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ORIGINAL ARTICLE

An adjustable method for decision making problems with neutrosophic soft sets

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Abstract Neutrosophic soft sets was developed to address uncertainty by taking information into account and using a parameterization tool and neutrosophic logic to analyze three membership functions: truth, indeterminacy and falsehood. For uncertainty issues that arise in the majority of real-world scenarios, neutrosophic soft sets work better. In this paper, using level soft sets of neutrosophic soft sets, we provide an adjustable method for decision-making based on neutrosophic soft sets and provide some illustrated instances. We introduce and explore characteristics of level soft sets. Additionally, we provide the weighted neutrosophic soft sets and explore their potential use in decision-making.

Keywords Soft sets · Neutrosophic soft sets · Level soft sets · Decision making · Choice value

1 Introduction

There is a lot of ambiguity, imprecision, and uncertainty in the actual world. In actuality, rather than being specific, the majority of the notions we encounter in daily life are ambiguous. On the other hand, accurate reasoning is impossible in classical mathematics if all mathematical concepts are not precise. Because many real-world issues in a variety of disciplines, including biology, economics, engineering, environmental science, social science, and medical science, involve data with varying degrees of uncertainty, scientists and engineers have developed an interest in modeling vagueness in recent years. It's possible that using traditional mathematical techniques to tackle these issues won't work. Numerous well-known and frequently helpful mathematical approaches to modeling vagueness exist, including fuzzy set theory (Zadeh 1965), intuitionistic fuzzy set theory (Atanassov 1986), rough set theory (Pawlak 1982), vague set theory (Gau and Buehrer 1993), and interval mathematics (Gorzalzany 1987). However, each of these theories has unique challenges, which are highlighted in Molodtsov (1999). Soft set theory was introduced by Molodtsov (1999) in 1999 as a new mathematical tool for handling uncertainty that is unaffected by the issues that plague current approaches. Numerous disciplines, including probability theory, measurement theory, Riemann integration, Perron integration, game theory, operations research, and smoothness of functions, have found application in this theory (Molodtsov 1999, 2004). Neutrosophic theory, an extensive extension of intuitionistic and fuzzy sets, was created by Smarandache (2004). The three values of truth, indeterminate, and falsehood make up neutrosophic sets. Neutrosophic theory is extensively combined with MCDM techniques and utilized in a variety of domains where decision-making is required, including supply chain, engineering, finance, supplier selection, and agriculture. Neutrosophic based MCDM has been recommended by researchers as the optimal option for managing complex scenarios. A neutrosophic soft set (NS-set), a broad hybrid structure based on the fusion of the soft and neutrosophic theories, was proposed by Maji (2012, 2013) in 2013. Deli and Broumi (2015) refined and adjusted this idea thereafter in 2015. These days, scientists are working to develop novel ideas to address vagueness, imprecision, and ambiguity.

This paper aims to expand on the framework that Jiang et al. (2011) established. More specifically, we use level

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soft sets of neutrosophic soft sets to provide an adjustable method to neutrosophic soft sets (Ozturk et al. 2019; Smarandache 2004) based decision making. Stated differently, we intend to expand upon the other decision-making methodology (Feng et al. 2010; Jiang et al. 2011) to include the neutrosophic scenario.

2 Preliminaries

Definition 1 Deli and Broumi (2015) Let U be a initial universe set and $\mathfrak{F}$ be a collection of parameters. The set of all neutrosophic sets of U is denoted by $P(U)$. Subsequently, a set defined by a set valued function F, which represents a mapping $\rho : \mathfrak{F} \to P(U)$, is known as the approximation function of the neutrosophic soft set $(\rho, \mathfrak{F})$ over U. Put differently, the NS sets (briefly NS sets) may be expressed as a set of ordered pairs as it is a parameterized family of certain components of the set $P(U)$.

$$(\rho, \mathfrak{F}) = \left\{ \left(\eth, < \Theta, T_{\rho(\eth)}(\Theta), I_{\rho(\eth)}(\Theta), F_{\rho(\eth)}(\Theta) > \right) : \eth \in \mathfrak{F}, \Theta \in U \right\}$$

where, $T_{\rho(\eth)}(\Theta), I_{\rho(\eth)}(\Theta), F_{\rho(\eth)}(\Theta) \in [0, 1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $F(e)$. Since supremum of each T, I, F is so the inequality $0 \leq T_{\rho(\eth)}(\Theta) + I_{\rho(\eth)}(\Theta) + F_{\rho(\eth)}(\Theta) \leq 3$ is obvious.

Definition 2 Maji (2012) Let $(\rho, \mathfrak{F})$ and $(\varpi, \mathfrak{F})$ be two NS sets over the universe set U. $(\rho, \mathfrak{F})$ is said to be neutrosophic soft subset of $(\varpi, \mathfrak{F})$ if $T_{\rho(\eth)}(\Theta) \leq T_{\varpi(\eth)}(\Theta)$, $I_{\rho(\eth)}(\Theta) \leq I_{\varpi(\eth)}(\Theta)$, $F_{\rho(\eth)}(\Theta) \geq F_{\varpi(\eth)}(\Theta)$, $\forall \eth \in \mathfrak{F}$, $\forall \Theta \in U$. It is denoted by $(\rho, \mathfrak{F}) \subseteq (\varpi, \mathfrak{F})$.

$(\rho, \mathfrak{F})$ is said to be neutrosophic soft equal to $(\varpi, \mathfrak{F})$ if $(\rho, \mathfrak{F})$ is neutrosophic soft subset of $(\varpi, \mathfrak{F})$ and $(\varpi, \mathfrak{F})$ is neutrosophic soft subset of $(\rho, \mathfrak{F})$. It is denoted by $(\rho, \mathfrak{F}) = (\varpi, \mathfrak{F})$.

Definition 3 Ozturk et al. (2019) Let $\left(\rho_1, \mathfrak{F}\right)$ and $\left(\rho_2, \mathfrak{F}\right)$ be two NS sets over the universe set U. Then their union is denoted by $\left(\rho_1, \mathfrak{F}\right) \cup \left(\rho_2, \mathfrak{F}\right) = \left(\rho_3, \mathfrak{F}\right)$ and is defined by:

$$\left(\rho_3, \mathfrak{F}\right) = \left\{ \left(\eth, < \Theta, T_{\rho_3(\eth)}(\Theta), I_{\rho_3(\eth)}(\Theta), F_{\rho_3(\eth)}(\Theta) > : \Theta \in U \right) : \eth \in \mathfrak{F} \right\}.$$

where

$$T_{\rho_3(\eth)}(\Theta) = \max \left\{ T_{\rho_1(\eth)}(\Theta), T_{\rho_2(\eth)}(\Theta) \right\}$$
$$I_{\rho_3(\eth)}(\Theta) = \max \left\{ I_{\rho_1(\eth)}(\Theta), I_{\rho_2(\eth)}(\Theta) \right\}$$
$$F_{\rho_3(\eth)}(\Theta) = \min \left\{ F_{\rho_1(\eth)}(\Theta), F_{\rho_2(\eth)}(\Theta) \right\}.$$

