OPSEARCH
https://doi.org/10.1007/s12597-025-01018-5

APPLICATION ARTICLE

A comparative analysis of two different decision-making methods in neutrosophic soft rough set environments

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Received: 7 January 2025 / Accepted: 27 August 2025
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Abstract

Neutrosophic soft rough sets are a valuable mathematical tool for dealing with uncertainty, inconsistency and ambiguity. The aim of this paper is to develop new decision-making methods to deal with uncertainties in real-world problems using neutrosophic soft rough sets. First, this paper defines several concepts used in decision-making methods on neutrosophic soft rough sets, including neutrosophic soft rough matrices, neutrosophic soft rough numbers, sum of neutrosophic soft rough numbers, cumulative geometric operators, discriminant measures and ranking concepts. Furthermore, using these newly defined concepts, we develop two different algorithms for decision-making methods. First, we introduce the multi-criteria decision-making method (MCDM) to determine the best option among several alternatives. In this method, we present an algorithm that uses the sum operator and ranking concepts associated with neutrosophic soft rough numbers. Traditional sharp techniques are generally ineffective in solving multi-criteria decision-making (MCDM) problems due to language and interpretation issues. To address this, we propose an alternative method known as the Technique for Similarity to Ideal Solution Rank Preference (TOPSIS). This method helps to identify the decision option that is closest to the positive ideal solution and farthest from the negative ideal solution. In this decision-making method, we present an algorithm that uses concepts such as neutrosophic soft rough matrices, neutrosophic soft rough numbers, and the cumulative geometric operator of neutrosophic soft rough numbers along with a separation measure. Finally, we provide a numerical example on medical diagnosis, illustrated by a hypothetical case study applying the proposed decision-making methods. Additionally, we compare the results obtained from these methods.

Keywords Neutrosophic soft rough set · Neutrosophic soft rough matrix · Multi-criteria decision making · TOPSIS · Medical diagnosis

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Published online: 22 September 2025

Abbreviations

TOPSIS	Technique for order preference by similarity to ideal solution
MCDM	Multi-criteria decision making
$NS(\Psi)$	Collection of all neutrosophic sets
$NSS(\Psi, \Xi)$	Collection of all neutrosophic soft sets
D_{NSR}	Neutrosophic soft rough matrix
AGO	Accumulated geometric operator
NSS	Neutrosophic soft set
NSN	Neutrosophic soft number
NSR	Neutrosophic soft rough number

1 Introduction

In 1982, Pawlak proposed the rough set, which is an extremely useful tool for dealing with uncertain, partial and imprecise problems, and this model and its derivatives are still one of the most widespread and powerful tools for dealing with uncertain and indistinguishable data [38]. A rough set is a bounded region within a set that expresses uncertainty. According to Pawlak, an equivalence relation can be used to replace any uncertain concept with lower and upper approximations of crisp sets. The equivalence relation is an extremely strict requirement in real applications and restricts the use of rough sets in practical settings. Therefore, many researchers have proposed various rough set generalization models using coverage, similarity, tolerance, preference, dominance relations and different neighborhood operators instead of the equivalence relation [19, 20, 51, 52, 56, 65]. In 1999, Molodtsov [27] introduced a novel technique called soft set, in which the preferences for each alternative were expressed in separate parameters, thereby allowing a solution to be found for the stated limitations. Numerous researchers have examined the soft set and used it in a variety of contexts [3, 7, 24, 33–36, 40, 49]. In the same year, Smarandache introduced the idea of the neutrosophic set [53]. The neutrosophic set is expressed by three functions: a membership function, an ambiguity function and a non-membership function, which are independently related, with the membership, ambiguity and non-membership function values belonging to $]^{-}0, 1^{+}[$. Neutrosophic sets play an important role in dealing with well-known uncertain situations across a range of realistic scenarios [1, 4, 11, 14, 18, 22, 30, 41, 54, 57, 58, 62, 63]. In 2010, P.K. Maji introduced the neutrosophic soft set idea [23]. Ö ztürk et al. and most researchers have extensively studied neutrosophic soft sets [17, 21, 29, 31, 32, 50, 60, 61]. Researchers are actively exploring the application of neutrosophy in decision-making problems [39, 42, 44–47]. Neutrosophic set theory offers a more flexible structure than classical and fuzzy approaches in modeling problems involving uncertainty, contradiction and incomplete information. In this context, neutrosophic rough set theory provides a more powerful representation by combining the lower and upper approximations of approximate sets with the truth, uncertainty and falsity components of the neutrosophic set. Tripathy et al. [55] discussed this combination in the context of neutrosophic rough topological spaces and made significant contributions to the integration

of the theory with topological structures by defining the concept of a single-valued neutrosophic rough continuous transformation.

Broumi et al. developed a hybrid structure called rough neutrosophic soft sets by combining rough sets and neutrosophic soft sets [9]. Redefining neutrosophic soft rough sets has been done by numerous researchers [6, 10, 12, 25, 43]. Nevertheless, the techniques for operations on neutrosophic soft rough sets are not universal and have certain drawbacks. In light of these shortcomings, Yolcu et al. proposed a novel concept for neutrosophic soft rough sets [59]. The neutrosophic soft rough set and neutrosophic soft rough topology have numerous applications in decision making problems. Several researchers have introduced the ideas of neutrosophic matrices, rough matrices, and neutrosophic rough matrices. Mondal et al. [28] in 2016 proposed rough neutrosophic TOPSIS for multi-attribute group decision making. Martina and Deepa [26] in 2023 proposed application of multi-valued rough neutrosophic set and matrix in multi-criteria decision making . Also, many researchers have studied the decision making methods on rough sets and their combinations [2, 5, 15, 16, 37, 48, 64]. However, there has been no investigation into the neutrosophic soft rough matrix and its operations and applications. Applying the concept of decision-making to the neutrosophic soft rough set also resolves the indeterminacy issue. Based on the study [59], neutrosophic soft rough matrix operations and applications are discussed in this paper. Also, we define neutrosophic soft rough number, the sum operation and accumulated geometric operator of neutrosophic soft rough number, seperation measure within the larger and smaller solution and rank concepts. In addition to, decision making methods for neutrosophic soft rough sets are presented.

The contributions of this paper can be summarized as follows:

1. Since membership functions are completely or partially ignored in previously described neutrosophic soft rough set approaches [6, 10, 25], this work aims to show how to apply neutrosophic soft rough set and matrix to multi-criteria decision making (MCDM) and TOPSIS methods, based on the work [59] where membership functions are fully used in a functional case.
2. An algorithm for multi-criteria decision making (MCDM) is presented to obtain an objective and comprehensive evaluation result to express and combine the evaluation information of multi-criteria decision makings.
3. TOPSIS method is presented for ease of use and easy understanding and interpretation of the results. TOPSIS method provides the opportunity to evaluate alternatives by numerating the evaluations made with linguistic variables by giving membership function and using the algorithm. It is aimed to determine the decision option with the shortest distance to the positive ideal solution and the farthest distance to the negative ideal solution. For this purpose, an algorithm for TOPSIS method is presented and an application example is given.
4. The validity and applicability of the proposed multi-criteria decision making (MCDM) and TOPSIS approaches are investigated using a real case study. It is shown how to apply the neutrosophic soft rough set and matrix to the multi-criteria decision making (MCDM) and TOPSIS methods.
5. Our present research connects a rough set and a neutrosophic soft set in circumstances requiring medical decision-making.

This paper is organized as follows: Sect. 2 provides the basic definitions used throughout the paper. Section 3.1 defines matrices, number, sum operators and accumulated geometric operations on neutrosophic soft rough sets and the concepts of separation axiomson neutrosophic soft rough sets. It also summarizes the multi-criteria decision making and TOPSIS methods and their steps on neutrosophic soft rough sets. Moreover, provides an illustrative example to apply these two methods. In the last section, the results of the multi-criteria decision making and TOPSIS methods on neutrosophic soft rough sets are compared. Some abbreviations used throughout the study are as follows.

2 Preliminaries

This section includes the basic definitions that we will use throughout the study.

Definition 1 [53] Let Ψ be the initial universe. Then, a neutrosophic set F on the universe Ψ is defined as

$$F = \{\langle \psi, \rho_F(\psi), \varkappa_F(\psi), \varsigma_F(\psi)\rangle : \psi \in \Psi\}$$

where $\rho, \varkappa, \varsigma : \Psi \to]^{-}0, 1^{+}[$ and $^{-}0 \leq \rho_F(\psi) + \varkappa_F(\psi) + \varsigma_F(\psi) \leq 3^{+}$. Where $\rho, \varkappa$ and ς represent the degree of membership, degree of indeterminacy and degree of nonmembership for some $\psi \in \Psi$, respectively.

