



# Number Sense Across the Transition from Preschool to Elementary School: A Latent Profile Analysis

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## Abstract

Empirical research on number sense to date has been conducted with preschoolers and elementary school students. Despite their contributions to the literature, these studies have used variable-centered analytic approaches that may prevent distinct number sense profiles for preschoolers and first graders from being identified. This study aimed to determine the number and proportions of distinct number sense profiles, the variables influencing these profiles, and the characteristics of the children with different profiles in the transition from preschool to elementary school. We conducted a quantitative study using a cross-sectional design for this purpose and used latent profile analysis to identify and describe distinct number sense profiles for preschoolers and first-grade students. A total of 1688 children, 848 from preschool and 840 from the first grade of elementary school, took part in the study. A Number Sense Screener was used to determine number sense profiles. The data collection tool used in the study was found to be reliable. Based on the findings of the study, we identified three profiles for both the preschoolers and first graders, namely, moderate very low, moderate low, and moderate high. We examined family and household income variables to help predict the profiles. We then took the characteristics and components of number sense that caused the differentiation in the profiles as revealed by the results of the study and discussed them in light of the literature.

**Keywords** Number sense · Preschool education · Elementary school · Mathematics teaching · Mathematics learning

## Introduction

The importance of math education in early childhood is widely acknowledged globally (Lee & Pant, 2017; Moomaw, 2015). Supporting children's math education from preschool

to first grade is crucial for their future academic success (Lee, 2010; Schacter & Jo, 2017). One way of improving children's math skills in elementary and secondary school is to foster number sense skills, as demonstrated by longitudinal studies on how math skills improve during early childhood (Aubrey & Dahl, 2014; Jordan et al., 2009).

Number sense refers to an understanding of numbers and their relationships that evolves through both nature and nurture (Berch, 2005; Wilson et al., 2009). It encompasses not just an inherent grasp of numbers, but also the pre-verbal and verbal abilities that develop before elementary school (Dyson et al., 2015; Gersten et al., 2005; Rousselle & Noël, 2007). Number sense is, therefore, an innate math skill in the early years that requires nurturing (Jordan et al., 2006).

The significance of number sense in mathematics education is also widely acknowledged globally (Yang & Li, 2008). Many countries have incorporated number sense and its development in their education programs (Chen et al., 2013). Countries such as Singapore, Hong Kong, and Taiwan, which excel in exams such as TIMMS and PISA, place particular emphasis in their curricula on developing number sense from an early stage (Ministry of Education,

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2012; Curriculum Development Council, 2000; Ministry of Education in Taiwan, 2003). The United States focuses on number sense in its mathematics education (National Council of Teachers of Mathematics [NCTM], 2000). Türkiye includes number skill development in its preschool education program (Milli Eğitim Bakanlığı [MEB], 2013), but fails to place sufficient emphasis on the basic components of number sense in its elementary school mathematics curriculum (Çetin & Öztürk, 2020). Research shows that many preschool children start school without having developed number sense early on (Berch, 2005) and that number sense performance among elementary students varies by component (Yang & Li, 2008). Despite the global emphasis on number sense, studies have shown that children in China have a better grasp of number sense than children in Finland (Aunio et al., 2006). Nonetheless, basic number proficiency in kindergarten has been found to affect mathematics proficiency until at least the third grade of elementary school and beyond (Jordan et al., 2006, 2010a, 2010b).

Research and elementary school curricula tend to approach children as a single group and apply the same methods regardless of their initial number sense abilities, but children's number sense abilities vary greatly (Aunio et al., 2006; Jordan et al., 2006; Yang & Li, 2008). Furthermore, research has shown that number sense in elementary school lays the foundation for learning more formal mathematical concepts and skills (Jordan et al., 2007). Studies have found that number sense is a key predictor of math achievement (Hassinger-Das et al., 2014; Jordan et al., 2009) and is crucial for understanding all aspects of formal mathematics (Feigenson et al., 2013). Understanding why some children are better prepared to learn math than others is crucial because a strong number sense is linked to a positive attitude toward numbers, awareness of how they are used, and the ability to interpret and think about them flexibly (Schneider & Thompson, 2000).