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Definition 4 Ozturk et al. (2019) Let $\left(\rho_1, \mathfrak{F}\right)$ and $\left(\rho_2, \mathfrak{F}\right)$ be two NS sets over the universe set U. Then their intersection is denoted by $\left(\rho_1, \mathfrak{F}\right) \cap \left(\rho_2, \mathfrak{F}\right) = \left(\rho_3, \mathfrak{F}\right)$ and is defined by:

$$(\rho_3, \mathfrak{F}) = \left\{ \left(\eth, < \Theta, T_{\rho_3(\eth)}(\Theta), I_{\rho_3(\eth)}(\Theta), F_{\rho_3(\eth)}(\Theta) > : \Theta \in U \right) : \eth \in \mathfrak{F} \right\}.$$

where

$$T_{\rho_3(\eth)}(\Theta) = \min \left\{ T_{\rho_1(\eth)}(\Theta), T_{\rho_2(\eth)}(\Theta) \right\}$$
$$I_{\rho_3(\eth)}(\Theta) = \min \left\{ I_{\rho_1(\eth)}(\Theta), I_{\rho_2(\eth)}(\Theta) \right\}$$
$$F_{\rho_3(\eth)}(\Theta) = \max \left\{ F_{\rho_1(\eth)}(\Theta), F_{\rho_2(\eth)}(\Theta) \right\}.$$

3 Main results

By expanding the intuitionistic fuzzy soft set-based decision making method and the fuzzy soft set-based decision making approach, we provide an adjustable technique for NS set based decision making issues in this part (Feng et al. 2010; Jiang et al. 2011).

Definition 5 Let (ρ, A) be a NS set over the universe U and $\mathfrak{F}$ be a set of parameters and $A \subseteq \mathfrak{F}$. For $s, t, h \in [0, 1]$, the $(s, t, h)-$level NS set of (ρ, A) is a crisp soft set $L((\rho, A); s, t, h) = (\rho_{(s,t,h)}, A)$ defined by

$$\rho_{(s,t,h)}(\eth) = L(\rho(\eth); s, t, h)$$
$$= \left\{ \Theta \in U : T_{\rho(\eth)}(\Theta) \geq s, \mathrm{I}_{\rho(\eth)}(\Theta) \geq t \text{ and } F_{\rho(\eth)}(\Theta) \leq h \right\}$$

for all $\eth \in A$.

Example 1 Let us examine a NS set (ρ, A) that characterizes the attractiveness of the mobile phones that Mr. P is contemplating buying.

Suppose that five mobile phones in the domain $U = \{mp_1, mp_2, mp_3, mp_4, mp_5\}$ under consideration, and that $\mathfrak{F} = \{\eth_1, \eth_2, \eth_3, \eth_4, \eth_5, \eth_6\}$, $A = \{\eth_1, \eth_2, \eth_3, \eth_4, \eth_5\} \subseteq \mathfrak{F}$ is a set of decision parameters. The $\eth_i (i = \overline{1, 5})$ stand for parameters "cheap", "expensive", "ios operating system", "Amoled display", "high battery capacity", respectively.

Suppose that

$$\rho(\eth_1) = \{\langle mp_1, .8, .6, .1\rangle, \langle mp_2, 0.6, .5, .1\rangle, \langle mp_3, .7, .5, .2\rangle, \langle mp_4, .4, .6, .2\rangle, \langle mp_5, .7, .8, .3\rangle\}$$
$$\rho(\eth_2) = \{\langle mp_1, .6, .3, .2\rangle, \langle mp_2, .5, .7, .1\rangle, \langle mp_3, .7, .5, .1\rangle, \langle mp_4, .5, .2, .3\rangle, \langle mp_5, .5, .6, .1\rangle\}$$
$$\rho(\eth_3) = \{\langle mp_1, .7, .8, .3\rangle, \langle mp_2, .2, .4, .1\rangle, \langle mp_3, .8, .4, .2\rangle, \langle mp_4, .7, .8, .1\rangle, \langle mp_5, .3, .5, .2\rangle\}$$
$$\rho(\eth_4) = \{\langle mp_1, .4, .3, .1\rangle, \langle mp_2, .5, .5, .3\rangle, \langle mp_3, .6, .3, .2\rangle, \langle mp_4, .5, .2, .3\rangle, \langle mp_5, .7, .6, .2\rangle\}$$
$$\rho(\eth_5) = \{\langle mp_1, .7, .6, .1\rangle, \langle mp_2, .7, .3, .2\rangle, \langle mp_3, .4, .5, .1\rangle, \langle mp_4, .4, .5, .1\rangle, \langle mp_5, .5, .2, .2\rangle\}$$

The NS set (ρ, A) is a parametrized family $\left\{ \rho(\eth_i), i = \overline{1, 5} \right\}$ of neutrosophic sets on U.

Table 1 Tabular representation of the neutrosophic soft set (ρ, A)

U	$\eth_1$	$\eth_2$	$\eth_3$	$\eth_4$	$\eth_5$
mp_1	.8, .6, .1	.6, .3, .2	.7, .8, .3	.4, .3, .1	.7, .6, .1
mp_2	.6, .5, .1	.5, .7, .1	.2, .4, .1	.5, .5, .3	.7, .3, .2
mp_3	.7, .5, .2	.7, .5, .1	.8, .4, .2	.6, .3, .2	.4, .5, .1
mp_4	.4, .6, .2	.5, .2, .3	.7, .8, .1	.5, .2, .3	.4, .5, .1
mp_5	.7, .8, .3	.5, .6, .1	.3, .5, .2	.7, .6, .2	.5, .2, .2

Table 2 (.5, .4, .6)–level NS set $L((\rho, A);.5, .4, .6)$.

U	$\eth_1$	$\eth_2$	$\eth_3$	$\eth_4$	$\eth_5$
mp_1	1	0	1	0	1
mp_2	1	1	0	1	0
mp_3	1	1	1	0	0
mp_4	0	0	1	0	0
mp_5	1	1	0	1	0

$$(\rho, A) = \left\{ \begin{array}{c} \text{cheap mobile phones} \\ = \{\langle mp_1, .8, .6, .1\rangle, \langle mp_2, .6, .5, .1\rangle, \langle mp_3, .7, .5, .2\rangle, \langle mp_4, .4, .6, .2\rangle, \langle mp_5, .7, .8, .3\rangle\}, \\ \text{expensive mobile phone} \\ = \{\langle mp_1, .6, .3, .2\rangle, \langle mp_2, .5, .7, .1\rangle, \langle mp_3, .7, .5, .1\rangle, \langle mp_4, .5, .2, .3\rangle, \langle mp_5, .5, .6, .1\rangle\}, \\ \text{mobile phone with ios operating system} \\ = \{\langle mp_1, .7, .8, .3\rangle, \langle mp_2, .2, .4, .1\rangle, \langle mp_3, .8, .4, .2\rangle, \langle mp_4, .7, .8, .1\rangle, \langle mp_5, .3, .5, .2\rangle\}, \\ \text{mobile phone with amoled display} \\ = \{\langle mp_1, .4, .3, .1\rangle, \langle mp_2, .5, .5, .3\rangle, \langle mp_3, .6, .3, .2\rangle, \langle mp_4, .5, .2, .3\rangle, \langle mp_5, .7, .6, .2\rangle\}, \\ \text{mobile phone with high battery capacity} \\ = \{\langle mp_1, .7, .6, .1\rangle, \langle mp_2, .7, .3, .2\rangle, \langle mp_3, .4, .5, .1\rangle, \langle mp_4, .4, .5, .1\rangle, \langle mp_5, .5, .2, .2\rangle\} \end{array} \right\}$$

Table 1 gives the tabular representation of the neutrosophic soft set (ρ, A).

