS}(\zeta) \subseteq \Omega$. Therefore $Int_{NSR}(\Omega) = 0_{(\Sigma,\Upsilon)} \cup \underline{apr}_{NS}(\zeta) \cup Bnd_{NS}(\zeta) = Bnd_{NS}(\zeta)$.

Theorem 4 *Let* $(\Sigma, \widetilde{\tau}_{NR}(\zeta), \Upsilon)$ *be a neutrosophic soft rough topological space over* Σ *and* $\Omega, \Delta \in NSS(\Sigma, \Upsilon)$. *Then,*

1. $Int_{NSR}\left(0_{(\Sigma,\Upsilon)}\right) = 0_{(\Sigma,\Upsilon)}$ *and* $Int_{NSR}\left(1_{(\Sigma,\Upsilon)}\right) = 1_{(\Sigma,\Upsilon)}$,
2. $Int_{NSR}(\Omega) \subseteq \Omega$,
3. Ω *is neutrosophic soft rough open set* $\Leftrightarrow Int_{NSR}(\Omega) = \Omega$,
4. $\Omega \subseteq \Delta \Rightarrow Int_{NSR}(\Omega) \subseteq Int_{NSR}(\Delta)$,
5. $Int_{NSR}(\Omega) \cap Int_{NSR}(\Delta) = Int_{NSR}(\Omega \cap \Delta)$,
6. $Int_{NSR}(\Omega) \cup Int_{NSR}(\Delta) \subseteq Int_{NSR}(\Omega \cup \Delta)$.

Proof 1. and 2. are clear from Definition 16.

3. Suppose that $Int_{NSR}(\Omega) = \Omega$. Since $Int_{NSR}(\Omega)$ is a neutrosophic soft rough open set Ω is neutrosophic soft rough open set. Conversely, if Ω is a neutrosophic soft rough open set, then the largest neutrosophic soft rough open set contained in Ω is Ω itself. Thus, $Int_{NSR}(\Omega) = \Omega$.
4. Suppose that $\Omega \subseteq \Delta$. By the condition 2, $Int_{NSR}(\Omega) \subseteq \Omega \subseteq \Delta$. Since $Int_{NSR}(\Omega)$ is the largest neutrosophic soft open set contained in Ω and so $Int_{NSR}(\Omega) \subseteq Int_{NSR}(\Delta)$.
5. Since $\Omega \cap \Delta \subseteq \Omega$ and $\Omega \cap \Delta \subseteq \Delta$, then $Int_{NSR}(\Omega \cap \Delta) \subseteq Int_{NSR}(\Omega)$ and $Int_{NSR}(\Omega \cap \Delta) \subseteq Int_{NSR}(\Delta)$. Hence $Int_{NSR}(\Omega \cap \Delta) \subseteq Int_{NSR}(\Omega) \cap Int_{NSR}(\Delta)$. On the other hand since $Int_{NSR}(\Omega) \subseteq \Omega$ and $Int_{NSR}(\Delta) \subseteq \Delta$, then $Int_{NSR}(\Omega) \cap Int_{NSR}(\Delta) \subseteq \Omega \cap \Delta$. Besides $Int_{NSR}(\Omega \cap \Delta) \subseteq \Omega \cap \Delta$ and it is the largest neutrosophic soft rough open set. Therefore, $Int_{NSR}(\Omega) \cap Int_{NSR}(\Delta) \subseteq Int_{NSR}(\Omega \cap \Delta)$. Thus, $Int_{NSR}(\Omega) \cap Int_{NSR}(\Delta) = Int_{NSR}(\Omega \cap \Delta)$.
6. Since $\Omega \subseteq \Omega \cup \Delta$ and $\Delta \subseteq \Omega \cup \Delta$, then $Int_{NSR}(\Omega) \subseteq Int_{NSR}(\Omega \cup \Delta)$ and $Int_{NSR}(\Delta) \subseteq Int_{NSR}(\Omega \cup \Delta)$. Therefore, $Int_{NSR}(\Omega) \cup Int_{NSR}(\Delta) \subseteq Int_{NSR}(\Omega \cup \Delta)$. □

Definition 17 Let $(\Sigma, \widetilde{\tau}_{NR}(\zeta), \Upsilon)$ be a neutrosophic soft rough topological space over Σ and $\Omega \in NSS(\Sigma, \Upsilon)$. Then, the neutrosophic soft rough closure of Ω is defined to be intersection of all neutrosophic soft rough closed supersets of Ω and we denoted by $cl_{NSR}(\Omega)$.