Determining children's number sense profiles is essential for providing effective mathematics education because their skills may differ. Profiles have been determined for children in Northern Ireland, England, Belgium, and Finland (Nanu et al., 2020) and for 6-year-old children in Cyprus (Pittalis et al., 2018), but no such analysis has been conducted for preschool or early school-aged children in Türkiye. Determining number sense profiles in Türkiye would be valuable for the literature. Knowing these profiles could help educators target their teaching methods to support children's specific skills. Ignoring this information could result in teaching children skills they have already mastered, thus wasting time and resources. Understanding the number sense profiles of normally developing children is vital because poor number sense development is one of the causes of dyscalculia, a learning disability in math (Can, 2020; Hannell, 2013; Üstün et al., 2019). The number sense profiles of children must be

taken into consideration when providing mathematics education, as this can impact their development and the fulfillment of their right to education (Hodgkin & Newell, 2007).

The use of technology, such as digital devices and educational software, in the development of children's number sense is still being explored. Technology can provide additional resources and opportunities for learning but must be properly integrated (Papadakis et al., 2018). Touchpad technology may improve number sense through hands-on experiences but more research is needed to determine its impact. The effectiveness of computer-based learning varies depending on the software and how it is used. Teachers should choose technology tools that align with learning goals and assess how they affect student learning. Some studies (Papadakis et al., 2016a; 2016b; 2018; 2021) suggest that hands-on activities should be combined with digital learning for optimal results. Digital apps should be tailored to children's number sense profiles to best support their development (Papadakis et al., 2021).

Children's abilities in mathematics are often broadly categorized as "good" or "bad" but this hides their strengths and weaknesses in specific numerical skills (Gray & Reeve, 2016). Some preschoolers are skilled counters but lack an understanding of cardinality, while others lack fundamental counting skills but have a grasp of how counting can indicate cardinality (Dowker, 2008). The development of math skills does not seem to follow a single sequence. Preschoolers may struggle with basic skills like counting but excel in seemingly more complex tasks like arithmetic (Dowker, 2008; Holmes & Dowker, 2013). Children's number sense profiles must be determined to accurately assess how their math skills are developing. The differentiation in the profiles of preschoolers when they start school reveals the quality of their education. The Turkish Ministry of National Education 2013 Preschool Education Program evaluates not just the child, but also the teacher and the program. The results of this study can help assess how the preschool program's mathematics activities affect the development of number sense.

This study aimed to examine the number sense profiles of preschoolers and first graders to guide the development of educational programs and digital apps that support the development of their math skills. The goal was to identify the skills that differentiate their profiles as this could indicate early math difficulties. The study also aimed to determine the effect of socioeconomic status and parents' education levels on children's number sense profiles. The findings are expected to provide a guide for designing educational activities and digital apps that support the development of number sense in children.

To evaluate the original points of the research mentioned above, we first need to define and examine the components of number sense. To do this, we explain the sub-dimensions

of the NSS (Number Sense Screener) used to reveal the number sense profiles of children aged 60–71 months and 72–83 months under the literature review heading below.

## Literature Review

### Definition and Components of Number Sense

Many studies have explored the characteristics and components of number sense (Howell & Kemp, 2005; Reys & Yang, 1998). Number sense is generally defined as involving the concepts of basic number knowledge and algebraic ability. But various theoretical frameworks exist, leading to different interpretations (Berch, 2005). Reys and Yang (1998) argue that number sense encompasses a person's general understanding of numbers and operations and their ability to use this knowledge in various ways. Number sense, thus, refers to a child's general understanding of numbers and algebraic operations, as well as the ability to use numbers in everyday life (Yang, 2005). Number sense can also be understood based on brain development and cognitive elements, meaning it is the ability to use numbers at different levels depending on the child's thinking flexibility (Schneider & Thompson, 2000). As children grow and their brains develop, their ability to use number sense improves sequentially (Aunio et al., 2006; Jordan et al., 2010a, 2010b). Factors such as working memory, inhibitory control, and attention shifting are all crucial for math performance throughout one's life (Bull & Lee, 2014; Diamond, 2013). Number sense is explained as the gradual development of a child's mathematics skills. According to this view, number sense is the foundation for math skills that emerge with the development of counting knowledge followed by arithmetic abilities (Jordan et al., 2009; Örstegren & Traff, 2013). Definitions of number sense place emphasis on biological ability while stating that environmental factors, including education, also support the development of number sense (Ívrendi, 2011). Number sense is an innate and evolving ability that develops before children enter elementary school (Dyson et al., 2015). Pittalis et al. (2018) stated that the algebraic dimension of number sense develops after children start school and receive an education. The development of number sense in children is also considered as an indicator for early intervention. Poor development of number sense can lead to dyscalculia, a math learning disability, in otherwise normally developing children (Can, 2020; Hannell, 2013; Üstün et al., 2019). Skills related to number sense are often used as indicators in dyscalculia diagnostic tests (Reeve & Humberstone, 2012). Griffin (2004) describes number sense as "easy to identify but difficult to describe and teach." The ability to perform complex calculations, understand the

meaning of numbers, and have an intuitive feel for numbers are all part of number sense (Berch, 2005).