Now, we takes $s = .5, t = .4$ and $h = .6$, then we have the following;

$$L\big(\rho(\eth_1);.5, .4, .6\big) = \{mp_1, mp_2, mp_3, mp_5\},$$
$$L\big(\rho(\eth_2);.5, .4, .6\big) = \{mp_2, mp_3, mp_5\},$$
$$L\big(\rho(\eth_3);.5, .4, .6\big) = \{mp_1, mp_3, mp_4\},$$
$$L\big(\rho(\eth_4);.5, .4, .6\big) = \{mp_2, mp_5\},$$
$$L\big(\rho(\eth_5);.5, .4, .6\big) = \{mp_1\}$$

The soft set $L((\rho, A);.5, .4, .6) = (F_{(.5,.4,.6)}, A)$, is the $(.5, .4, .6)$–level NS set of (ρ, A), where the set-valued mapping $F_{(.5,.4,.6)} : A \to NSS(U)$ defines $F_{(.5,.4,.6)}(\eth_i) = L(\rho(\eth_i);.5, .4, .6)$ for $i = \overline{1, 5}$. The tabular representation of the $(.5, .4, .6)$–level NS set $L((\rho, A);.5, .4, .6)$. is provided in Table 2.

Now, we show some properties of the (s, t, h)–level NS sets.

Theorem 1 *Let $\mathfrak{I}$ be a set of parameters and $A \subseteq \mathfrak{I}$ and let (ρ, A) be a NS set over U. $L((\rho, A);s_1, t_1, h_1)$ and $L((\rho, A);s_2, t_2, h_2)$ are, respectively, $(s_1, t_1, h_1)-$ level NS set and $(s_2, t_2, h_2)-$ level NS set of (ρ, A) where $s_1, t_1, h_1, s_2, t_2, h_2 \in [0, 1]$. We have $L((\rho, A);s_1, t_1, h_1) \subseteq L((\rho, A);s_2, t_2, h_2)$. if $s_1 \le s_2, t_1 \le t_2$ and $h_1 \ge h_2$.*

Proof Let $L((\rho, A);s_1, t_1, h_1) \subseteq (F_{(s_1,t_1,h_1)}, A)$, where $F_{(s_1,t_1,h_1)}(\eth) = L\big(\rho(\eth);s_1, t_1, h_1\big) = \big\{\Theta \in U : T_{\rho(\eth)}(\Theta) \ge s_1, \mathrm{I}_{\rho(\eth)}(\Theta) \ge t_1 \text{ and } T_{\rho(\eth)}(\Theta) \le h_1\big\}$ for all $\eth \in A$. Let $L((\rho, A);s_2, t_2, h_2) \subseteq (F_{(s_2,t_2,h_2)}, A)$, where $F_{(s_2,t_2,h_2)}(\eth) = L\big(\rho(\eth);s_2, t_2, h_2\big) = \big\{\Theta \in U : T_{\rho(\eth)}(\Theta) \ge s_2, \mathrm{I}_{\rho(\eth)}(\Theta) \ge t_2 \text{ and } T_{\rho(\eth)}(\Theta) \le h_2\big\}$ for all $\eth \in A$.It is clear that $A \subseteq A$. In the following we will prove that $\forall \eth \in A$, $F_{(s_1,t_1,h_1)}(\eth) \subseteq F_{(s_2,t_2,h_2)}(\eth)$. Since $s_1 \le s_2, t_1 \le t_2$ and $h_1 \le h_2$,then for all $\eth \in A$, we have following

$$\big\{\Theta \in U : T_{\rho(\eth)}(\Theta) \ge s_1, \mathrm{I}_{\rho(\eth)}(\Theta) \ge t_1 \text{ and } F_{\rho(\eth)}(\Theta) \le h_1\big\} \subseteq \big\{\Theta \in U : T_{\rho(\eth)}(\Theta) \ge s_2, \mathrm{I}_{\rho(\eth)}(\Theta) \ge t_2 \text{ and } F_{\rho(\eth)}(\Theta) \le h_2\big\}.$$

Since $F_{(s_1,t_1,h_1)}(\eth) = \big\{\Theta \in U : T_{\rho(\eth)}(\Theta) \ge s_1, \mathrm{I}_{\rho(\eth)}(\Theta) \ge t_1 \text{ and } T_{\rho(\eth)}(\Theta) \le h_1\big\}$ and $F_{(s_2,t_2,h_2)}(\eth) = \big\{\Theta \in U : T_{\rho(\eth)}(\Theta) \ge s_2, \mathrm{I}_{\rho(\eth)}(\Theta) \ge t_2 \text{ and } F_{\rho(\eth)}(\Theta) \le h_2\big\}$, we have $F_{(s_1,t_1,h_1)}(\eth) \subseteq F_{(s_2,t_2,h_2)}(\eth)$. Therefore, $L((\rho, A);s_1, t_1, h_1) \subseteq L((\rho, A);s_2, t_2, h_2)$. □

Theorem 2 *Let (ρ, A) and (ϖ, B) NS sets over U, where $\mathfrak{I}$ is a set of parameters and $A, B \subseteq \mathfrak{I}$. $L((\rho, A);s, t, h)$ and $L((\varpi, B);s, t, h)$ are $(s, t, h)-$ level NS sets of (ρ, A) and χ, respectively, where $s, t, h \in [0, 1]$. If $(\rho, A) \subseteq \chi$, then we have $L((\rho, A);s, t, h) \subseteq L((\varpi, B);s, t, h)$.*