Example 5 Let consider the neutrosophic soft rough topology $\widetilde{\tau}_{NR}(\zeta)$ given in 3. Suppose that any $\Omega \in NSS(\Sigma, \Upsilon)$ be defined as follows;

$$\Omega = \left\{ \begin{array}{l} \left(\sigma_1 < \frac{n_1}{0.1,0.1,0.8}, \frac{n_2}{0.1,0.3,0.9}, \frac{n_3}{0.2,0.1,0.7}, \frac{n_4}{0.2,0.2,0.6} >\right), \\ \left(\sigma_2 < \frac{n_1}{0.1,0.1,0.7}, \frac{n_2}{0.2,0.4,0.9}, \frac{n_3}{0.1,0.5,0.7}, \frac{n_4}{0.1,0.3,0.7} >\right), \\ \left(\sigma_3 < \frac{n_1}{0.1,0.2,0.8}, \frac{n_2}{0.3,0.5,0.6}, \frac{n_3}{0.1,0.1,0.7}, \frac{n_4}{0.2,0.2,0.4} >\right) \end{array} \right\}$$

Obviously, $\left(0_{(\Sigma,\Upsilon)}\right)^c$, $\left(1_{(\Sigma,\Upsilon)}\right)^c$, $\left(\underline{apr}_{NS}(\zeta)\right)^c$, $\left(\overline{apr}_{NS}(\zeta)\right)^c$, $(Bnd_{NS}(\zeta))^c$ are all neutrosophic soft rough closed sets over $(\Sigma, \widetilde{\tau}_{NR}(\zeta), \Upsilon)$. We can calculate these sets as follow,

$$\left(0_{(\Sigma,\Upsilon)}\right)^c = \left(1_{(\Sigma,\Upsilon)}\right),\ \left(1_{(\Sigma,\Upsilon)}\right)^c = \left(0_{(\Sigma,\Upsilon)}\right)$$

$$\left(\underline{apr}_{NS}(\zeta)\right)^c = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{1,1,0}, \frac{n_2}{1,1,0}, \frac{n_3}{0.5,0.6,0.5}, \frac{n_4}{1,1,0} >\right), \\ \left(\sigma_2 < \frac{n_1}{1,1,0}, \frac{n_2}{1,1,0}, \frac{n_3}{1,1,0}, \frac{n_4}{0.5,0.7,0.3} >\right), \\ \left(\sigma_3 < \frac{n_1}{1,1,0}, \frac{n_2}{1,1,0}, \frac{n_3}{1,1,0}, \frac{n_4}{1,1,0} >\right) \end{array} \right\}$$

$$\left(\overline{apr}_{NS}(\zeta)\right)^c = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.2,0.1,0.7}, \frac{n_2}{0.1,0.4,0.9}, \frac{n_3}{0.3,0.3,0.6}, \frac{n_4}{0.2,0.3,0.5} >\right), \\ \left(\sigma_2 < \frac{n_1}{0.1,0.2,0.6}, \frac{n_2}{0.3,0.5,0.8}, \frac{n_3}{0.1,0.7,0.5}, \frac{n_4}{0.1,0.4,0.6} >\right), \\ \left(\sigma_3 < \frac{n_1}{0.1,0.3,0.7}, \frac{n_2}{1,1,0}, \frac{n_3}{0.1,0.2,0.8}, \frac{n_4}{0.2,0.3,0.3} >\right) \end{array} \right\}$$

$$(Bnd_{NS}(\zeta))^c = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.7,0.9,0.2}, \frac{n_2}{0.1,0.6,0.9}, \frac{n_3}{0.5,0.4,0.5}, \frac{n_4}{0.2,0.3,0.5} >\right), \\ \left(\sigma_2 < \frac{n_1}{0.1,0.2,0.6}, \frac{n_2}{0.3,0.5,0.8}, \frac{n_3}{0.1,0.7,0.5}, \frac{n_4}{0.3,0.4,0.4} >\right), \\ \left(\sigma_3 < \frac{n_1}{0.1,0.3,0.7}, \frac{n_2}{1,1,0}, \frac{n_3}{0.1,0.2,0.8}, \frac{n_4}{0.2,0.3,0.3} >\right) \end{array} \right\}$$

Then, $\Omega \subseteq \left(0_{(\Sigma,\Upsilon)}\right)^c, \left(\underline{apr}_{NS}(\zeta)\right)^c, \left(\overline{apr}_{NS}(\zeta)\right)^c, (Bnd_{NS}(\zeta))^c$. Therefore $cl_{NSR}(\Omega) \subseteq \left(0_{(\Sigma,\Upsilon)}\right)^c \cap \left(\underline{apr}_{NS}(\zeta)\right)^c \cap \left(\overline{apr}_{NS}(\zeta)\right)^c \cap (Bnd_{NS}(\zeta))^c = \left(\overline{apr}_{NS}(\zeta)\right)^c$.