According to Berch (2005), the disparities between various conceptions of number sense may be explained by the fact that cognitive scientists and math educators have distinctly different meanings for the concept. One of the empirical studies that best explains the number sense skills of children in early childhood is the one carried out by Jordan et al. (2006), as pointed out by Pittalis et al. (2018). Jordan et al. (2006) demonstrated with empirical evidence that number sense is a two-dimensional concept. They explained the first dimension using the concepts of "low order" and "high order," described earlier by Berch (2005) referring to the characteristics of number sense. According to Berch (2005), number sense can be achieved through "conceptual sense-making" in math education, from the "low order" features based on biological sensory organs to the "high order" features. The second dimension proposed by Jordan et al. (2006) is based on the latent structure (unidimensional, bidimensional, and integrated bidimensional) of children's central conceptual structures, proposed by Okamoto and Case (1996). According to Jordan et al. (2009), number sense consists of early number knowledge and arithmetic skills. Early number knowledge is rooted in biological abilities and consists of primary and secondary biological abilities. Primary biological abilities are the number sense abilities that children develop in their early years before receiving education. Secondary biological abilities develop with formal math education and help improve primary biological abilities. The results of the study by Pittalis et al. (2018) and Örstegren and Traff (2013) support the claims made by Jordan et al. (2009) that number sense is biologically based (early number knowledge) and can be improved through education (arithmetic skills). Going by the results of the study by Jordan et al., (2006, 2007, 2009, Jordan et al., 2010a), number sense components can be categorized under the dimensions of early number knowledge and arithmetic skills. The components of early number knowledge that children develop through innate and informal learning (counting, number recognition, number comparisons, and nonverbal calculation) are the "low order" number sense (Berch, 2005). The components of number sense that develop through educational activities (story problems and number combinations) are the "high order" number sense. This study will compare the number sense profiles of first-grade children who have started or received preschool education, and then discuss the development of primary biological abilities and the secondary biological abilities acquired through preschool education. It will define the latent profiles of the early number knowledge and arithmetic skills components based on the children's number sense components. Determining the number sense components that play a role in the differentiation of children's number sense profiles by identifying

developmental indicators will contribute to the literature. We will then discuss the role of preschool education in the development of the “high order” number sense in the context of these results.

This study examined children’s number sense profiles based on counting, number recognition, number comparisons, nonverbal calculation, story problems, and number combinations, which are referred to by Jordan et al. (2006) as the subcomponents of number sense for preschool children. The number sense subcomponents are:

### Counting (COUNT)

Counting and the concept of numbers are built on the principles of one-to-one correspondence, sequence recognition, and the understanding of counting rules such as the stable order principle and cardinality. Before attending preschool, children typically learn these basic principles through everyday experiences (Geary et al., 2018). Later, they develop more advanced skills like counting backwards, double counting, counting sets larger than 10, and performing mathematical operations (Baroody, 1987; Purpura et al., 2013; Siegler & Booth, 2004).

### Number Recognition (NREC)

Jordan et al., (2012a, 2012b) found that children primarily use numbers to label quantities of three or fewer but can read and recognize numbers. They also use numbers to count objects in a set (Jordan et al., 2010a, 2010b).

### Number Comparisons (NCOMP)

Landerl et al. (2004) found that numerical comparison involves understanding the sizes of numerical amounts and comparing them. Children need to understand the relationship between numbers and their relative quantities (Jordan et al., 2012a, 2012b). Children can identify numbers that come immediately after a given number, and they can determine which number is greater or smaller in two given numbers.

### Nonverbal Calculation (NVCAL)

Klein and Bisanz (2000) defined non-verbal calculation as the ability to handle addition-subtraction using objects. Quantity comparison is the ability to compare magnitudes in symbolic form (Reys & Yang, 1998). Preschoolers can solve non-verbal calculations with objects, such as determining the total number of objects after some are hidden (Huttenlocher et al., 1994; Levine et al., 1992).

### Story Problems (SPROB)

Verbal problems require the ability to solve problems without the use of manipulatives by mentally grasping, evaluating, and understanding the meaning of operations (Levine et al., 1992; McIntosh et al., 1992). Preschool children have limited success in solving arithmetic problems presented to them verbally (“*Sue has  $m$  cents. Jim gives her  $n$  more cents. How many cents does Sue now have?*”). This is due to not being able to grasp the vocabulary and syntactic structure of the problem or finding it difficult to mentally depict quantities in the absence of concrete objects (Jordan et al., 2006).