Definition 2 [13] Let Ψ represent an initial set of universes and Ξ denote a collection of parameters. The set $NS(\Psi)$ is defined as the collection of all neutrosophic sets within Ψ. Consequently, a neutrosophic soft set $\left(\widetilde{F}, \Xi\right)$ defined over Ψ can be described as a set that is determined by a set-valued function $\widetilde{F}$, which functions as a mapping $\widetilde{F} : \Xi \to NS(\Psi)$. This function $\widetilde{F}$ is referred to as the approximate function of the neutrosophic soft set $\left(\widetilde{F}, \Xi\right)$. To elaborate, the neutrosophic soft set represents a parametrized group of certain elements belonging to the set $NS(\Psi)$, and thus can be expressed as a series of ordered pairs.

$$\left(\widetilde{F}, \Xi\right) = \left\{\left(\gamma, \left\langle \psi, \rho_{\widetilde{F}(\gamma)}(\psi), \varkappa_{\widetilde{F}(\gamma)}(\psi), \varsigma_{\widetilde{F}(\gamma)}(\psi)\right\rangle : \psi \in \Psi\right) : \gamma \in \Xi\right\}.$$

where $\rho_{\widetilde{F}(\gamma)}(\psi), \varkappa_{\widetilde{F}(\gamma)}(\psi), \varsigma_{\widetilde{F}(\gamma)}(\psi) \in [0, 1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $\widetilde{F}(\gamma)$. Since supremum of each ρ, $\varkappa$,ς is 1 so the inequality $0 \leq \rho_{\widetilde{F}(\gamma)}(\psi) + \varkappa_{\widetilde{F}(\gamma)}(\psi) + \varsigma_{\widetilde{F}(\gamma)}(\psi) \leq 3$ is obvious.

Definition 3 [8] Let $\left(\widetilde{F},\Xi\right)$ be neutrosophic soft set over the universe set Ψ. The complement of $\left(\widetilde{F},\Xi\right)$ is denoted by $\left(\widetilde{F},\Xi\right)^{c}$ and is defined by:

$$\left(\widetilde{F},\Xi\right)^{c}=\left\{\left(\gamma,\left\langle\psi,\varsigma_{\widetilde{F}(\gamma)}\left(\psi\right),1-\varkappa_{\widetilde{F}(\gamma)}\left(\psi\right),\rho_{\widetilde{F}(\gamma)}\left(\psi\right)\right\rangle:\psi\in\Psi\right):\gamma\in\Xi\right\}.$$

Obvious that, $\left(\left(\widetilde{F},\Xi\right)^{c}\right)^{c}=\left(\widetilde{F},\Xi\right)$.

Definition 4 [23] . Let $\left(\widetilde{F}_1,\Xi\right)$ and $\left(\widetilde{F}_2,\Xi\right)$ be two neutrosophic soft sets over the universe set Ψ . $\left(\widetilde{F}_1,\Xi\right)$ is said to be neutrosophic soft subset of $\left(\widetilde{F}_2,\Xi\right)$ if $\rho_{\widetilde{F}_1(\gamma)}\left(\psi\right)\leq\rho_{\widetilde{F}_2(\gamma)}\left(\psi\right)$, $\varkappa_{\widetilde{F}_1(\gamma)}\left(\psi\right)\leq\varkappa_{\widetilde{F}_2(\gamma)}\left(\psi\right)$,and $\varsigma_{\widetilde{F}_1(\gamma)}\left(\psi\right)\geq\varsigma_{\widetilde{F}_2(\gamma)}\left(\psi\right)$, for all $\gamma\in\Xi$, for all $\psi\in\Psi$. It is denoted by $\left(\widetilde{F}_1,\Xi\right)\subseteq\left(\widetilde{F}_2,\Xi\right)$.

$\left(\widetilde{F}_1,\Xi\right)$ is said to be neutrosophic soft equal to $\left(\widetilde{F}_2,\Xi\right)$ if $\left(\widetilde{F}_1,\Xi\right)$ is neutrosophic soft subset of $\left(\widetilde{F}_2,\Xi\right)$ and $\left(\widetilde{F}_2,\Xi\right)$ is neutrosophic soft subset of $\left(\widetilde{F}_1,\Xi\right)$. It is denoted by $\left(\widetilde{F}_1,\Xi\right)=\left(\widetilde{F}_2,\Xi\right)$.

Definition 5 [29] Let $\left(\widetilde{F}_1,\Xi\right)$ and $\left(\widetilde{F}_2,\Xi\right)$ be two neutrosophic soft sets over the universe set Ψ. Then their union is denoted by $\left(\widetilde{F}_1,\Xi\right)\cup\left(\widetilde{F}_2,\Xi\right)=\left(\widetilde{F}_3,\Xi\right)$ and is defined by:

$$\left(\widetilde{F}_3,\Xi\right)=\left\{\left(\gamma,\left\langle\psi,\rho_{\widetilde{F}_3(\gamma)}\left(\psi\right),\varkappa_{\widetilde{F}_3(\gamma)}\left(\psi\right),\varsigma_{\widetilde{F}_3(\gamma)}\left(\psi\right)\right\rangle:\psi\in\Psi\right):\gamma\in\Xi\right\}$$

where

$$\rho_{\widetilde{F}_3(\gamma)}\left(\psi\right)=\max\left\{\rho_{\widetilde{F}_1(\gamma)}\left(\psi\right),\rho_{\widetilde{F}_2(\gamma)}\left(\psi\right)\right\},$$

$$\varkappa_{\widetilde{F}_3(\gamma)}\left(\psi\right)=\max\left\{\varkappa_{\widetilde{F}_1(\gamma)}\left(\psi\right),\varkappa_{\widetilde{F}_2(\gamma)}\left(\psi\right)\right\},$$

$$\varsigma_{\widetilde{F}_3(\gamma)}\left(\psi\right)=\min\left\{\varsigma_{\widetilde{F}_1(\gamma)}\left(\psi\right),\varsigma_{\widetilde{F}_2(\gamma)}\left(\psi\right)\right\}.$$

Definition 6 [29] Let $\left(\widetilde{F}_1,\Xi\right)$ and $\left(\widetilde{F}_2,\Xi\right)$ be two neutrosophic soft sets over the universe set Ψ. Then their intersection is denoted by $\left(\widetilde{F}_1,\Xi\right)\cap\left(\widetilde{F}_2,\Xi\right)=\left(\widetilde{F}_3,\Xi\right)$ and is defined by:

$$\left(\widetilde{F}_3, \Xi\right) = \left\{\left(\gamma, \left\langle \psi, \rho_{\widetilde{F}_3(\gamma)}(\psi), \varkappa_{\widetilde{F}_3(\gamma)}(\psi), \varsigma_{\widetilde{F}_3(\gamma)}(\psi)\right\rangle : \psi \in \Psi\right) : \gamma \in \Xi\right\}.$$

where

$$\begin{aligned}
\rho_{\widetilde{F}_3(\gamma)}(\psi) &= \min\left\{\rho_{\widetilde{F}_1(\gamma)}(\psi), \rho_{\widetilde{F}_2(\gamma)}(\psi)\right\}, \\
\varkappa_{\widetilde{F}_3(\gamma)}(\psi) &= \min\left\{\varkappa_{\widetilde{F}_1(\gamma)}(\psi), \varkappa_{\widetilde{F}_2(\gamma)}(\psi)\right\}, \\
\varsigma_{\widetilde{F}_3(\gamma)}(\psi) &= \max\left\{\varsigma_{\widetilde{F}_1(\gamma)}(\psi), \varsigma_{\widetilde{F}_2(\gamma)}(\psi)\right\}.
\end{aligned}$$

Definition 7 [29] 1. A neutrosophic soft set $\left(\widetilde{F}, \Xi\right)$ over the universe set Ψ is said to be null neutrosophic soft set if $\rho_{\widetilde{F}_3(\gamma)}(\psi) = 0$, $\varkappa_{\widetilde{F}_3(\gamma)}(\psi) = 0$, $\varsigma_{\widetilde{F}_3(\gamma)}(\psi) = 1$; for all $\gamma \in \Xi$, for all $\psi \in \Psi$. It is denoted by $0_{(\Psi,\Xi)}$.