Theorem 5 *Let* $(\Sigma, \widetilde{\tau}_{NR}(\zeta), \Upsilon)$ *be a neutrosophic soft rough topological space over* Σ *and* $\Omega \in NSS(\Sigma, \Upsilon)$. *Then,*

1. $cl_{NSR}\left(0_{(\Sigma,\Upsilon)}\right) = 0_{(\Sigma,\Upsilon)}$ *and* $cl_{NSR}\left(1_{(\Sigma,\Upsilon)}\right) = 1_{(\Sigma,\Upsilon)}$,
2. $\Omega \subseteq cl_{NSR}(\Omega)$,
3. Ω *is a neutrosophic soft rough closed set* $\Leftrightarrow \Omega = cl_{NSR}(\Omega)$,
4. $cl_{NSR}(cl_{NSR}(\Omega)) = cl_{NSR}(\Omega)$,
5. $\Omega \subseteq \Delta \Rightarrow cl_{NSR}(\Omega) \subseteq cl_{NSR}(\Delta)$,
6. $cl_{NSR}(\Omega \cup \Delta) = cl_{NSR}(\Omega) \cup cl_{NSR}(\Delta)$,
7. $cl_{NSR}(\Omega \cap \Delta) \subseteq cl_{NSR}(\Omega) \cap cl_{NSR}(\Delta)$.

Proof 1. and 2. are clear from Definition 17.

3. Consider $\Omega = cl_{NSR}(\Omega)$. Since $cl_{NSR}(\Omega)$ is a neutrosophic soft rough closed set, so Ω is a neutrosophic soft rough closed set over Σ. Conversely, suppose that Ω be a neutrosophic soft closed set over Ω. Then, Ω is neutrosophic soft rough closed superset of Ω. So that $\Omega = cl_{NSR}(\Omega)$.
4. Let $cl_{NSR}(\Omega) = \Delta$. Then, Δ is a neutrosophic soft rough closed set. Hence, by the condition 3, $cl_{NSR}(\Delta) = \Delta$. Therefore, $cl_{NSR}(cl_{NSR}(\Omega)) = cl_{NSR}(\Omega)$.
5. We know that $\Omega \subseteq cl_{NSR}(\Omega)$ and $\Delta \subseteq cl_{NSR}(\Delta)$ and so, $\Omega \subseteq \Delta \subseteq cl_{NSR}(\Delta)$. Since $cl_{NSR}(\Omega)$ is the smallest neutrosophic soft rough closed set containing Ω, then $cl_{NSR}(\Omega) \subseteq cl_{NSR}(\Delta)$.
6. Since $\Omega \subseteq \Omega \cup \Delta$ and $\Delta \subseteq \Omega \cup \Delta$, then $cl_{NSR}(\Omega) \subseteq cl_{NSR}(\Omega \cup \Delta)$ and $cl_{NSR}(\Delta) \subseteq cl_{NSR}(\Omega \cup \Delta)$. So $cl_{NSR}(\Omega) \cup cl_{NSR}(\Delta) \subseteq cl_{NSR}(\Omega \cup \Delta)$. On the other

hand, since $\Omega \subseteq cl_{NSR}(\Omega)$ and $\Delta \subseteq cl_{NSR}(\Delta)$, then $\Omega \cup \Delta \subseteq cl_{NSR}(\Omega) \cup cl_{NSR}(\Delta)$. Besides $cl_{NSR}(\Omega \cup \Delta)$ is the smallest neutrosophic soft rough closed set that containing $\Omega \cup \Delta$. Therefore, $cl_{NSR}(\Omega \cup \Delta) \subseteq cl_{NSR}(\Omega) \cup cl_{NSR}(\Delta)$. Hence, $cl_{NSR}(\Omega \cup \Delta) = cl_{NSR}(\Omega) \cup cl_{NSR}(\Delta)$.