### Number Combinations (NCOMB)

The ability of students to perform arithmetic without physical objects is linked to their ability to store and manipulate numerical information in their working memory and their understanding of mathematical operations (Jordan et al., 2007). In the case of first graders, their focus is mainly on addition and subtraction. Children can understand that a number can be broken down into other numbers, for example, 6 can be expressed as combinations of  $3 + 3$ ,  $5 + 1$ ,  $1 + 5$ , and  $4 + 2$  (Confer, 2005).

We examined theoretical studies and research on the development of number sense and its components and discussed the role of early childhood education and the family under the heading Children’s development of number sense skills, below.

## Development of Children’s Number Sense

Early childhood is when children start to use their number sense skills. They develop these skills through object and spatial interactions. As they grow, they use contextually-based mathematic skills to steadily improve their number sense competencies such as comparison, ordering, calculation, measurement, and classification. Numerical operations and arithmetic skills in support of children’s number sense development start to form logically based on counting principles at the age of three (Gelman & Meck, 1983). When children develop and improve number sense skills in their early years, this helps them to perceive and solve real-world problems and adapt to life later on. Preschool mathematics education is thus crucial for children’s growth. Many countries have programs, such as Pre-K Mathematics and Building Blocks in the United States, the Early Numeracy Research Project in Australia, and First Steps in Australia, aimed at improving early childhood mathematics education. Research shows that these programs have made significant contributions to early mathematics education (Clements & Sarama, 2011; Sarama & Clements, 2009a; Clarke & Clarke, 2004; Perry & Dockett, 2007; Willis et al., 2004). These

projects include such skills as quantity creation, counting, and symbolic thinking, which are important for developing number sense (Moomaw, 2015).

At this point, we analyzed and interpreted various developmental theories (cognitive and social development) to explain and support the development of number sense. We also discussed the theoretical perspective using the research results for supporting the development of number sense.

Piaget's theory of cognitive development explains the progression of counting skills in preschool and elementary school. According to Piaget, children learn by interacting with their surroundings during preschool, when they are in their pre-operational stage as characterized by egocentric, one-dimensional, and non-reversible thinking. Piaget's number conservation experiments show how children's counting knowledge evolves from preschool to elementary school, starting with one-to-one matching and equivalent set formation (Piaget, 1970). Children who start elementary school understand that the number of objects in a group stays constant even if their positions change and this is known as number conservation. However, preschoolers cannot grasp number conservation due to the effects of egocentric, one-dimensional, and non-reversible thinking. By the time children reach the concrete operational stage in elementary school, they have acquired number retention and their ability to mentally manipulate numbers has become more sophisticated, allowing them to perform arithmetic operations using mental symbols (Wadsworth, 1996).

According to Berch (2005), the “low order” dimension of number sense comprises innate abilities that develop without education. Okamoto and Case (1996) explain the biological development of children's number sense by using the concept of “numerical discrimination.” The basic numerical information that children acquire goes on to form the foundation for the development of their number sense later. The results of the study by Pittalis et al. (2018) on the dimensions of number sense along with those by Jordan et al., (2006, 2007, 2009, 2009) show that the first dimension, early number knowledge, is the early intuitive number knowledge that develops as a biological ability in a child around the age of 6–6.5. Early number sense continues to develop gradually starting with a child's entry into first grade. In this respect, the step-by-step development of a child's number sense echoes Piaget's theory. According to Jean Piaget, number sense involves the gradual development of an individual's comprehension of quantities and the connections between numbers. It begins with a natural understanding of such concepts as “greater” or “smaller” and eventually evolves into a grasp of symbolic representation and fundamental arithmetic operations. Number conservation is considered a crucial aspect of number sense and is achieved through the growth of logical thinking and the ability to mentally manipulate symbols (Wadsworth, 1996).

According to Vygotsky (1978), number sense is the ability to understand and manipulate mathematical concepts and relationships. Vygotsky believed that this ability develops through social interaction and cognitive apprenticeship, where more knowledgeable individuals support and scaffold the learning of less knowledgeable individuals. In the context of number sense, scaffolding can involve breaking down complex mathematical problems into simpler parts, providing concrete examples, or using gestures and other forms of representation to support understanding (Vygotsky, 1962). According to Berch (2005), the “high order” dimension of number sense develops through educational activities. According to Okamoto and Case (1996), supporting children's number sense through education contributes to the development of verbal sequential factors. Pittalis et al. (2018) state that the arithmetic skills of the number sense dimensions develop with formal education in children who start first grade at the average age of 6–6.5. This shows just how important education is for enhancing children's understanding of numbers during their early years. Teachers will find it easier to establish a learning environment when they know the number sense profiles of the children whose number sense development they are supporting. This results in the need to determine how many number sense profiles the children have and their characteristics. Having highlighted the importance of the environment in which the child lives for their number sense development, we shall now explain the development of number sense by linking it to Bronfenbrenner's ecological theory.