Proof Let $L((\rho, A);s, t, h) = (\rho_{(s,t,h)}, A)$, where $\rho_{(s,t,h)}(\eth) = L(\rho(\eth);s, t, h) = \big\{\Theta \in U : T_{\rho(\eth)}(\Theta) \ge s, \mathrm{I}_{\rho(\eth)}(\Theta) \ge t \text{ and } F_{\rho(\eth)}(\Theta) \le h\big\}$ for all $\eth \in A$. Let $L((\varpi, B);s, t, h) = (\varpi_{(s,t,h)}, A)$, where $\varpi_{(s,t,h)}(\eth) = L(\varpi(\eth);s, t, h) = \big\{\Theta \in U : T_{\varpi(\eth)}(\Theta) \ge s, \mathrm{I}_{\varpi(\eth)}(\Theta) \ge t \text{ and } F_{\varpi(\eth)}(\Theta) \le h\big\}$ for all $\eth \in A$. Obviously, $A \subseteq A$. In the following we will prove that $\forall \eth \in A$, $\rho_{(s,t,h)}(\eth) \subseteq \varpi_{(s,t,h)}(\eth)$. Since $(\rho, A) \subseteq (\varpi, B)$,then we have the following

$$T_{\rho(\eth)}(\Theta) \le T_{\varpi(\eth)}(\Theta),$$
$$\mathrm{I}_{\rho(\eth)}(\Theta) \le I_{\varpi(\eth)}(\Theta) \text{ and } F_{\rho(\eth)}(\Theta) \ge F_{\varpi(\eth)}(\Theta) \text{ for all } \Theta \in U \text{ and } \eth \in A.$$

Assume that $\Theta \in \rho_{(s,t,h)}(\eth)$. Since $\rho_{(s,t,h)}(\eth) = \big\{\Theta \in U : T_{\rho(\eth)}(\Theta) \ge s, \mathrm{I}_{\rho(\eth)}(\Theta) \ge t \text{ and } F_{\rho(\eth)}(\Theta) \le h\big\}$,

then we have that $T_{\rho(\eth)}(\Theta) \geq s$, $I_{\rho(\eth)}(\Theta) \geq t$ and $F_{\rho(\eth)}(\Theta) \leq h$. Since $T_{\rho(\eth)}(\Theta) \leq T_{\varpi(\eth)}(\Theta)$, $I_{\rho(\eth)}(\Theta) \leq I_{\varpi(\eth)}(\Theta)$ and $F_{\rho(\eth)}(\Theta) \geq F_{\varpi(\eth)}(\Theta)$, Thus $T_{\varpi(\eth)}(\Theta) \geq s$, $I_{\varpi(\eth)}(\Theta) \geq t$ and $F_{\varpi(\eth)}(\Theta) \leq h$. Hence, $\Theta \in \{T_{\varpi(\eth)}(\Theta) \geq s, I_{\varpi(\eth)}(\Theta) \geq t \text{ and } F_{\varpi(\eth)}(\Theta) \leq h\}$. Since $\varpi_{(s,t,h)}(\eth) = \{\Theta \in U : T_{\varpi(\eth)}(\Theta) \geq s, \mathrm{I}_{\varpi(\eth)}(\Theta) \geq t \text{ and } F_{\varpi(\eth)}(\Theta) \leq h\}$, then we have $\Theta \in \varpi_{(s,t,h)}(\eth)$. Thus, We have that $\rho_{(s,t,h)}(\eth) \subseteq \varpi_{(s,t,h)}(\eth)$. Consequently, $L((\rho,A);s,t,h) \subseteq L(\chi;s,t,h)$. □

Definition 6 Let (ρ,A) be a NS set over U and $\mathfrak{F}$ be a set of parameters and $A \subseteq \mathfrak{F}$. Let $\lambda : A \to [0,1]^3$ be an neutrosophic set in A which is called a threshold neutrosophic set. The level soft set of (ρ,A) with respect to λ is a crisp soft set $L((\rho,A);\lambda) = (F_\lambda, A)$ defined by $F_\lambda(\eth) = L(\rho(\eth);\lambda(\eth)) = \{\Theta \in U : T_{\rho(\eth)}(\Theta) \geq T_\lambda(\eth), \mathrm{I}_{\rho(\eth)}(\Theta) \geq FI_\lambda(\eth) \text{ and } F_{\rho(\eth)}(\Theta) \leq F_\lambda(\eth)\}$ for all $\eth \in A$.

To illustrate this idea, let us consider the following example.

Definition 7 Let (ρ,A) be a NS set over U and $\mathfrak{F}$ be a set of parameters and $A \subseteq \mathfrak{F}$. Based on the neutrosophic soft set (ρ,A), we can define a neutrosophic set $mid_{(\rho,A)} : A \to [0,1] \times [0,1] \times [0,1]$ by

$$T_{mid_{(\rho,A)}}(\eth) = \frac{1}{|U|} \sum_{\Theta \in U} T_{\rho(\eth)}(\Theta), \mathrm{I}_{mid_{(\rho,A)}}(\eth) = \frac{1}{|U|} \sum_{\Theta \in U} I_{\rho(\eth)}(\Theta) \text{ and}$$

$$F_{mid_{(\rho,A)}}(\eth) = \frac{1}{|U|} \sum_{\Theta \in U} F_{\rho(\eth)}(\Theta)$$

for all $\eth \in A$. The mid-threshold of the NS set (ρ,A) is known as the neutrosophic set $mid_{(\rho,A)}$. Furthermore, the mid-level soft set of (ρ,A) is identified as $L((\rho,A);mid_{(\rho,A)})$, which is the level soft set of (ρ,A) with respect to the mid-threshold neutrosophic set $mid_{(\rho,A)}$. The mid-level decision rule that follows refers to the use of the mid-threshold and the mid-level soft set in decision making based on NS sets.

Example 2 Let us consider Example 1. (ρ,A) with its tabular form given by Table 1. It is clear that the mid-threshold of (ρ,A) is a neutrosophic set

$$mid_{(\rho,A)} = \left\{ \begin{array}{c} \langle \eth_1, .64, .6, .18\rangle, \langle \eth_2, .56, .46, .16\rangle, \langle \eth_3, .54, .58, .18\rangle, \\ \langle \eth_4, .54, .38, .22\rangle, \langle \eth_5, .54, .42, .14\rangle \end{array} \right\}$$

and mid-level soft set of (ρ,A) is a soft set $L((\rho,A);mid)$ with its tabular represantion given by Table 3.