7. Since $\Omega \cap \Delta \subseteq cl_{NSR}(\Omega) \cap cl_{NSR}(\Delta)$ and $cl_{NSR}(\Omega \cap \Delta)$ is the smallest neutrosophic soft rough closed set that containing $\Omega \cap \Delta$. Then, $cl_{NSR}(\Omega \cap \Delta) \subseteq cl_{NSR}(\Omega) \cap cl_{NSR}(\Delta)$. □

5 An application to NSRSs

Rough sets, particularly rough sets on two separate universes, have applications in many domains, including multi-attribute decision making, clustering, data mining, and medical diagnosis, as an effective way to cope with erroneous information and data. In this section, we present a real example on decision making problems using the newly defined neutrosophic soft rough sets. Proposed algorithm is as follows:

Algorithm

Step 1: Input Neutrosophic soft sets

Step 2: By Definition 12, we calculate the lower and upper neutrosophic soft approximation.

Step 3: By Definition 11, we use cosine similarity measure and compute $CS(\wp_i)$ $i = 1, 2, .., k$ for $\left(\widetilde{N}_i, \Upsilon\right)$ and upper approximation.

Step 4: We sort the $CS(\wp_i)$ values.

Step 5: The optimal decision is chosen as the largest of the $CS(\wp_i)$ values.

5.1 Illustrative example

With this example, it is aimed to make an evaluation among the experts with the help of the upper approach obtained by the chief expert opinion. Let $\Sigma = \{n_1, n_2, n_3, n_4\}$ be a denotes a set of patients, $\Upsilon = \{\sigma_1, \sigma_2, \sigma_3\}$ be a set of diseases and $S = \{S_1, S_2, S_3\}$ be a set of experts. We consider following neutrosophic soft sets denotes experts opinion;

Step 1:

$$S_1 = \left(\widetilde{N}_1, \Upsilon\right) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.3,0.6,0.7}, \frac{n_2}{0.5,0.6,0.6}, \frac{n_3}{0.5,0.3,0.4} >\right), \\ \left(\sigma_2 < \frac{n_1}{0.4,0.6,0.3}, \frac{n_3}{0.2,0.2,0.4}, \frac{n_4}{0.2,0.4,0.4} >\right) \end{array} \right\}$$

$$S_2 = \left(\widetilde{N}_2, \Upsilon\right) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.6,0.8,0.1}, \frac{n_2}{0.6,0.5,0.3}, \frac{n_4}{0.3,0.5,0.7} >\right), \\ \left(\sigma_3 < \frac{n_1}{0.6,0.6,0.2}, \frac{n_3}{0.3,0.7,0.2} >\right) \end{array} \right\}$$

$$S_3 = \left(\widetilde{N}_3, \Upsilon\right) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_2}{0.7,0.3,0.2}, \frac{n_3}{0.4,0.5,0.3} >\right), \\ \left(\sigma_2 < \frac{n_2}{0.6,0.3,0.4}, \frac{n_3}{0.3,0.2,0.4}, \frac{n_4}{0.3,0.2,0.2} >\right), \\ \left(\sigma_3 < \frac{n_1}{0.4,0.5,0.6}, \frac{n_3}{0.3,0.3,0.2}, \frac{n_4}{0.2,0.5,0.2} >\right) \end{array} \right\}$$

for the another neutrosophic soft set $\zeta \subset NSS(\Sigma, \Upsilon)$ with respect to $\left(\Sigma, \widetilde{N}, \Upsilon\right)$ denotes chief expert assessment;

$$\zeta = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.8,0.7,0.8}, \frac{n_2}{0.8,0.7,0.2}, \frac{n_3}{0.7,0.6,0.2}, \frac{n_4}{0.4,0.6,0.3} >\right), \\ \left(\sigma_2 < \frac{n_1}{0.5,0.7,0.2}, \frac{n_2}{0.7,0.4,0.2}, \frac{n_3}{0.6,0.4,0.1}, \frac{n_4}{0.5,0.7,0.1} >\right), \\ \left(\sigma_3 < \frac{n_1}{0.8,0.7,0.4}, \frac{n_3}{0.6,0.4,0.1}, \frac{n_4}{0.4,0.6,0.1} >\right), \\ \left(\sigma_4 < \frac{n_1}{0.3,0.4,0.4}, \frac{n_2}{0.5,0.2,0.3}, \frac{n_3}{0.4,0.2,0.5}, \frac{n_4}{0.5,0.4,0.7} >\right) \end{array} \right\}$$