Bronfenbrenner's ecological theory of development states that a child's number sense development is influenced by various systems in their environment including direct interactions with family and teachers (microsystem), interactions between different microsystems such as family and school (mesosystem), larger societal context such as cultural norms (exosystem), and overarching cultural and historical context (macrosystem) (Bronfenbrenner, 1979). We examined the number sense profiles of the children participating in this study in terms of their parents and the education they received and explained the ecological theory by detailing the concepts of the microsystem and mesosystem.

The microsystem, consisting of family and teachers, plays a crucial role in developing a child's number sense by providing supportive and engaging learning environments. The child's own interests and abilities also contribute to their development (Bronfenbrenner, 1979). The family provides the child with their first social experiences and models of mathematical thinking, influencing the development of their number sense. Before children enter school and receive formal education (around 6.5 years of age on average), their number sense typically develops through informal interactions, influenced by their innate cognitive abilities and social environment (Saxe et al., 1987). The preschool years

are crucial for the growth of number sense, which matures through experience and knowledge, unique to each child. From 4 to 7, most children learn to understand counting, addition, and subtraction, to recognize numbers, and they start to grasp the relative magnitude of numbers (Reys & Yang, 1998). This process is thought to stem primarily from the child's experiences with numbers in their home environment. During this critical period for number sense development, children learn through playing games that involve working with tangible objects (Griffin, 2004; Louange, 2007), by interacting with older children and adults (Robinson et al., 2002), and by engaging directly with their surroundings (Clements & Sarama, 2014; Jordan, 2007; McIntosh et al., 1992). The range of educational activities that parents do with their children varies based on socio-demographic characteristics such as the parents' education level (Aikens & Barbarin, 2008; Sénéchal, 2006; Son & Morrison, 2010). Parents' education levels affect their knowledge, attitudes, and behaviors toward child-rearing and have a multi-directional impact on the child's overall development (Zadeh et al., 2010). Family socio-economic status can also affect the opportunities available to the child (Şirin, 2005). A study by Jordan et al., (2012a, 2012b) found that number sense interventions positively affect number proficiency and mathematics achievement among low socio-economic level children at risk. This could lead to differences in number sense skills at the start of preschool education based on the children's socio-demographic characteristics. Other factors such as the quality of preschool education and teacher qualifications can also affect mathematical performance among children (Dahlberg et al., 1999), which is why this study aimed to examine the effect of family income and education levels on children's number sense profiles, identifying the role of family factors in number sense development.

In the mesosystem, the relationship between family and school, as well as the quality of the educational experience provided, can either support or hinder the development of number sense. Formal education, through the curriculum and instructional strategies, can also support or hinder this development.

The quality of preschool education is crucial for the development of a child's mathematical abilities, especially counting skills. Research shows that children who receive

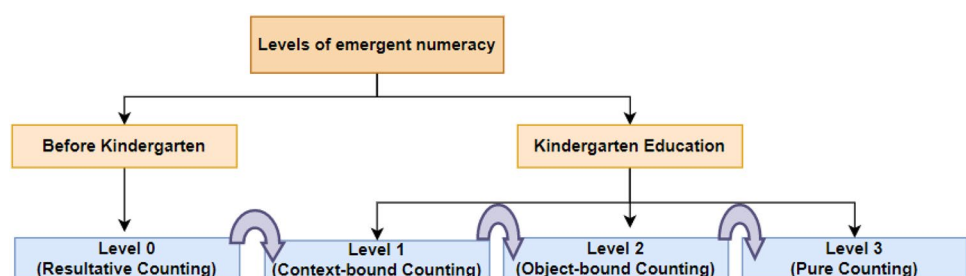
higher-quality preschool education have better mathematical skills in elementary school (Lehr et al., 2016). Early childhood education also affects a child's development positively (Melhuish et al., 2015). Children in countries with earlier start times for schooling perform better in mathematics compared with their peers (Kavkler et al., 2000; Luyten, 2006; Wolke et al., 2015). Developing counting skills in preschool helps set the foundation for other mathematical skills before elementary school (Papadakis et al., 2021). The study of horizontal and vertical mathematization can be applied to understanding children's transition from preschool to elementary school. Horizontal mathematization involves moving from real-world objects to mental symbols, while vertical mathematization involves creating an imaginary world through symbols and thoughts (Freudenthal, 1991; 1983). So, this study looked at the role of preschool education in the development of children's number sense as they make the transition from preschool to elementary school.