2. A neutrosophic soft set $\left(\widetilde{F}, \Xi\right)$ over the universe set Ψ is said to be absolute neutrosophic soft set if $\rho_{\widetilde{F}_3(\gamma)}(\psi) = 1$, $\varkappa_{\widetilde{F}_3(\gamma)}(\psi) = 1$, $\varsigma_{\widetilde{F}_3(\gamma)}(\psi) = 0$; for all $\gamma \in \Xi$, for all $\psi \in \Psi$. It is denoted by $1_{(\Psi,\Xi)}$.

Clearly, $0^c_{(\Psi,\Xi)} = 1_{(\Psi,\Xi)}$ and $1^c_{(\Psi,\Xi)} = 0_{(\Psi,\Xi)}$.

Definition 8 [38] Let Ψ be the initial universe and Ξ be a set of parameters. Then, lower, upper, and boundary approximations of Ξ are defined as

$$\begin{aligned}
\Re_{_}(\Xi) &= \bigcup_{\psi\in\Psi}\{\Re(\psi) : \Re(\psi) \subseteq \Xi\}, \\
\Re^{-}(\Xi) &= \bigcup_{\psi\in\Psi}\{\Re(\psi) : \Re(\psi) \cap \Xi \neq \emptyset\},
\end{aligned}$$

and

$$B\Re(\Xi) = \Re^{-}(\Xi) \setminus \Re_{_}(\Xi),$$

respectively. Where $\Re$ is an indiscernibility relation $\Re \subseteq \Psi \times \Psi$ which indicates our information about elements of Ψ. The set Ξ is said to be defined if $\Re^{-}(\Xi) = \Re_{_}(\Xi)$. If $\Re^{-}(\Xi) \neq \Re_{_}(\Xi)$ i.e $B\Re(\Xi) \neq \varnothing$, the set Ξ is rough set w.r.t $\Re$.

Definition 9 [59] Let Ψ denote a non-empty universal set, Ξ represent a set of parameters, and $NS(\Psi)$ indicate the collection of all neutrosophic sets over Ψ. Consider the pair $\left(\widetilde{F}, \Xi\right)$ as a neutrosophic soft set over Ψ, where $\widetilde{F}$ is a function defined as $\widetilde{F} : \Xi \to NS(\Psi)$. Subsequently, the triplet $\left(\Psi, \widetilde{F}, \Xi\right)$ is referred to as a neutro-

sophic soft approximation space. The lower and upper neutrosophic soft approximations of a set $\left(\widetilde{K},\Xi\right) \subseteq NSS\left(\Psi,\Xi\right)$ with respect to $\left(\Psi,\widetilde{F},\Xi\right)$ are symbolized as $\underline{apr}_{NS}\left(\widetilde{K},\Xi\right)$ and $\overline{apr}_{NS}\left(\widetilde{K},\Xi\right)$ respectively, with their definitions outlined as follows:

$$\underline{apr}_{NS}\left(\widetilde{K},\Xi\right) = \left\{\left(\gamma_i, \left\langle \psi, \underline{\rho}_{\gamma_i}\left(\psi\right), \underline{\varkappa}_{\gamma_i}\left(\psi\right), \underline{\varsigma}_{\gamma_i}\left(\psi\right)\right\rangle\right), \forall\gamma_i \in \Xi, \forall\psi \in \Psi\right\}$$
$$\overline{apr}_{NS}\left(\widetilde{K},\Xi\right) = \left\{\left(\gamma_i, \left\langle \psi, \overline{\rho}_{\gamma_i}\left(\psi\right), \overline{\varkappa}_{\gamma_i}\left(\psi\right), \overline{\varsigma}_{\gamma_i}\left(\psi\right)\right\rangle\right), \forall\gamma_i \in \Xi, \forall\psi \in \Psi\right\}$$

where,

$$\underline{\rho}_{\gamma_i}\left(\psi\right) = \left\{\bigwedge \rho_{\gamma_i}\left(\psi\right) : \rho_{\gamma_i}\left(\psi\right) \in \left(\widetilde{K},\Xi\right) \cap \left(\widetilde{F}_i,\Xi\right); \forall\left(\widetilde{F}_i,\Xi\right) \subseteq \left(\widetilde{K},\Xi\right), \forall\gamma_i \in \Xi, \forall\psi \in \Psi\right\}$$
$$\underline{\varkappa}_{\gamma_i}\left(\psi\right) = \left\{\bigwedge_{\gamma_i} \varkappa\left(\psi\right) : \varkappa_{\gamma_i}\left(\psi\right) \in \left(\widetilde{K},\Xi\right) \cap \left(\widetilde{F}_i,\Xi\right); \forall\left(\widetilde{F}_i,\Xi\right) \subseteq \left(\widetilde{K},\Xi\right), \forall\gamma_i \in \Xi, \forall\psi \in \Psi\right\}$$
$$\underline{\varsigma}_{\gamma_i}\left(\psi\right) = \left\{\bigvee \varsigma_{\gamma_i}\left(\psi\right) : \varsigma_{\gamma_i}\left(\psi\right) \in \left(\widetilde{K},\Xi\right) \cap \left(\widetilde{F}_i,\Xi\right); \forall\left(\widetilde{F}_i,\Xi\right) \subseteq \left(\widetilde{K},\Xi\right), \forall\gamma_i \in \Xi, \forall\psi \in \Psi\right\}$$
$$\overline{\rho}_{\gamma_i}\left(\psi\right) = \left\{\bigvee \rho_{\gamma_i}\left(\psi\right) : \rho_{\gamma_i}\left(\psi\right) \in \left(\widetilde{K},\Xi\right) \cup \left(\widetilde{F}_i,\Xi\right); \forall\left(\widetilde{F}_i,\Xi\right) \in NSS\left(\Psi,\Xi\right), \forall\gamma_i \in \Xi, \forall\psi \in \Psi\right\}$$
$$\overline{\varkappa}_{\gamma_i}\left(\psi\right) = \left\{\bigvee_{\gamma_i} \varkappa\left(\psi\right) : \varkappa_{\gamma_i}\left(\psi\right) \in \left(\widetilde{K},\Xi\right) \cup \left(\widetilde{F}_i,\Xi\right); \forall\left(\widetilde{F}_i,\Xi\right) \in NSS\left(\Psi,\Xi\right), \forall\gamma_i \in \Xi, \forall\psi \in \Psi\right\}$$
$$\overline{\varsigma}_{\gamma_i}\left(\psi\right) = \left\{\bigwedge \varsigma_{\gamma_i}\left(\psi\right) : \varsigma_{\gamma_i}\left(\psi\right) \in \left(\widetilde{K},\Xi\right) \cup \left(\widetilde{F}_i,\Xi\right); \forall\left(\widetilde{F}_i,\Xi\right) \in NSS\left(\Psi,\Xi\right), \forall\gamma_i \in \Xi, \forall\psi \in \Psi\right\}$$

where "$\bigwedge$" and "$\bigvee$" mean "min" and "max" operators, respectively. It is easy see that $\underline{apr}_{NS}\left(\widetilde{K},\Xi\right)$ and $\overline{apr}_{NS}\left(\widetilde{K},\Xi\right)$ are two neutrosophic soft set on $NSS\left(\Psi,\Xi\right)$.

3 Main results

In this section, the concepts used in decision making methods on neutrosophic soft rough set are defined. Also, decision making methods for neutrosophic soft rough sets are presented.

Definition 10 Neutrosophic soft rough matrix is defined $D_{NSR} = \left\langle \underline{apr}_{ij}\left(\widetilde{K},\Xi\right), \overline{apr}_{ij}\left(\widetilde{K},\Xi\right)\right\rangle$ of order $m \times n$ as follows

$$D_{NSR} = \begin{array}{c} \\ \psi_1 \\ \psi_2 \\ \vdots \\ \psi_j \end{array} \begin{array}{c} \begin{array}{cccc} \gamma_1 & \gamma_2 & \cdots & \gamma_i \end{array} \\ \begin{pmatrix} \left\langle \underline{apr}_{11}, \overline{apr}_{11}\right\rangle & \left\langle \underline{apr}_{12}, \overline{apr}_{12}\right\rangle & \cdots & \left\langle \underline{apr}_{1n}, \overline{apr}_{1n}\right\rangle \\ \left\langle \underline{apr}_{21}, \overline{apr}_{21}\right\rangle & \left\langle \underline{apr}_{22}, \overline{apr}_{22}\right\rangle & \cdots & \left\langle \underline{apr}_{2n}, \overline{apr}_{2n}\right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \left\langle \underline{apr}_{m1}, \overline{apr}_{m1}\right\rangle & \left\langle \underline{apr}_{m2}, \overline{apr}_{m2}\right\rangle & \cdots & \left\langle \underline{apr}_{mn}, \overline{apr}_{mn}\right\rangle \end{pmatrix} \end{array}$$