Definition 8 Let (ρ,A) be a NS set over U and $\mathfrak{F}$ be a set of parameters and $A \subseteq \mathfrak{F}$. Based on the neutrosophic soft set (ρ,A), we can define a neutrosophic set $tpbt_{(\rho,A)} : A \to [0,1] \times [0,1] \times [0,1]$ by

$$T_{tpbt_{(\rho,A)}}(\eth) = \max_{\Theta \in U} T_{\rho(\eth)}(\Theta), I_{tpbt_{(\rho,A)}}(\eth) = \max_{\Theta \in U} I_{\rho(\eth)}(\Theta) \text{ and}$$

$$F_{tpbt_{(\rho,A)}}(\eth) = \min_{\Theta \in U} F_{\rho(\eth)}(\Theta) \text{ for all } \eth \in A.$$

We can define an neutrosophic set $tptp_{(\rho,A)} : A \to [0,1] \times [0,1] \times [0,1]$ by

$$T_{tptp_{(\rho,A)}}(\eth) = \max_{\Theta \in U} T_{\rho(\eth)}(\Theta), I_{tptp_{(\rho,A)}}(\eth) = \max_{\Theta \in U} I_{\rho(\eth)}(\Theta) \text{ and}$$

$$F_{tptp_{(\rho,A)}}(\eth) = \max_{\Theta \in U} F_{\rho(\eth)}(\Theta) \text{ for all } \eth \in A.$$

We can define an neutrosophic set $btbt_{(\rho,A)} : A \to [0,1] \times [0,1] \times [0,1]$ by

$$T_{btbt_{(\rho,A)}}(\eth) = \min_{\Theta \in U} T_{\rho(\eth)}(\Theta), I_{btbt_{(\rho,A)}}(\eth) = \min_{\Theta \in U} I_{\rho(\eth)}(\Theta) \text{ and}$$

$$F_{btbt_{(\rho,A)}}(\eth) = \min_{\Theta \in U} F_{\rho(\eth)}(\Theta) \text{ for all } \eth \in A.$$

We can define an neutrosophic set $bttp_{(\rho,A)} : A \to [0,1] \times [0,1] \times [0,1]$ by

$$T_{bttp_{(\rho,A)}}(\eth) = \min_{\Theta \in U} T_{\rho(\eth)}(\Theta), I_{bttp_{(\rho,A)}}(\eth) = \min_{\Theta \in U} I_{\rho(\eth)}(\Theta) \text{ and}$$

$$F_{bttp_{(\rho,A)}}(\eth) = \max_{\Theta \in U} F_{\rho(\eth)}(\Theta) \text{ for all } \eth \in A.$$

The neutrosophic set $topbottom_{(\rho,A)}$ (briefly $tpbt_{(\rho,A)}$) is called the top-bottom-threshold (tpbt threshold) of the NS set (ρ,A), the neutrosophic set $toptop_{(\rho,A)}$ (briefly $tptp_{(\rho,A)}$) is called the top-top-threshold (tptp threshold) of the NS set (ρ,A), the neutrosophic set $bottombottom_{(\rho,A)}$ (briefly $btbt_{(\rho,A)}$) is called the bottom-bottom-threshold (btbt threshold) of the NS set (ρ,A). Also, the level soft set of (ρ,A) w.r.t. the top-bottom-threshold (tpbt threshold) of the neutrosophic set $topbottom_{(\rho,A)}$, (briefly $tpbt_{(\rho,A)}$) namely $L((\rho,A);topbottom_{(\rho,A)})$ is called top-bottom-level soft set of (ρ,A) and simply denoted $L((\rho,A);topbottom)$. The level soft set of (ρ,A) w.r.t. the top-top-threshold (tptp threshold) of the neutrosophic set $toptop_{(\rho,A)}$, (briefly $tptp_{(\rho,A)}$,) namely $L((\rho,A);toptop_{(\rho,A)})$ is called tptp-level soft set of (ρ,A) and simply denoted $L((\rho,A);toptop)$. The level soft set of (ρ,A) w.r.t. the btbt threshold of the neutrosophic set $btbt_{(\rho,A)}$, namely $L((\rho,A);btbt_{(\rho,A)})$ is called bottom-bottom-level soft set of (ρ,A) and simply denoted $L((\rho,A);btbt)$.The level soft set of (ρ,A) w.r.t. the bottom-top-threshold of the neutrosophic set $bottomtop_{(\rho,A)}$, namely $L((\rho,A);bottomtop_{(\rho,A)})$ is

Table 3 $L((\rho,A);mid)$

U	$\eth_1$	$\eth_2$	$\eth_3$	$\eth_4$	$\eth_5$
mp_1	1	0	0	0	1
mp_2	0	1	0	0	0
mp_3	0	1	0	0	0
mp_4	0	0	1	0	0
mp_5	0	0	0	1	0

Table 4 $L((\rho,A);tpbt)$

U	$\eth_1$	$\eth_2$	$\eth_3$	$\eth_4$	$\eth_5$
mp_1	0	0	0	0	1
mp_2	0	0	0	0	0
mp_3	0	0	0	0	0
mp_4	0	0	0	0	0
mp_5	0	0	0	0	0

called bottom-top-level soft set of (ρ,A) and simply denoted $L((\rho,A);bottomtop)$.

The use of the ttbt threshold threshold and thettbt threshold soft set in NS sets based decision making are what the ttbt threshold-level decision rule entails in the following. Using the tptp-threshold and taking into account the tptp-level soft set in NS sets based decision making is what the tptp-level decision rule entails. When making decisions based on NS sets, the btbt-level decision rule will need applying the btbt threshold and taking the bottom-bottom-level soft set into account.

Example 3 Let us consider Example 1. (ρ,A) with its tabular form given by Table 1. It is clear that the tpbt threshold of (ρ,A) is a neutrosophic set

$$tpbt_{(\rho,A)} = \left\{ \begin{array}{c} \langle \eth_1, .8, .8, .1\rangle, \langle \eth_2, .7, .7, .1\rangle, \langle \eth_3, .8, .8, .1\rangle, \\ \langle \eth_4, .7, .6, .1\rangle, \langle \eth_5, .7, .6, .1\rangle \end{array} \right\}$$

and the top–bottom-level soft set of (ρ,A) is a soft set $L((\rho,A);tpbt)$ with its tabular representation given by Table 4.

It is clear that the tptp threshold of (ρ,A) is a neutrosophic set

$$tptp_{(\rho,A)} = \left\{ \begin{array}{c} \langle \eth_1, .8, .8, .3\rangle, \langle \eth_2, .7, .7, .3\rangle, \langle \eth_3, .8, .8, .3\rangle, \\ \langle \eth_4, .7, .6, .3\rangle, \langle \eth_5, .7, .6, .2\rangle \end{array} \right\}$$

and the top–top-level soft set of (ρ,A) is a soft set $L((\rho,A);tptp)$ with its tabular representation given by Table 5.

It is clear that the btbt threshold of (ρ,A) is a neutrosophic set

$$btbt_{(\rho,A)} = \left\{ \begin{array}{c} \langle \eth_1, .4, .5, .1\rangle, \langle \eth_2, .5, .2, .1\rangle, \langle \eth_3, .2, .4, .1\rangle, \\ \langle \eth_4, .4, .2, .1\rangle, \langle \eth_5, .4, .2, .1\rangle \end{array} \right\}$$

and the btbt-level soft set of (ρ,A) is a soft set $L((\rho,A);btbt)$ with its tabular representation given by Table 6.