The development of children's number sense is described by Heuvel-Panhuizen (2008) in three general levels (see Fig. 1).

According to Heuvel-Panhuizen (2008), children's number sense is considered to be at level 0 prior to entering preschool education. This sense can vary based on the child's exposure to mathematical stimuli. However, through preschool education, this sense can develop and reach levels 1, 2, and 3. There are four levels of number sense, both before and during preschool, as shown in Fig. 1. The first level in preschool is when children can count in certain situations and do basic addition/subtraction. At the second level, they can count objects and do basic addition/subtraction by removing objects. At the third level, they can focus on a missing number and perform mental addition/subtraction calculations.

Heuvel-Panhuizen (2008) states that children's mathematical concept knowledge before starting preschool education may vary from child to child. For example, some children may be able to count well, while others may not be able to. Children may have understood the meanings of numbers to different degrees. But the number sense developed during preschool education may differ when starting elementary school. Investigating the number sense profiles of children before starting preschool education and their number sense

**Fig. 1** Levels of emergent numeracy



profiles when they start elementary school will reveal which profiles have changed. The change in number sense can be interpreted based on the education received during preschool education. According to Heuvel-Panhuizen (2008), number sense is expected to develop during early childhood education. We expect to contribute to the literature by investigating this development indicator in Turkish children. Furthermore, both preschool and elementary school children's number sense development can be monitored and evaluated through various assessment methods to guide their future education and development.

Supporting children's counting skills systematically in preschool education is crucial for their number sense development. Gelman and Gallistel's (1978) counting principles have been studied as part of this development. These principles provide guidelines and methods for teaching young children about numbers and counting. They believe that children have an inherent number sense that can be developed through exposure to counting and numerical patterns. The principles aim to help children build a strong foundation for future mathematical learning by teaching counting in a meaningful and systematic way. The one-by-one principle is related to number sense as it establishes the foundation for counting and number recognition by counting objects one by one. The fixed order principle helps children develop consistency and stability in counting by counting in a consistent order. The cardinal number principle stresses that the last number counted represents the total number of objects, helping children see numbers as stable and abstract entities. The separation principle, also known as abstraction, teaches that the quantity of objects remains the same despite changes in order and helps children understand the independence of numbers from physical objects. The principle of the insignificance of order shows that the order in which objects are counted does not affect their quantity and helps children develop a more flexible and abstract understanding of numbers and the concepts of equality and commutative property (Gelman & Gallistel, 1978).

The use of technology, such as computer- and tablet-assisted education, can have a significant effect on children's understanding of numbers, as shown in a study by Papadakis et al., (2018). But not all digital games played by children contain educational content, as found in the research conducted by Gözümlü and Kandır (2021) and Gözümlü (2022). To optimize the effectiveness of digital applications in education, developers, educators, and parents need to understand children's number sense profiles. This can be achieved through adapting the content of digital games to suit their skills and interests. Implementing realistic mathematics education activities and digital applications that are tailored to each child's development and interests can create an effective learning environment, as shown by another study conducted by Papadakis et al.,

(2016b). Determining children's number sense profiles can increase the effectiveness of digital applications in the learning environment.

## Number Sense Profiles

To help children understand numbers, we need to identify their number sense profiles. Several studies have looked at children's number and math abilities and found common participant profiles. Wu et al. (2015) found three profiles—high, typical, and low-achieving—with the high-achieving group scoring highest on all scales in their study of low-income preschoolers. Scalise et al. (2021) found four numeracy skill profiles through a latent profile analysis of 6 measures (verbal counting, object counting, cardinality, numeral identification, and symbolic magnitude comparison) in preschoolers: (I) “poor math abilities,” (II) “strong math abilities,” (III) “moderate abilities,” and (IV) “strong counting and numeracy skills, but weak magnitude skills.” Gray and Reeve (2016) assessed children's math abilities through such tasks as object counting, verbal counting, knowledge of Arabic numerals, cardinality, ordinal relationships, and nonverbal arithmetic, and found five distinct numeracy skill profiles (poor abilities, average abilities, good abilities but poor counting, good arithmetic, and excellent abilities) in a sample of middle-income Australian preschoolers. Cahoon et al. (2021) identified five developmental pathways for learning math through a longitudinal study tracking early numeracy skills in preschoolers (43–54 months) at three points in time. Pitalis et al. (2018) followed 6-year-olds ( $n = 204$ ) from the 1st to the 2nd grade to study the development of early number sense and found that basic math development contributes to the latent structure of traditional math and algebraic arithmetic. To support children's number sense, activities should include various number sense skills, and teachers or program developers need to consider children's number sense profiles.