Where $\overline{apr}_{ij}\left(\widetilde{K},\Xi\right)$ is an upper approximation $\underline{apr}_{ij}\left(\widetilde{K},\Xi\right)$ is a lower approximation of the neutrosophic soft rough set $\left(\widetilde{K},\Xi\right)$. It can be expressed as

$$D_{NSR}=\left\langle \underline{apr}_{ij}\left(\widetilde{K},\Xi\right),\overline{apr}_{ij}\left(\widetilde{K},\Xi\right)\right\rangle=\left\langle \langle \underline{\rho}_{\gamma_i}(\psi_j),\underline{\varkappa}_{\gamma_i}(\psi_j),\underline{\varsigma}_{\gamma_i}(\psi_j)\rangle,\langle \overline{\rho}_{\gamma_i}(\psi_j),\overline{\varkappa}_{\gamma_i}(\psi_j),\overline{\varsigma}_{\gamma_i}(\psi_j)\rangle\right\rangle$$
$$\underline{apr}_{ij}\left(\widetilde{K},\Xi\right)=\left\{\left(\gamma_i,\left\langle\psi_j,\underline{\rho}_{\gamma_i}(\psi_j),\underline{\varkappa}_{\gamma_i}(\psi_j),\underline{\varsigma}_{\gamma_i}(\psi_j)\right\rangle\right),\forall\gamma_i\in\Xi,\forall\psi_j\in\Psi\right\}$$
$$\overline{apr}_{ij}\left(\widetilde{K},\Xi\right)=\left\{\left(\gamma_i,\langle\psi_j,\overline{\rho}_{\gamma_i}(\psi_j),\overline{\varkappa}_{\gamma_i}(\psi_j),\overline{\varsigma}_{\gamma_i}(\psi_j)\rangle\right),\forall\gamma_i\in\Xi,\forall\psi_j\in\Psi\right\}$$

where $\underline{\rho}_{\gamma_i}(\psi_j)$,$\underline{\varkappa}_{\gamma_i}(\psi_j)$ and $\underline{\varsigma}_{\gamma_i}(\psi_j)$ are the lower approximation values of truth, indeterminacy, and false membership values respectively, $\overline{\rho}_{\gamma_i}(\psi_j),\overline{\varkappa}_{\gamma_i}(\psi_j)$ and $\overline{\varsigma}_{\gamma_i}(\psi_j)$ are the upper approximation values of truth, indeterminacy, and false membership matrices of the neutrosophic soft rough matrix D_{NSR}. With the conditions:

$$0\le\underline{\rho}_{\gamma_i}(\psi_j)+\underline{\varkappa}_{\gamma_i}(\psi_j)+\underline{\varsigma}_{\gamma_i}(\psi_j)\le 3,$$
$$0\le\overline{\rho}_{\gamma_i}(\psi_j)+\overline{\varkappa}_{\gamma_i}(\psi_j)+\overline{\varsigma}_{\gamma_i}(\psi_j)\le 3.$$

Definition 11 According to the ith alternative and j^{th} attribute, the neutrosophic soft rough number is defined by $\left\langle \underline{apr}_{ij},\overline{apr}_{ij}\right\rangle$.Where

$$\left\langle \underline{apr}_{ij},\overline{apr}_{ij}\right\rangle=\left\langle \underline{apr}_{ij}(\underline{\rho}_{\gamma_i},\underline{\varkappa}_{\gamma_i},\underline{\varsigma}_{\gamma_i}),\overline{apr}_{ij}(\overline{\rho}_{\gamma_i},\overline{\varkappa}_{\gamma_i},\overline{\varsigma}_{\gamma_i})\right\rangle=\langle\widetilde{\rho}_{\gamma_i},\widetilde{\varkappa}_{\gamma_i},\widetilde{\varsigma}_{\gamma_i}\rangle$$

Definition 12 Let Ψ be a non-empty universe set, Ξ be a set of parameters and $\underline{apr}_{ij}=(\underline{\rho}_{\gamma_i},\underline{\varkappa}_{\gamma_i},\underline{\varsigma}_{\gamma_i})$ and $\overline{apr}_{ij}=(\overline{\rho}_{\gamma_i},\overline{\varkappa}_{\gamma_i},\overline{\varsigma}_{\gamma_i})$ be two neutrosophic soft rough numbers. The sum operator of neutrosophic soft rough is defined as follows. The sum operator converts neutrosophic soft rough into neutrosophic soft numbers and is defined as

$$R_{\left(\widetilde{K},\Xi\right)}=\underline{apr}_{ij}\oplus\overline{apr}_{ij}=\left\{\left\langle\begin{array}{c}\psi_j,\left(\underline{\rho}_{\gamma_i}(\psi_j)+\overline{\rho}_{\gamma_i}(\psi_j)-\underline{\rho}_{\gamma_i}(\psi_j).\overline{\rho}_{\gamma_i}(\psi_j)\right),\\ \left(\underline{\varkappa}_{\gamma_i}(\psi_j).\overline{\varkappa}_{\gamma_i}(\psi_j)\right),\left(\underline{\varsigma}_{\gamma_i}(\psi_j).\overline{\varsigma}_{\gamma_i}(\psi_j)\right)\end{array}\right\rangle,\forall\gamma_i\in\Xi,\forall\psi_j\in\Psi\right\}.$$

Definition 13 Accumulated geometric operator (AGO)

Assume a neutrosophic soft rough number in the form $\left\langle \underline{apr}_{ij}\left(\underline{\rho}_{\gamma_i},\underline{\varkappa}_{\gamma_i},\underline{\varsigma}_{\gamma_i}\right),\overline{apr}_{ij}\left(\overline{\rho}_{\gamma_i},\overline{\varkappa}_{\gamma_i},\overline{\varsigma}_{\gamma_i}\right)\right\rangle$. We transform the neutrosophic soft rough number into the neutrosophic soft number (NSN) using the accumulated geometric operator(AGO). The operator is expressed as follows.

$$\left\langle \underline{apr}_{ij}\left(\underline{\rho}_{\gamma_i},\underline{\varkappa}_{\gamma_i},\underline{\varsigma}_{\gamma_i}\right),\overline{apr}_{ij}\left(\overline{\rho}_{\gamma_i},\overline{\varkappa}_{\gamma_i},\overline{\varsigma}_{\gamma_i}\right)\right\rangle=\left\langle\left(\underline{\rho}_{\gamma_i}\times\overline{\rho}_{\gamma_i}\right)^{0.5},\left(\underline{\varkappa}_{\gamma_i}\times\overline{\varkappa}_{\gamma_i}\right)^{0.5},\left(\underline{\varsigma}_{\gamma_i}\times\overline{\varsigma}_{\gamma_i}\right)^{0.5}\right\rangle$$

Definition 14 Let $\underline{apr}_{ij} = \left\langle \underline{\rho}_{\gamma_i}, \underline{\varkappa}_{\gamma_i}, \underline{\varsigma}_{\gamma_i} \right\rangle$ and $\overline{apr}_{ij} = \left\langle \overline{\rho}_{\gamma_i}, \overline{\varkappa}_{\gamma_i}, \overline{\varsigma}_{\gamma_i} \right\rangle$ be two neutrosophic soft rough numbers. Therefore, the following definition applies to the seperation measure between $\underline{apr}_{ij}$ and $\overline{apr}_{ij}$.

$$D\left(\underline{apr}_{ij}, \overline{apr}_{ij}\right) = \bigcup_{e_i} \left|\underline{\rho}_{\gamma_i} - \overline{\rho}_{\gamma_i}\right| + \left|\underline{\varkappa}_{\gamma_i} - \overline{\varkappa}_{\gamma_i}\right| + \left|\underline{\varsigma}_{\gamma_i} - \overline{\varsigma}_{\gamma_i}\right|$$

Seperation measure within the larger solution and smaller solution D^+ and D^-, respectively.