It is clear that the bttp threshold of (ρ,A) is a neutrosophic set

Table 5 $L((\rho,A);tptp)$

U	$\eth_1$	$\eth_2$	$\eth_3$	$\eth_4$	$\eth_5$
mp_1	0	0	0	0	1
mp_2	0	0	0	0	0
mp_3	0	0	0	0	0
mp_4	0	0	0	0	0
mp_5	0	0	0	1	0

Table 6 $L((\rho,A);btbt)$

U	$\eth_1$	$\eth_2$	$\eth_3$	$\eth_4$	$\eth_5$
mp_1	1	0	0	1	1
mp_2	1	1	1	0	0
mp_3	0	1	0	0	1
mp_4	0	0	1	0	1
mp_5	0	1	0	0	0

Table 7 $L((\rho,A);bttp)$

U	$\eth_1$	$\eth_2$	$\eth_3$	$\eth_4$	$\eth_5$
mp_1	1	1	1	1	1
mp_2	1	1	1	1	1
mp_3	1	1	1	1	1
mp_4	1	1	1	1	1
mp_5	1	1	1	1	1

$$bttp_{(\rho,A)} = \{\langle \eth_1, .4, .5, .5\rangle, \langle \eth_2, .5, .2, .8\rangle, \langle \eth_3, .2, .4, .6\rangle, \langle \eth_4, .4, .2, .6\rangle, \langle \eth_5, .4, .2, .8\rangle\}$$

and the bttp-level soft set of (ρ,A) is a soft set $L((\rho,A);bttp)$ with its tabular representation given by Table 7.

From Table 7, one can see that all the tabular entries are equal to 1. In other words, the bttp threshold can always be satisfied.

Theorem 3 *Let (ρ,A) be a NS set over U and $\mathfrak{F}$ be a set of parameters and $A \subseteq \mathfrak{F}$. Let $\delta_1 : A \to [0,1]\times[0,1]\times[0,1]$ and $\delta_2 : A \to [0,1]\times[0,1]\times[0,1]$ be two threshold neutrosophic sets. $L((\rho,A);\delta_1) = (\rho_{\delta_1},A)$ and $L((\rho,A);\delta_2) = (\rho_{\delta_2},A)$ are the level soft sets of (ρ,A) with respect to δ_1 and δ_2, respectively. If $T_{\delta_2}(\eth) \le T_{\delta_1}(\eth)$, $I_{\delta_2}(\eth) \le I_{\delta_1}(\eth)$ and $F_{\delta_2}(\eth) \ge F_{\delta_1}(\eth)$ for all $\eth \in A$, then we have $L((\rho,A);\delta_1) \subseteq L((\rho,A);\delta_2)$.*

Proof It is clear that from the Theorem 1. □

Theorem 4 *Let (ρ,A) and (ϖ,B) be NS sets over U, where $A \subseteq \mathfrak{F}$ and $\mathfrak{F}$ is a set of parameters. Let $\delta : A \to [0,1]\times[0,1]\times[0,1]$ be a threshold neutrosophic sets. $L((\rho,A);\delta) = (\rho_\delta,A)$ and $L((\varpi,B);\delta) = (\varpi_\delta,A)$ are the level soft sets of (ρ,A) and (ϖ,B) with respect*

to δ, respectively. If $(\rho, A) \subseteq (\varpi, B)$, then we have $L((\rho, A); \delta) \subseteq L((\varpi, B); \delta)$.

Proof It is clear that from the Theorem 2. □

It is clear that level soft sets with respect to a neutrosophic set generalize (s, t, h)-level soft set by substituting a function on the parameter set A, namely, a neutrosophic set $\delta : A \to [0,1] \times [0,1] \times [0,1]$, for a constant value $(s, t, h) \in [0,1]$. Let $\delta' = \left\{\langle \eth, T_{\delta'}(\eth), I_{\delta'}(\eth), F_{\delta'}(\eth)\rangle\right\}$be the constant neutrosophic set in A given by $T_{\delta'}(\eth) = s, I_{\delta'}(\eth) = t$ and $F_{\delta'}(\eth) = h$ for all $\eth \in A$. Then we immediately have $L((\rho, A); \delta) = L((\rho, A); s, t, h)$. Thus, the level soft set with respect to the constant neutrosophic set δ' coincides with the (s, t, h)-level soft set. That is, the (s, t, h)-level soft set may be considered as a level soft set with respect to a constant neutrosophic set, i.e, the (s, t, h)-level soft set is a special case of level soft sets with respect to a neutrosophic set.

Theorem 5 *Let (ρ, A) be a NS set over U and $\mathfrak{F}$ be a set of parameters and $A \subseteq \mathfrak{F}$. $L((\rho, A); mid)$, $L((\rho, A); tpbt)$, $L((\rho, A); tptp)$ and $L((\rho, A); btbt)$ are the mid-level soft sets, the tpbt-level soft set, the tptp-level soft set and the btbt-level soft set of (ρ, A), respectively. Then we have the following properties*:

1. $L((\rho, A); mid) \subseteq L((\rho, A); tpbt)$
2. $L((\rho, A); tptp) \subseteq L((\rho, A); tpbt)$
3. $L((\rho, A); btbt) \subseteq L((\rho, A); tpbt)$

Proof We will prove only (1). (2) and (3) proved analogously.

(1) Let $L((\rho, A); tpbt) = (F_{tpbt}, A)$, where $T_{tpbt_{(\rho,A)}}(\eth) = \max\limits_{\Theta \in U} T_{\rho(\eth)}(\Theta), I_{tpbt_{(\rho,A)}}(\eth) = \max\limits_{\Theta \in U} I_{\rho(\eth)}(\Theta)$ and $F_{tpbt_{(\rho,A)}}(\eth) = \min\limits_{\Theta \in U} F_{\rho(\eth)}(\Theta)$ for all $\eth \in A$. Let $L((\rho, A); mid) = (F_{mid}, A)$,where$T_{mid_{(\rho,A)}}(\eth) = \frac{1}{|U|} \sum\limits_{\Theta \in U} T_{\rho(\eth)}(\Theta)$, $I_{mid_{(\rho,A)}}(\eth) = \frac{1}{|U|} \sum\limits_{\Theta \in U} I_{\rho(\eth)}(\Theta)andF_{mid_{(\rho,A)}}(\eth) = \frac{1}{|U|} \sum\limits_{\Theta \in U} F_{\rho(\eth)}(\Theta)$. Obviously, $A \subseteq A$. Now, we will prove thatt $\forall \eth \in A$, $F_{tpbt}(\eth) \subseteq F_{mid}(\eth)$. Since $\max\limits_{\Theta \in U} T_{\rho(\eth)}(\Theta) \geq \frac{1}{|U|} \sum\limits_{\Theta \in U} T_{\rho(\eth)}(\Theta)$, $\max\limits_{\Theta \in U} I_{\rho(\eth)}(\Theta) \geq \frac{1}{|U|} \sum\limits_{\Theta \in U} I_{\rho(\eth)}(\Theta)$ and $\min\limits_{\Theta \in U} F_{\rho(\eth)}(\Theta) \leq \frac{1}{|U|} \sum\limits_{\Theta \in U} F_{\rho(\eth)}(\Theta)$, then for all $\eth \in A$ we have $T_{tpbt_{(\rho,A)}}(\eth) \geq T_{mid_{(\rho,A)}}(\eth)$, $I_{tpbt_{(\rho,A)}}(\eth) \geq I_{mid_{(\rho,A)}}(\eth)$ and $F_{tpbt_{(\rho,A)}}(\eth) \leq F_{mid_{(\rho,A)}}(\eth)$. Thus, we have the following