Nanu et al. (2020) examined the effects of the elementary school years on the math skills of 652 children between the ages of 4 and 7 in four European countries. They reported that the formal education years significantly affected math skill profiles, but country differences and chronological age made little impact. Their findings showed that the formal education children receive at an early age significantly supports children with a low math skills profile. Nanu et al. (2020) recommended that future research investigate the impact of early mathematics education on children's mathematics profiles, particularly for those with low profiles. This could be done by examining the differences in mathematics profiles among the preschoolers who receive mathematics education in their preschool years.

## The Present Study

This study is useful for identifying and determining the number sense profiles of preschool and first-grade children and understanding the significance of number sense components in transitioning between profiles. While earlier research has studied children's early mathematics abilities (e.g., Gray & Reeve, 2016; Nanu et al., 2020; Scalise et al., 2021; Wu et al., 2015), this is the first study to specifically profile number sense skills and components in Turkish preschool and first-grade children. Knowing the number sense profiles of Turkish children will provide valuable insight into their specific strengths and weaknesses here and help support early mathematics achievement. The findings of this study will have significant implications for early intervention research, allowing researchers to determine which children require intervention or what educational interventions will support the transition to different profiles. This study aims to address four main questions.

How many number sense profiles do preschoolers and first graders have?

How are number sense profiles distributed among preschoolers and first graders?

Do parental variables significantly predict the number sense profiles of preschoolers and first graders?

What are the profile types for the number sense of preschoolers and first graders?

## Method

This study aims to determine the latent profiles of preschoolers (60–71 months) and first graders of elementary schools (72–83 months) children's number senses and compare the number sense profiles of preschool children and elementary school first-grade children. To do this, the study utilized the development design model, which is appropriate for quantitative research.

With development design, the researcher examines how characteristics within a study group change over time. There are two types of development design models: cross-sectional and longitudinal. The method for analyzing the number sense profiles of preschoolers and first graders through cross-sectional design in number sense research involves comparing two different groups of children at different ages or stages of development using the same parameters. In this instance, the first group is preschoolers (60–71 months) and the second group is first graders (72–83 months). Participants are chosen based on specific criteria and data are collected to analyze and compare their number sense profiles. One criterion for participation in

this study was that the first graders must have attended a preschool institution (Leedy & Ormrod, 2001). The composition of the research groups is explained below.

## Participants

This study was conducted with 1688 children attending state-run preschools and elementary schools in Afyonkarahisar, Türkiye. Table 1 shows the demographic information of the participating children.

From Table 1 we can see that the highest percentage of preschoolers is females (51.2%), as is the highest percentage of first graders (53.8%). When examining how many siblings the children have, we see that 54.6% of the preschoolers and 51.2% of the first graders have two siblings. As for the household incomes of the children, 66.7% of the preschoolers and 54.6% of first graders come from a household with a second rank income (TL 3001–TL 7000). Looking at the education levels of the children's mothers, we see that 38.8% of the preschoolers' mothers and 64.9% of the first graders' mothers have a secondary school education. Regarding the fathers' education levels, 40.3% of the first graders' fathers have a university education, and 47.1% of first graders' fathers have a secondary school diploma. When examining the highest proportional distributions of the participant children, we can see that while gender, birth order, household income, and mother's education level are similar, only the father's education level is significantly different.

**Table 1** Demographic statistics for the participants

Variable	Category	Preschoolers		First graders	
		60–71 Months		72–83 Months	
		N	%	N	%
Gender	Female	414	48.8	388	46.2
	Male	434	51.2	452	53.8
Number of siblings	1 Sibling	161	19.0	101	12.0
	2 Siblings	463	54.6	430	51.2
	3 Or more	224	26.4	309	36.8
Household income	TL 3000 or below	126	14.9	247	29.4
	TL 3001—TL 7000	566	66.7	459	54.6
	TL 7001 or above	156	18.4	134	16.0
Mother's education	Secondary	329	38.8	545	64.9
	High School	260	30.7	163	19.4
	University	259	30.5	132	15.7
Father's education	Secondary	218	25.7	396	47.1
	High School	288	34.0	251	29.9
	University	342	40.3	193	23.0
Total		848		840	

Minimum sample size is a key factor in latent profile analysis. Nylund et al. (2007) argue that a sample size of at least 500 is needed to determine the correct number of latent profiles. As 848 preschoolers and 840 first graders participated in this research, we can say that the criterion for minimum sample size was met.