$$D^+\left(d_{\gamma_i}\left(\psi_j\right), d^+_{\gamma_i}\right) = \bigcup_{e_i} \left|\widetilde{\rho}_{\gamma_i} - \rho^+_{\gamma_i}\right| + \left|\widetilde{\varkappa}_{\gamma_i} - \varkappa^+_{\gamma_i}\right| + \left|\widetilde{\varsigma}_{\gamma_i} - \varsigma^+_{\gamma_i}\right|$$
$$D^-\left(d_{\gamma_i}\left(\psi_j\right), d^-_{\gamma_i}\right) = \bigcup_{e_i} \left|\widetilde{\rho}_{\gamma_i} - \rho^-_{\gamma_i}\right| + \left|\widetilde{\varkappa}_{\gamma_i} - \varkappa^-_{\gamma_i}\right| + \left|\widetilde{\varsigma}_{\gamma_i} - \varsigma^-_{\gamma_i}\right|$$

where, $d_{\gamma_i}\left(\psi_j\right) \in NSN$ and

$$d^+_{\gamma_i} = \left(\rho^+_{\gamma_i}, \varkappa^+_{\gamma_i}, \varsigma^+_{\gamma_i}\right) = \left(\left(\max_i \widetilde{\rho}_{\gamma_i}\left(\psi_j\right)\right), \left(\max_i \widetilde{\varkappa}_{\gamma_i}\left(\psi_j\right)\right), \left(\max_i \widetilde{\varsigma}_{\gamma_i}\left(\psi_j\right)\right)\right),$$
$$d^-_{\gamma_i} = \left(\rho^-_{\gamma_i}, \varkappa^-_{\gamma_i}, \varsigma^-_{\gamma_i}\right) = \left(\left(\min_i \widetilde{\rho}_{\gamma_i}\left(\psi_j\right)\right), \left(\min_i \widetilde{\varkappa}_{\gamma_i}\left(\psi_j\right)\right), \left(\min_i \widetilde{\varsigma}_{\gamma_i}\left(\psi_j\right)\right)\right).$$

Definition 15 Let $\Psi = \{\psi_1, \psi_2, \psi_3, \ldots, \psi_j\}$ and $\Xi = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_i\}$ be a parameter set. Rank all alternatives is defined for each row as follows:

$$\Omega_{s_{\gamma_i}(\psi_j)} = \frac{2}{3} - \frac{s_{\gamma_1}\left(\psi_j\right)}{3} - \frac{s_{\gamma_2}\left(\psi_j\right)}{3} + \frac{s_{\gamma_3}\left(\psi_j\right)}{3} - \frac{s_{\gamma_4}\left(\psi_j\right)}{3} - \frac{s_{\gamma_5}\left(\psi_j\right)}{3} + \ldots + \frac{s_{\gamma_i}\left(\psi_j\right)}{3}$$

where

$$s_{\gamma_i}\left(\psi_j\right) = \frac{\rho_{R_{\left(\widetilde{K},\Xi\right)}}\left(\psi\right)}{\sqrt{\left(\rho_{R_{\left(\widetilde{K},\Xi\right)}}\left(\psi\right)\right)^2 + \left(\varkappa_{R_{\left(\widetilde{K},\Xi\right)}}\left(\psi\right)\right)^2 + \left(\varsigma_{R_{\left(\widetilde{K},\Xi\right)}}\left(\psi\right)\right)^2}}$$

where $R_{\left(\widetilde{K},\Xi\right)} = \underline{apr}_{NS}\left(\widetilde{K}, \Xi\right) \oplus \overline{apr}_{NS}\left(\widetilde{K}, \Xi\right)$.

3.1 Decision making methods for neutrosophic soft rough sets

In this section, a problem for a neutrosophic soft rough set will be presented and two different algorithms will be applied to this problem, namely the multi-criteria decision making method and the TOPSIS method, respectively.

We take i alternatives $\Psi = \{\psi_1, \psi_2, \ldots, \psi_j\}$ and j criteria $\Xi = \{\gamma_1, \gamma_2, \ldots, \gamma_i\}$ for solving decision making problem. The ratings are taken by the performance of the every alternative over each criterion. The weights of each criteria are asigned by an expert $\{w_1, w_2, \ldots, w_i\}$. In this process a single person evaluated the alternatives. The ratings are given by the linguistic variable then it transformed into neutrosophic soft numbers.

We present an Algorithm 1 for multi-criteria decision making method. This algorithm is obtained by reconstructing the concepts of neutrosophic soft rough sets redefined by Yolcu et al. [59], by adapting them to the work [65].

Algorithm 1:

Step 1: Input Neutrosophic soft sets

Step 2: By Definition 9, we calculate the lower and upper neutrosophic soft approximation.

Step 3: Using Definition 12, the value of $R_{\left(\widetilde{K},\Xi\right)} == \underline{apr}_{NS}\left(\widetilde{K},\Xi\right) \oplus \overline{apr}_{NS}\left(\widetilde{K},\Xi\right)$ is calculated.

Step 4: $s_{\gamma_i}(\psi_j)$ is calculated using the following formula.

$$s_{\gamma_i}(\psi_j) = \frac{\rho_{R_{\left(\widetilde{K},\Xi\right)}}(\psi)}{\sqrt{\left(\rho_{R_{\left(\widetilde{K},\Xi\right)}}(\psi)\right)^2 + \left(\varkappa_{R_{\left(\widetilde{K},\Xi\right)}}(\psi)\right)^2 + \left(\varsigma_{R_{\left(\widetilde{K},\Xi\right)}}(\psi)\right)^2}}$$

Step 5: Rank of each alternative is calculated using Definition 15, and the most suitable choice is obtained.

In accordance with the aforementioned procedure, the decision-making process can be obtained based on the ranking. Within Step 4, $s_{\gamma_i}(\psi_j)$ represents the cosine similarity evaluation between $R_{\left(\widetilde{K},\Xi\right)}(\psi)$ and the proposed ideal solution $(1, 0, 0)$ as introduced by Ye [57].

We present an Algorithm 2 for TOPSIS method. This algorithm is obtained by reconstructing the concepts of neutrosophic soft rough sets defined by Yolcu et al. [59], by adapting them to the work of [26].

Algorithm 2:

Step 1: Build the neutrosophic soft rough decision matrix. We form a neutrosophic soft rough set for each alternative over each criterion.

By this we built the following matrix,

$$D_{NSR} = \begin{array}{c} \\ \psi_1 \\ \psi_2 \\ \vdots \\ \psi_j \end{array} \begin{array}{c} \begin{array}{cccc} \gamma_1 & \gamma_2 & \cdots & \gamma_i \end{array} \\ \begin{pmatrix} \left\langle \underline{apr}_{11}, \overline{apr}_{11} \right\rangle & \left\langle \underline{apr}_{12}, \overline{apr}_{12} \right\rangle & \cdots & \left\langle \underline{apr}_{1n}, \overline{apr}_{1n} \right\rangle \\ \left\langle \underline{apr}_{21}, \overline{apr}_{21} \right\rangle & \left\langle \underline{apr}_{22}, \overline{apr}_{22} \right\rangle & \cdots & \left\langle \underline{apr}_{2n}, \overline{apr}_{2n} \right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \left\langle \underline{apr}_{m1}, \overline{apr}_{m1} \right\rangle & \left\langle \underline{apr}_{m2}, \overline{apr}_{m2} \right\rangle & \cdots & \left\langle \underline{apr}_{mn}, \overline{apr}_{mn} \right\rangle \end{pmatrix} \end{array}$$

where, $\underline{apr}_{ij}$ is lower approximation values and $\overline{apr}_{ij}$ upper approximation values of the neutrosophic soft rough set $\left(\widetilde{K}, \Xi\right)$.

Step 2: The neutrosophic soft rough numbers found in each cell of the neutrosophic soft rough decision matrix are converted to neutrosophic soft numbers (NSN).

Let $\left\langle \underline{apr}_{ij} \left(\underline{\rho}_{\gamma_i}, \underline{\varkappa}_{\gamma_i}, \underline{\varsigma}_{\gamma_i} \right), \overline{apr}_{ij} \left(\overline{\rho}_{\gamma_i}, \overline{\varkappa}_{\gamma_i}, \overline{\varsigma}_{\gamma_i} \right) \right\rangle$ be a neutrosophic soft rough set. We use the accumulated geometric operator (AGO) (Definition 13) to transform the neutrosophic soft number :

$$\left\langle \underline{apr}_{ij}, \overline{apr}_{ij} \right\rangle = \left\langle \left(\underline{\rho}_{\gamma_i} \times \overline{\rho}_{\gamma_i} \right)^{0.5}, \left(\underline{\varkappa}_{\gamma_i} \times \overline{\varkappa}_{\gamma_i} \right)^{0.5}, \left(\underline{\varsigma}_{\gamma_i} \times \overline{\varsigma}_{\gamma_i} \right)^{0.5} \right\rangle = \langle \widetilde{\rho}_{\gamma_i}, \widetilde{\varkappa}_{\gamma_i}, \widetilde{\varsigma}_{\gamma_i} \rangle$$

Step 3: Determining the weights of criteria.