$$\left\{\Theta \in U : T_{\rho(\eth)}(\Theta) \geq T_{mid_{(\rho,A)}}(\eth), \mathrm{I}_{\rho(\eth)}(\Theta) \geq I_{mid_{(\rho,A)}}(\eth) \text{and } F_{\rho(\eth)}(\Theta) \leq F_{mid_{(\rho,A)}}(\eth)\right\} \subseteq \left\{\Theta \in U : T_{\rho(\eth)}(\Theta) \geq T_{tpbt_{(\rho,A)}}(\eth), \mathrm{I}_{\rho(\eth)}(\Theta) \geq I_{tpbt_{(\rho,A)}}(\eth) \text{ and } F_{\rho(\eth)}(\Theta) \leq F_{tpbt_{(\rho,A)}}(\eth)\right\}$$

Since

$$F_{tpbt}(\eth) = \left\{\begin{array}{c} \Theta \in U : T_{\rho(\eth)}(\Theta) \geq T_{tpbt_{(\rho,A)}}(\eth), \\ \mathrm{I}_{\rho(\eth)}(\Theta) \geq I_{tpbt_{(\rho,A)}}(\eth) \text{ and} \\ F_{\rho(\eth)}(\Theta) \leq F_{tpbt_{(\rho,A)}}(\eth) \end{array}\right\}$$

and

$$F_{mid}(\eth) = \left\{\begin{array}{c} \Theta \in U : T_{\rho(\eth)}(\Theta) \geq T_{mid_{(\rho,A)}}(\eth), \\ \mathrm{I}_{\rho(\eth)}(\Theta) \geq I_{mid_{(\rho,A)}}(\eth) \text{ and} \\ F_{\rho(\eth)}(\Theta) \leq F_{mid_{(\rho,A)}}(\eth) \end{array}\right\}$$

then the following

$$\rho_{mid_{(\rho,A)}}(\eth) \subseteq \rho_{tpbt_{(\rho,A)}}(\eth).$$

Therefore, $L((\rho, A); mid) \subseteq L((\rho, A); tpbt)$. □

Table 8 L$((\rho, A); mid)$ with choice value

U	$\eth_1$	$\eth_2$	$\eth_3$	$\eth_4$	$\eth_5$	*ChoiceValue*
mp_1	1	0	0	0	1	$c_1 = 2$
mp_2	0	1	0	0	0	$c_2 = 1$
mp_3	0	1	0	0	0	$c_3 = 1$
mp_4	0	0	1	0	0	$c_4 = 1$
mp_5	0	0	0	0	1	$c_5 = 1$

3.1 Decision making algorithm

Algorithm 1 Adjustable decision making algorithm

1. Input the resultant NS set (ρ, A).
2. Input a threshold neutrosophic set $\delta : A \to [0,1]^3$ (or give a threeshold $(s,t,h) \in [0,1]^3$; For making decisions, one can select the btbt-level decision rule, the tptp-level decision rule, the mid-level decision rule, or the tpbt-level decision rule.
3. Compute the level soft set $L((\rho, A); \delta)$ w.r.t neutrosophic set δ (or the (s,t,h)-level soft set $L((\rho, A); s,t,h)$; or the mid-level soft set $L((\rho, A); mid)$; or the tpbt threshold-level soft set $L((\rho, A); tpbt)$; or the tptp-level soft set $L((\rho, A); tptp)$; or the btbt level soft set $L((\rho, A); btbt)$)
4. Present the level soft set $L((\rho, A); \delta)$ (or $L((\rho, A); s,t,h)$; or $L((\rho, A); mid)$; or $L((\rho, A); tpbt)$; or $L((\rho, A); tptp)$; or $L((\rho, A); btbt)$) in tabular form and compute the choice value c_i of q_i, $\forall i$.
5. The optimal decision is to select q_k if $c_k = \max_i c_i$.
6. If k has more than one value then any one of q_k may be chosen.

Remark 1 The process described above is an extension of intuitionistic fuzzy soft set and fuzzy soft set decision making algorithms, which are neutrosophic in nature (Feng et al. 2010; Jiang et al. 2011). That is, although NS sets are used in Algorithm 1 for decision-making, fuzzy soft sets and intuitionistic fuzzy soft sets, respectively, are the foundation of the decision-making process in Feng et al. (2010); Jiang et al. (2011). Examine the following example to illustrate Algorithm 1's fundamental idea.

Example 4 Now that we have a tabular representation of the neutrosophic soft set (ρ, A) in Table 1, let's go back and review the decision-making problem. Using the mid-threshold $mid_{(\rho,A)}$ in conjunction with the mid-level decision rule, we can solve this problem and arrive at the midlevel soft set $L((\rho, A); mid)$ with choice values and tabular representation as shown in Table 8. Table 8 indicates that mp1 is the best option since c_1 is the maximum choice value. The tptp-level soft set $L((\rho, A); tptp)$ with choice values and tabular representation is in Table 9 if we approach this problem using the tptp-level decision rule. We will also utilize the tptp-threshold $tptp_{(\rho,A)}$. Table 9 indicates that $c_1 = 1$ and $c_5 = 1$ are the maximum choice values. Therefore, mp_1 and mp_5 might also be chosen as the best options.

4 Weighted NS sets based decision making

Definition 9 Let $NS(U)$ be the set of all neutrosophic sets in the universe U. Let $\mathfrak{T}$ be a set of parameters and $A \subseteq \mathfrak{T}$. A weighted NS set is a triple (ρ, A, w), where (ρ, A) is a NS set over U and $w : A \to [0,1]$ is a weight function specifying the weight $\omega_i = w(\eth_i)$ for each attribute $\eth_i \in A$.

It is possible to see each NS set as a weighted NS set. The idea of weighted NS sets offers a mathematical framework for modeling and assessing decision-making problems in which not all choice criteria are equally essential, much to weighted fuzzy soft sets (Feng et al. 2010; Jiang et al. 2011). The differences in parameter importance are represented by the weight function in a weighted neutrosophic soft set.