Step 2:

The lower and upper neutrosophic soft approximations of ζ are calculated as,

$$\underline{apr}_{NS}(\zeta) = \left\{ \left(\sigma_1 < \frac{n_2}{0.5, 0.3, 0.6}, \frac{n_3}{0.4, 0.3, 0.4} >\right), \left(\sigma_2 < \frac{n_3}{0.2, 0.2, 0.4}, \frac{n_4}{0.2, 0.2, 0.4} >\right) \right\}$$

$$\overline{apr}_{NS}(\zeta) = \left\{ \begin{array}{c} \left(\sigma_1 < \frac{n_1}{0.8,0.7,0.8}, \frac{n_2}{0.8,0.7,0.1}, \frac{n_3}{0.7,0.6,0.2}, \frac{n_4}{0.4,0.6,0.3} >\right), \\ \left(\sigma_2 < \frac{n_1}{0.5,0.7,0.2}, \frac{n_2}{0.7,0.4,0.2}, \frac{n_3}{0.6,0.4,0.1}, \frac{n_4}{0.5,0.7,0.1} >\right), \\ \left(\sigma_3 < \frac{n_1}{0.8,0.7,0.2}, \frac{n_3}{0.6,0.7,0.1}, \frac{n_4}{0.5,0.4,0.7} >\right) \end{array} \right\}$$

Step 3: By Definition 11, we can compute the cosine similarity measure between $\left(\widetilde{N}_i, \Upsilon\right)$ $i = 1, 2, 3$ and upper approximation as follows:

$$CS(\wp_1)\left(\left(\widetilde{N}_1, \Upsilon\right), \overline{apr}_{NS}(\zeta)\right) = 0.769314$$
$$CS(\wp_2)\left(\left(\widetilde{N}_2, \Upsilon\right), \overline{apr}_{NS}(\zeta)\right) = 0.770527$$
$$CS(\wp_3)\left(\left(\widetilde{N}_3, \Upsilon\right), \overline{apr}_{NS}(\zeta)\right) = 0.789153$$

Step 4: $CS(\wp_3) > CS(\wp_2) > CS(\wp_1)$

Step 5: So, the optimal decision is $CS(\wp_1)$. That is, the S_3 expert made the closest assessment to the chief expert.

6 Discussion

In this section, we will compare the proposed neutrosophic soft rough approach with other approaches that combine rough set with neutrosophic soft sets [14, 38–40]. In [38] study, although neutrosophic soft rough set theory was studied, membership functions were completely ignored. The structure is built like a classic rough soft set structure. Therefore, this does not fit well with the neutrosophic set structure. In the study of [14], membership functions were similarly ignored. In the study of [14], the structure was constructed using the full soft set. In the study of [40], membership functions were used, but membership functions were not included in any process while defining the membership lower and upper approaches. Therefore, membership functions do not affect the result of the transaction. [39] defined the concepts of lower and upper approach with the help of the concept of right neighborhood. When the study is examined, the structure is built in such a way that the membership functions do not affect the structure. As it is known, the neutrosophic set structure consists of three membership functions. These membership functions form the basis of the neutrosophic soft set structure. Therefore, these membership functions cannot be ignored when it is desired to create a hybrid structure using any set with a neutrosophic soft set. When other approaches

on neutrosophic soft rough sets are examined, it is seen that the basic operations on the set are defined in a way that is almost accordance with the classical rough set theory. In other words, the solves of membership functions are ignored in some basic operations. This situation prevents the structure from being neutrosophic. In the proposed model, each membership function is used more effectively and these membership functions are used for all basic operations on neutrosophic soft rough set. It can also be easily seen that this model can more fully represent the problem parameters of the soft set. These features make proposed model completely different from other studies in this respect.

7 Conclusion

We introduced a new neutrosophic soft rough set method with neutrosophic soft rough topology. The characteristics of the lower and upper neutrosophic soft rough approximations are specified and validated. Then, neutrosophic soft rough topological spaces were defined and basic concepts such as open set, closed set, interior and closure are presented on the spaces. The properties of newly defined concepts were examined are verified. In addition, the study was supported by many examples. We hope that this study will encourage further research on neutrosophic soft rough sets in order to provide a generic from a work for their practical use.

Declarations

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethical approval Not applicable.

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