We use a variety of methods to determine the weights of the criterion. Let $s = \langle \widetilde{\rho}_{\gamma_i}, \widetilde{\varkappa}_{\gamma_i}, \widetilde{\varsigma}_{\gamma_i} \rangle$ taken as a neutrosophic soft number. The evaluated values are given by the expert for the criteria. Then the ith criterion's weight can be defined as

$$w_{\gamma_i}(\psi_j) = \bigcup_{\widetilde{\rho}_{\gamma_i}, \widetilde{\varkappa}_{\gamma_i}, \widetilde{\varsigma}_{\gamma_i}} \frac{1 - \sqrt{\left\{ \left(1 - \widetilde{\rho}_{\gamma_i}(\psi_j)\right)^2 + \widetilde{\varkappa}_{\gamma_i}(\psi_j)^2 + \widetilde{\varsigma}_{\gamma_i}(\psi_j)^2 \right\} / 3}}{\sum_{i=1}^{m} \left(1 - \sqrt{\left\{ \left(1 - \widetilde{\rho}_{\gamma_i}(\psi_j)\right)^2 + \widetilde{\varkappa}_{\gamma_i}(\psi_j)^2 + \widetilde{\varsigma}_{\gamma_i}(\psi_j)^2 \right\} / 3} \right)} \quad (1)$$

and $\sum_{i=1}^{m} w_{\gamma_i} = 1$.

Step 4: Determine the larger and smaller ideal solution for neutrosophic soft number.

$$d^{+}_{\gamma_i} = \left(\rho^{+}_{\gamma_i}, \varkappa^{+}_{\gamma_i}, \varsigma^{+}_{\gamma_i}\right) = \left(\left(\max_i \widetilde{\rho}_{\gamma_i}(\psi_j) \right), \left(\max_i \widetilde{\varkappa}_{\gamma_i}(\psi_j) \right), \left(\max_i \widetilde{\varsigma}_{\gamma_i}(\psi_j) \right) \right),$$

$$d^{-}_{\gamma_i} = \left(\rho^{-}_{\gamma_i}, \varkappa^{-}_{\gamma_i}, \varsigma^{-}_{\gamma_i}\right) = \left(\left(\min_i \widetilde{\rho}_{\gamma_i}(\psi_j) \right), \left(\min_i \widetilde{\varkappa}_{\gamma_i}(\psi_j) \right), \left(\min_i \widetilde{\varsigma}_{\gamma_i}(\psi_j) \right) \right).$$

Step 5: Calculate the separation measures between the neutrosophic soft numbers of every alternative from larger ideal solution and smaller ideal solution by 14 .

Table 1 The sum operator $R_{\left(\widetilde{K},\Xi\right)}$

$R_{\left(\widetilde{K},\Xi\right)}$	γ_1	γ_2	γ_3
ψ_1	0.8, 0, 0.2	0.82, 0.54, 0.08	0.7, 0, 0.1
ψ_2	0.84, 0.24, 0.5	0.6, 0, 0.1	0, 0, 1
ψ_3	0.82, 0.14, 0.16	0.5, 0, 0.3	0.4, 0, 0.2
ψ_4	0, 0, 1	0.6, 0.35, 0.05	0.8, 0, 0.2

Table 2 Cosine similarity measure $s_{\gamma_i}(\psi_j)$

$s_{\gamma_i}(\psi_j)$	γ_1	γ_2	γ_3	$\Omega_{s_{\gamma_i}(\psi_j)}$
ψ_1	0.8432	0.8324	0.9899	0.4381
ψ_2	0.8345	0.9863	0	0.0598
ψ_3	0.9679	0.8574	0.8944	0.3563
ψ_4	0	0.8615	0.9701	0.7028

$$D^{+}\left(d_{\gamma_i}\left(\psi_j\right), d_{\gamma_i}^{+}\right) = \bigcup_{e_i} \left|\widetilde{\rho}_{\gamma_i} - \rho_{\gamma_i}^{+}\right| + \left|\widetilde{\varkappa}_{\gamma_i} - \varkappa_{\gamma_i}^{+}\right| + \left|\widetilde{\varsigma}_{\gamma_i} - \varsigma_{\gamma_i}^{+}\right|$$

$$D^{-}\left(d_{\gamma_i}\left(\psi_j\right), d_{\gamma_i}^{-}\right) = \bigcup_{e_i} \left|\widetilde{\rho}_{\gamma_i} - \rho_{\gamma_i}^{-}\right| + \left|\widetilde{\varkappa}_{\gamma_i} - \varkappa_{\gamma_i}^{-}\right| + \left|\widetilde{\varsigma}_{\gamma_i} - \varsigma_{\gamma_i}^{-}\right|$$

Step 6: Determine the neutrosophic soft rough ranking coefficient by the following formula

$$R_i^{+} = \sum_{e_i=1}^{n} w_{\gamma_i}.D^{+}$$

$$R_i^{-} = \sum_{e_i=1}^{n} w_{\gamma_i}.D^{-}$$

Step 7: The final ranking is calculated using the formula below.

$$R_i = \frac{R_i^{-}}{R_i^{+} + R_i^{-}}$$

The choices are listed from lowest to highest based on this value.

3.2 Illustrative example

In a particular hospital, some patients come with the symptoms of cholecystitis . Each one has a different condition. Doctors treat their condition along with their symptoms. The problem is to find out the condition of the patients and which ones have serious cholecystitis stages. By applying the proposed method, we will solve the problem and rank the patients according to their conditions Tables 1 and 2. There are four patients

$\Psi = \{\psi_1, \psi_2, \psi_3, \psi_4\}$ to take as alternatives. We take $\Xi = \{\gamma_1, \gamma_2, \gamma_3\}$ symptoms to identify the condition of cholecystitis. Here symptoms are taken as criteria

γ_1 - Severe pain in the right upper quadrant,

γ_2 - Nausea vomiting

γ_3 - Icterus.

Let's apply Algorithm 1 to this problem.

Step 1: Let the sets $\left(\widetilde{F}_1, \Xi\right)$, $\left(\widetilde{F}_2, \Xi\right)$ and $\left(\widetilde{F}_3, \Xi\right)$ be the evaluations of 3 different doctors on patients with symptoms of cholecystitis.

$$\left(\widetilde{F}_1, \Xi\right) = \left\{ \begin{array}{c} (\gamma_1, \langle\psi_1, 0.4, 0.5, 0.3\rangle, \langle\psi_2, 0.2, 0.3, 0.5\rangle, \langle\psi_3, 0.7, 0.3, 0.2\rangle), \\ (\gamma_2, \langle\psi_1, 0.4, 0.6, 0.8\rangle, \langle\psi_3, 0.3, 0.2, 0.5\rangle, \langle\psi_4, 0.2, 0.5, 0.5\rangle). \end{array} \right\}$$

$$\left(\widetilde{F}_2, \Xi\right) = \left\{ \begin{array}{c} (\gamma_1, \langle\psi_1, 0.7, 0.7, 0.3\rangle, \langle\psi_2, 0.4, 0.8, 0.1\rangle, \langle\psi_3, 0.4, 0.2, 0.8\rangle), \\ (\gamma_2, \langle\psi_1, 0.7, 0.9, 0.3\rangle, \langle\psi_3, 0.5, 0.6, 0.7\rangle, \langle\psi_4, 0.3, 0.7, 0.2\rangle), \\ (\gamma_3, \langle\psi_1, 0.6, 0.3, 0.2\rangle, \langle\psi_4, 0.8, 0.3, 0.2\rangle). \end{array} \right\}$$

$$\left(\widetilde{F}_3, \Xi\right) = \left\{ \begin{array}{c} (\gamma_1, \langle\psi_2, 0.8, 0.6, 0.1\rangle, \langle\psi_3, 0.5, 0.7, 0.4\rangle), \\ (\gamma_2, \langle\psi_1, 0.5, 0.7, 0.3\rangle, \langle\psi_2, 0.6, 0.8, 0.2\rangle, \langle\psi_4, 0.4, 0.6, 0.1\rangle), \\ (\gamma_3, \langle\psi_1, 0.7, 0.4, 0.5\rangle, \langle\psi_3, 0.3, 0.6, 0.3\rangle, \langle\psi_4, 0.4, 0.5, 0.8\rangle). \end{array} \right\}$$

Another set of neutrosophic soft $\left(\widetilde{K}, \Xi\right) \subset NSS(\Psi, \Xi)$ expressing the expected Doctor's assessment on patients with severe cholecystitis symptoms on $\left(\Psi, \widetilde{F}, \Xi\right)$ is given below.