Table 9 L($L((\rho, A); tptp)$ with choice value

U	$\eth_1$	$\eth_2$	$\eth_3$	$\eth_4$	$\eth_5$	*ChoiceValue*
mp_1	0	0	0	0	1	$c_1 = 1$
mp_2	0	0	0	0	0	$c_2 = 0$
mp_3	0	0	0	0	0	$c_3 = 0$
mp_4	0	0	0	0	0	$c_4 = 0$
mp_5	0	0	0	1	0	$c_5 = 1$

Table 10 (ρ, A, w)

U	$\eth_1, \omega_1 = .7$	$\eth_2, \omega_2 = .4$	$\eth_3, \omega_3 = .5$	$\eth_4, \omega_4 = .6$	$\eth_5, \omega_5 = .7$
mp_1	.8, .6, .1	.6, .3, .2	.7, .8, .3	.4, .3, .1	.7, .6, .1
mp_2	.6, .5, .1	.5, .7, .1	.2, .4, .1	.5, .5, .3	.7, .3, .2
mp_3	.7, .5, .2	.7, .5, .1	.8, .4, .2	.6, .3, .2	.4, .5, .1
mp_4	.4, .6, .2	.5, .2, .3	.7, .8, .1	.5, .2, .3	.4, .5, .1
mp_5	.7, .8, .3	.5, .6, .1	.3, .5, .2	.7, .6, .2	.5, .2, .2

We now demonstrate the flexible technique to weighted neutrosophic soft sets-based decision making utilizing level soft sets.

The weighted fuzzy soft set and weighted intuitionistic fuzzy soft set decision making approach (see Feng et al. 2010; Jiang et al. 2011) are obviously extensions of the neutrosophic algorithm 2. In other words, it is possible that Algorithm 2 was created to deal with problems involving weighted NS sets in decision-making. In particular, we take parameter weights into account and compute the weighted choice value c_i' as opposed to the choice value c_i. Like in Algorithm 1, in Algorithm 2, if too many "ideal options" are reached in the final stage, one can go back to the second step and change the threshold (or decision rule) that was previously applied to modify the optimal decision. In order to illustrate the previously described idea, consider Example 1.

Example 5 Assume that Mr. X set the following weights for the parameters in A. For the parameters "cheap", $\omega_1 = .7$; "expensive", $\omega_2 = .4$; "ios operating system", $\omega_3 = .5$; "Amoled display", $\omega_4 = .6$; "high battery capacity", $\omega_5 = .7$.

Algorithm 2 Adjustable weighted decision making algorithm

1. Input the weighted NS set (ρ, A).
2. Input a threshold neutrosophic set $\delta : A \to [0,1] \times [0,1] \times [0,1]$ (or give a threeshold $(s,t,h) \in [0,1] \times [0,1] \times [0,1]$; For making decisions, one can select the btbt-level decision rule, the tptp-level decision rule, the mid-level decision rule, or the tpbt-level decision rule.
3. Compute the level soft set $L((\rho, A); \delta)$ w.r.t neutrosophic set δ (or the (s,t,h)-level soft set $L((\rho, A); s,t,h)$; or the mid-level soft set $L((\rho, A); mid)$; or the tpbt threshold-level soft set $L((\rho, A); tpbt)$; or the tptp-level soft set $L((\rho, A); tptp)$; or the btbt level soft set $L((\rho, A); btbt)$)
4. Present the level soft set $L((\rho, A); \delta)$ (or $L((\rho, A); s,t,h)$; or $L((\rho, A); mid)$; or $L((\rho, A); tpbt)$; or $L((\rho, A); tptp)$; or $L((\rho, A); btbt)$) in tabular form and compute the weighted choice value c_i' of q_i, $\forall i$.
5. The optimal decision is to select q_k if $c_k' = \max_i c_i'$.
6. If k has more than one value then any one of q_k may be chosen.

Table 11 $L((\rho, A); btbt)$ with weighted choice

U	$\eth_1, \omega_1 = .7$	$\eth_2, \omega_2 = .4$	$\eth_3, \omega_3 = .5$	$\eth_4, \omega_4 = .6$	$\eth_5, \omega_5 = .7$	Weighted choice value (c_i')
mp_1	1	0	0	1	1	$c_1' = 2.0$
mp_2	1	1	1	0	0	$c_2' = 1.5$
mp_3	0	1	0	0	1	$c_3' = 1.1$
mp_4	0	0	1	0	1	$c_4' = .5$
mp_5	0	1	0	0	0	$c_5' = .4$

So we have a weight function $\omega : A \to [0, 1]$ and the NS set (ρ, A) in Example 1 are transformed into a weighted NS set (ρ, A, w) with its tabular representation, as shown in Table 10.

As an adjustable strategy, different rules may be used in decision-making problems. For example, if we apply a bottom–bottom-level decision rule to this situation, we will employ the btbt-threshold $tptp_{(\rho,A)}$, resulting in the mid-level soft set $L((\rho, A);btbt)$ with weighted choice values and a tabular representation, as shown in Table 11.

Table 11 shows that $c_1' = 2.0$ is the greatest weighted choice value, indicating that mp_1 is the ideal choice.

As a result, Mr. P should buy mp_1 as the best house after assigning weights to various factors.

5 Conclusion

We expand upon the methodology presented by Jiang et al. (2011) in this study. Using level soft sets of NS sets, we specifically describe an adjustable method for NS sets-based decision making. Additionally, we provide the concept of weighted neutrosophic soft sets and explore its potential uses in decision-making. In possible future research, It may be investigated the neutrosophic set combined with other types of soft sets such as: HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, SuperHyperSoft Set, TreeSoft Set and investigate them into the decision making process.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

References

Atanassov K (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20:87–96
Deli I, Broumi S (2015) Neutrosophic soft relations and some properties. Ann Fuzzy Math Inform 9(1):169–182
Feng F, Jun YB, Liu X, Li L (2010) An adjustable approach to fuzzy soft set based decision making. J Comput Appl Math 234(1):10–2
Gau WL, Buehrer DJ (1993) Vague sets. IEEE Trans Syst Man Cybern 23(2):610–614
Gorzalzany MB (1987) A method of inference in approximate reasoning based on interval-valued fuzzy sets. Fuzzy Sets Syst 21:1–17
Jiang Y, Tang Y, Chen Q (2011) An adjustable approach to intuitionistic fuzzy soft sets based decision making. Appl Math Model 35(2):824–836
Maji PK (2012) A NS set approach to a decision making problem. Ann Fuzzy Math Inform 3:313–319. https://doi.org/10.5281/zenodo.23151
Maji P (2013) NS set. Ann Fuzzy Math Inform 5:157–168
Molodtsov D (1999) Soft set theory-first results. Comput Math Appl 37:19–31
Molodtsov D (2004) The theory of soft sets. URSS Publishers, Moscow
Ozturk T, Gunduz Aras C, Bayramov S (2019) A new approach to operations on NS sets and to neutrosophic soft topological spaces. Commun Math Appl 10(3):481–493. https://doi.org/10.26713/cma.v10i3.1068
Pawlak Z (1982) Rough sets. Int J Inform Comput Sci 11:341–356
Smarandache F (2004) Neutrosophic set-a generalization of the intuitionistic fuzzy set. Int J Pure Appl Math 24(3):287
Zadeh LA (1965) Fuzzy set. Inform Control 8:338–353

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