$$\left(\widetilde{K}, \Xi\right) = \left\{ \begin{array}{c} (\gamma_1, \langle\psi_1, 0.8, 0.7, 0.2\rangle, \langle\psi_2, 0.7, 0.8, 0.1\rangle, \langle\psi_3, 0.6, 0.7, 0.2\rangle), \\ (\gamma_2, \langle\psi_1, 0.7, 0.9, 0.2\rangle, \langle\psi_2, 0.6, 0.7, 0.1\rangle, \langle\psi_3, 0.5, 0.7, 0.3\rangle, \langle\psi_4, 0.5, 0.7, 0.1\rangle), \\ (\gamma_3, \langle\psi_1, 0.7, 0.4, 0.1\rangle, \langle\psi_3, 0.4, 0.6, 0.2\rangle, \langle\psi_4, 0.8, 0.6, 0.2\rangle). \end{array} \right\}$$

Step 2: The lower and upper neutrosophic soft approximation of $\left(\widetilde{K}, \Xi\right)$ are calculated as,

$$\underline{apr}_{NS}\left(\widetilde{K}, \Xi\right) = \left\{ \begin{array}{c} (\gamma_1, \langle\psi_2, 0.2, 0.3, 0.5\rangle, \langle\psi_3, 0.4, 0.2, 0.8\rangle,), \\ (\gamma_2, \langle\psi_1, 0.4, 0.6, 0.8\rangle, \langle\psi_4, 0.2, 0.5, 0.5\rangle). \end{array} \right\}$$

$$\overline{apr}_{NS}\left(\widetilde{K}, \Xi\right) = \left\{ \begin{array}{c} (\gamma_1, \langle\psi_1, 0.8, 0.7, 0.2\rangle, \langle\psi_2, 0.8, 0.8, 0.1\rangle, \langle\psi_3, 0.7, 0.7, 0.2\rangle), \\ (\gamma_2, \langle\psi_1, 0.7, 0.9, 0.1\rangle, \langle\psi_2, 0.6, 0.8, 0.1\rangle, \langle\psi_3, 0.5, 0.7, 0.3\rangle, \langle\psi_4, 0.5, 0.7, 0.1\rangle), \\ (\gamma_3, \langle\psi_1, 0.7, 0.4, 0.1\rangle, \langle\psi_3, 0.4, 0.6, 0.2\rangle, \langle\psi_4, 0.8, 0.6, 0.2\rangle). \end{array} \right\}$$

Step 3: From Definition 12, the sum operator $R_{\left(\widetilde{K}, \Xi\right)} = \underline{apr}_{NS}\left(\widetilde{K}, \Xi\right) \oplus \overline{apr}_{NS}\left(\widetilde{K}, \Xi\right)$ is used to convert the neutrosophic soft rough numbers to neutrosophic soft numbers.

Step 4: To rank the neutrosophic soft numbers in the decision-making process, the cosine similarity measure is applied among the neutrosophic soft numbers. Using Definition 15, the cosine similarity measure $s_{\gamma_i}(\psi_j)$ and the rank $\Omega_{s_{\gamma_i}(\psi_j)}$ of each alternative are calculated for $i = 1, 2, 3$ and $j = 1, 2, 3, 4$.

Step 5: According to the principle of rank of all alternatives numerical size, we have;

Table 3 Linguistic terms for NSNs

Linguistic variable	Code	Neutrosophic soft number
Not Hot	NH	0.1, 0.1, 1
Not Very Hot	NVH	0.1, 0.2, 0.9
Medium Low	ML	0.2, 0.4, 0.8
Fair	F	0.7, 0.6, 0.6
Medium High	MH	0.8, 0.6, 0.5
Not Very Low	NVL	0.8, 0.7, 0.3
Not Low	NL	0.9, 0.8, 0.1

Table 4 Report of doctor for each patients

	γ_1	γ_2	γ_3
ψ_1	NL	F	MH
ψ_2	NVL	NL	NH
ψ_3	MH	NVL	F
ψ_4	F	MH	NVH

Table 5 NSR decision matrix

	γ_1	γ_2	γ_3
ψ_1	$\langle 0,0,1\rangle, \langle 0.8,0.7,0.2\rangle$	$\langle 0.4,0.6,0.8\rangle, \langle 0.7,0.9,0.1\rangle$	$\langle 0,0,1\rangle, \langle 0.7,0.4,0.1\rangle$
ψ_2	$\langle 0.2,0.3,0.5\rangle, \langle 0.8,0.8,0.1\rangle$	$\langle 0,0,1\rangle, \langle 0.6,0.8,0.1\rangle$	$\langle 0,0,1\rangle, \langle 0,0,1\rangle$
ψ_3	$\langle 0.4,0.2,0.8\rangle, \langle 0.7,0.7,0.2\rangle$	$\langle 0,0,1\rangle, \langle 0.5,0.7,0.3\rangle$	$\langle 0,0,1\rangle, \langle 0.4,0.6,0.2\rangle$
ψ_4	$\langle 0,0,1\rangle, \langle 0,0,1\rangle$	$\langle 0.2,0.5,0.5\rangle, \langle 0.5,0.7,0.1\rangle$	$\langle 0,0,1\rangle, \langle 0.8,0.6,0.2\rangle$

$$\psi_4 > \psi_1 > \psi_3 > \psi_2$$

Consequently, patient ψ_4 suffers from severe symptoms of cholecystitis. ψ_2 is less affected than the other patients.Table 4

Let's apply Algorithm 2 to this problem.

The doctor gives his report by linguistic variables, then we change it into neutrosophic soft numbers. Table 3 presents the values of linguistic variables

Doctors evaluated patients according to symptoms as shown in the table below.

Step 1: Build the neutrosophic soft rough decision matrix.

Step 2: Transforming NSR decision matrix into neutrosophic soft number (NSN). The conversion of the data in Table 5 (ψ_1, γ_1) cell to NSR is calculated as follows.

$$\begin{aligned} d_{\gamma_1}(\psi_1) &= \left\langle (0\times 0.8)^{0.5}, (0\times 0.7)^{0.5}, (1\times 0.2)^{0.5} \right\rangle \\ &= \left\langle (0)^{0.5}, (0)^{0.5}, (0.2)^{0.5} \right\rangle \\ &= \langle 0,0,0.4472\rangle \end{aligned}$$

Remaining values are calculated in the same way. The values are shown in Table 6:

Step 3: Calculate the weights values for each symptoms.

Table 6 Transforming NSR decision matrix into NSN

	γ_1	γ_2	γ_3
ψ_1	$\langle 0, 0, 0.4472\rangle$	$\langle 0.5291, 0.7348, 0.2828\rangle$	$\langle 0, 0, 0.3162\rangle$
ψ_2	$\langle 0.4, 0.4898, 0.2236\rangle$	$\langle 0, 0, 0.3162\rangle$	$\langle 0, 0, 1\rangle$
ψ_3	$\langle 0.5291, 0.3741, 0.4\rangle$	$\langle 0, 0, 0.5477\rangle$	$\langle 0, 0, 0.4472\rangle$
ψ_4	$\langle 0, 0, 1\rangle$	$\langle 0.3162, 0.5916, 0.2236\rangle$	$\langle 0, 0, 4472\rangle$

Table 7 Weights of symptoms

$w_{\gamma_i}(\psi_j)$	γ_1	γ_2	γ_3
ψ_1	0.2982	0.3816	0.32
ψ_2	0.4804	0.3545	0.1649
ψ_3	0.4511	0.2644	0.2843
ψ_4	0.1810	0.4562	0.3626
w_{γ_i}	$w_{\gamma_1} = 0.3527$	$w_{\gamma_2} = 0.3642$	$w_{\gamma_3} = 0.2830$

Table 8 Larger and Smaller Ideal Solutions

	γ_1	γ_2	γ_3
$d^+_{\gamma_i}$	$\langle 0.5291, 0.4898, 1\rangle$	$\langle 0.5291, 0.7348, 0.5477\rangle$	$\langle 0, 0, 1\rangle$
$d^-_{\gamma_i}$	$\langle 0, 0, 0.2236\rangle$	$\langle 0, 0, 0.2236\rangle$	$\langle 0, 0, 0.3162\rangle$

$$w_{\gamma_1}(\psi_1) = \frac{1 - \sqrt{\left((1-0)^2 + 0^2 + (0.4472)^2\right)/3}}{(0.3675 + 0.4703 + 0.3944)} = \frac{0.3675}{1.2322} = 0.2982$$

The remaining values are calculated in the same way and the weight for each attribute is calculated by taking the arithmetic average. The values are shown in Table 7

Step 4: Using the Definition 14 calculated the larger ideal solution and smaller ideal solution. For each symptom, we find the maximum and minimum ideal value. These values are shown in Table 8.

Step 5: Using the Definition 14 calculate the values D^+ and D^-.

$$D^+\left(d_{\gamma_1}(\psi_1), d^+_{\gamma_1}\right) = |0 - 0.5291| + |0 - 0.4898| + |0.4472 - 1| = 1.5717$$
$$D^-\left(d_{\gamma_1}(\psi_1), d^-_{\gamma_1}\right) = |0 - 0| + |0 - 0| + |0.4472 - 0.2236| = 0.2236$$

Similarly, calculated the seperation measure value for each patient on each symptom. The values of D^+ and D^- shown in Tables 9 and 10.

Step 6: Calculate the neutrosophic soft rough ranking coefficient.

Table 9 Seperation from $d^+_{\gamma_i}$

D^+	γ_1	γ_2	γ_3
ψ_1	1.5717	0.2649	0.6838
ψ_2	0.9055	1.4954	0
ψ_3	0.4843	1.2639	0.5528
ψ_4	1.0189	0.6802	0.5528

Table 10 Seperation from $d^-_{\gamma_i}$

D^-	γ_1	γ_2	γ_3
ψ_1	0.2236	1.3231	0
ψ_2	0.8898	0.0926	0.6838
ψ_3	1.0796	0.3241	0.131
ψ_4	0.7764	0.9078	0.131

Table 11 R_i^+ and R_i^-

R_1^+	0.8443
R_2^+	0.8639
R_3^+	0.7875
R_4^+	0.7635
R_1^-	0.5607
R_2^-	0.5410
R_3^-	0.5358
R_4^-	0.6415

Table 12 Order the patient

	Rankingcoefficient	*Order*
ψ_1	0.3990	*III*
ψ_2	0.3850	*IV*
ψ_3	0.4049	*II*
ψ_4	0.4565	*I*

$$\begin{aligned} R_1^+ &= (1.5717 \times 0.3527) + (0.2649 \times 0.3642) + (0.6838 \times 0.2830) \\ &= 0.8443 \\ R_1^- &= (0.2236 \times 0.3527) + (1.3231 \times 0.3642) + (0 \times 0.2830) \\ &= 0.5607 \end{aligned}$$

Similarly, the remaining values are calculated.These values are shown in Table 11.

Step 7: Calculate the final ranking.

$$R_1 = \frac{R_1^-}{R_1^+ + R_1^-} = \frac{0.5607}{0.8443 + 0.5607} = \frac{0.5607}{1.405} = 0.3990$$

The remaining values are calculated by same way. These values are shown in Table 12.

Where, R_1, R_2, R_3 and R_4 indicate the rank of patients ψ_1,ψ_2,ψ_3 and ψ_4, respectively.

Table 13 Comparative results

Patients	*MCDM* Result		Topsis result	
	Rank	Order	Ranking coefficient	Order
R_1	0.4381	*II*	0.3990	*III*
R_2	0.0598	*IV*	0.3850	*IV*
R_3	0.3563	*III*	0.4049	*II*
R_4	0.7028	*I*	0.4565	*I*

Table 12 shows each patient's ranking values and their order. The order of the patient's condition is

$$\psi_4 > \psi_3 > \psi_1 > \psi_2.$$

Patient ψ_4 suffers from severe cholecystitis symptoms. Patient ψ_2 was affected less than others.

4 Comparison analysis

In this section, we compare the proposed TOPSIS method and multi-criteria decision-making (MCDM) methods. Table 13 shows a comparison of the two methods.

Therefore, the results did not change in either method. As a result, more ψ_4 patients than ψ_2 patients were affected by cholecystitis symptoms. However, the results for ψ_1 and ψ_3 patients differed between the two methods. The MCDM method ranked ψ_1 as more important than ψ_3, while the TOPSIS method ranked ψ_3 as more important than ψ_1. This difference stems from the fundamental differences in how the two methods approach evaluation criteria. The MCDM method has high intuitive interpretive power but limited consistency control. The TOPSIS method makes decisions by considering the distances of each alternative from positive and negative ideal solutions. This method: Performs a more systematic and objective evaluation under uncertainty. Especially when combined with a neutrosophic structure, it offers a more holistic view by evaluating accuracy, uncertainty, and error rates together. Since it is based on proximity to the ideal state, it tends to produce more stable and consistent results in decision-making processes. According to the findings obtained in this study:

- The TOPSIS method provides more stable and mathematically consistent results, especially in situations where there is a high degree of uncertainty. The change in the middle ranks is due to the strong discriminating power of the method.
- The MCDM method offers a more intuitive, decision-maker-oriented, and flexible approach, and may be preferred in situations where expert opinions are dominant.

Applying the TOPSIS method on a neutrosophic soft rough set is powerful because it can mathematically process both proximity to the ideal solution and uncertainty. Applying the MCDM method on a neutrosophic soft rough set, on the other hand, is suitable for creating more customizable and flexible decision models. Therefore, the

use of the TOPSIS method is more appropriate in the problem presented in this study; however, the method should be chosen based on the decision-maker's needs and the complexity level of the problem.

5 Conclusion

A neutrosophic soft rough set aims to improve quality in various applications and neutrosophic soft rough matrix is used in decision making problems. We have shown in this paper how to apply the neutrosophic soft rough set and matrix to the multi-criteria decision making (MCDM) and TOPSIS methods. The task of cholecystitis patient condition determination is successfully solved by the methods we propose for neutrosophic soft rough sets. As a result, patient 4 has severe symptoms, and neutrosophic soft rough sets shows to be more useful for handling uncertainty situations. A detailed discussion is held regarding the outcomes and comparative analysis of the suggested algorithms. Numerous domains, including pattern recognition, machine learning, image processing, expert systems, and database knowledge discovery, make use of rough set theory. Our suggested techniques assist in resolving issues in numerous applications outside of the medical domain. Future research will employ the set and matrix that have been presented.

Real-world decision-making processes often contain incomplete, contradictory or uncertain information. This study offers a mathematical structure that allows managers to make more reliable decisions in such environments. Neutrosophic soft rough structures in particular provide a strong alternative in areas where traditional methods are inadequate. TOPSIS, which is based on proximity to the ideal solution in the decision-making process, offers managers a fast and systematic solution. This provides managers with a great advantage, especially in time-constrained situations or scenarios where a large number of alternatives are evaluated. The flexible structure of multi-criteria decision-making methods allows decision makers to adapt the system according to their own priorities. This can respond to sectoral needs by making decision models more situational and customized. Thanks to the quantification of uncertainties, managers can build more controlled and transparent decision processes. These models can be successfully applied in strategic decision areas. In this context, managers can provide institutional reliability and accountability by basing their decisions on mathematical foundations rather than intuitive ones. This study provides a new perspective to the literature by integrating neutrosophic soft rough set theory with topological structures. Thus, the applicability of uncertainty theories in wider areas increases and the theoretical infrastructure deepens with new definitions. It offers a new direction to academic studies in the field of multi-criteria decision making by revealing how decision-making techniques such as TOPSIS and MCDM can be used more effectively with different mathematical frameworks. This study provides a methodological basis in terms of determining the strengths and weaknesses of different methods. It provides a preliminary structure for comparison with more methods in future research.

Although this study demonstrates the effectiveness of both the TOPSIS and MCDM methods within the framework of neutrosophic soft rough sets, it is not

without limitations. One primary limitation is the dependence on predefined decision matrices and expert evaluations, which may introduce subjectivity and affect the reliability of the outcomes? Especially in highly ambiguous scenarios. Additionally, the study focuses on a comparative analysis within a specific problem context; Hence, the generalizability of the results to other domains or decision environments may be limited.

Another limitation lies in the computational complexity that may arise when handling large-scale problems with numerous criteria and alternatives, particularly in MCDM models where greater flexibility comes at the cost of increased modeling effort. As for future work, several promising directions can be explored. Firstly, the integration of optimization algorithms? Such as genetic algorithms, particle swarm optimization, or machine learning techniques? Could enhance the robustness and adaptability of the decision-making process. Secondly, hybrid decision-making models that combine the strengths of both TOPSIS and other MCDM techniques could be developed to balance precision and flexibility more effectively. Finally, applying the proposed approach to real-world problems in domains such as medical diagnosis, supply chain management, or risk assessment would provide practical validation and reveal further insights into its applicability.

Author contributions The authors contributed equally.

Funding The authors declare that no funds, grants, or other support were received during the preparation of this manuscript.

Data availability All data and materials in this study are original.

Declarations

Conflict of interest The authors have no relevant financial or non-financial interests to disclose.

Ethical approval and permission for participation Ethics approval and permission for participation are not applicable.

Publication permission This study does not require any publication permission.

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