### Data Collection Tools

The study used two data collection tools. The first tool was a Personal Information Form, and the second tool was the Number Sense Screener (NSS) for children aged 60–71 months and 72–83 months. The tools are described below.

#### Personal Information Form

The researchers created a personal information form that asks questions about the children and their parents. The form includes information such as the child’s gender, the number of siblings they have, the school they attend, the education level of the parents, and the household income.

#### Number Sense Screener (NSS) for Children Aged (60–71 Months and 72–83 Months)

“Number Sense Brief” is a condensed version of the assessment tool used to assess the early numeracy skills of first graders and preschoolers (Jordan et al., 2006, 2007) (33 items). The tool is made up of six subtests and takes between 15 and 20 min to complete. A reliability analysis of the 29-item NSS form conducted using the Rasch model (Jordan et al., 2012a, 2012b) showed a mean value of 0.85 for the entire sample.

The NSS was adapted to Turkish by İnal Kızıltepe (Kızıltepe, 2019) using a sample that included 658 children from 12 schools serving children from the lower-middle and upper socioeconomic levels. Internal consistency analyses were performed in a similar manner to those performed on

the original scales. The reliability coefficients for all the scales in the assessment tool ranged between 0.826 and 0.837 after combining Guttman's lambda-2, the Alpha coefficient, the Feldt-Gilmer coefficient, the Feldt-Brennan coefficient, and Raju's beta coefficient. DIF analysis showed that the Turkish version of the NSS did not contain any gender-biased items.

The tool was then adapted by Uyanık Aktulun (2019) to be used with Turkish children aged 72–83 months-old Turkish children using a sample of 672 children from the lower-middle and upper socio-economic levels. A reliability coefficient ranging from 0.876 to 0.884 was calculated for the entire NSS. The item statistics for this study were calculated based on Classical Test Theory, and are presented in Table 2.

Item Facility in Table 2 represents the mean score of an item relative to its maximum score, ranging between 0.71 and 0.89 for the preschoolers and 0.73 and 0.92 for the first graders. This indicates a good range of difficulty in the performance required from children (O’Sullivan & Stoyhoff, 2012; Rust, & Golombok, 2014). Item discrimination is an indicator of how much an item can discriminate between more and less capable children, based on the correlation between performance for that item and the rest of the assessment. A value of around 0.3 or above is usually taken to indicate a good level of discrimination. Item discrimination for the preschoolers ranged from 0.28 to 0.58 and for the first graders from 0.31 to 0.63. This indicates that the items worked very well. It also contributes to the assessment tool’s high reliability scores for the preschoolers (0.90) and the first graders (0.93) using Guttman’s lambda-2 (Rust, & Golombok, 2014).

### Data Collection

We collected the data for this study after obtaining the approval of the ethics committee at Afyon Kocatepe University (approval number 91909). We obtained consent forms from the parents of the participating children. The researchers informed the parents that the children’s personal

**Table 2** Internal consistency and item statistics regarding the sub-dimensions of the NSS

Variable	# of items	Preschoolers			First graders		
		Item		Guttman’s lambda-2	Item		Guttman’s lambda-2
		Facility	Discrimination		Facility	Discrimination	
NSS Total	29	0.71–0.89	0.28–0.58	0.90	0.73–0.92	0.31–0.63	0.93
COUNT	3	0.63–0.94	0.21–0.60	0.74	0.69–0.93	0.34–0.64	0.75
NREC	4	0.75–0.96	0.23–0.61	0.72	0.70–0.94	0.20–0.65	0.76
NCOMB	7	0.75–0.91	0.20–0.56	0.71	0.70–0.96	0.19–0.61	0.73
NVCAL	4	0.76–0.90	0.24–0.56	0.72	0.71–0.94	0.25–0.61	0.74
SPROB	5	0.63–0.94	0.23–0.63	0.74	0.74–0.96	0.27–0.59	0.80
NCOMP	6	0.66–0.93	0.22–0.60	0.84	0.71–0.90	0.19–0.58	0.89

information would only be used for research purposes and would not be shared with any third parties. The study employed four testers, each trained in how to administer the NSS. These testers were postgraduate students studying in the Department of Early Childhood Education and had experience working with children. The testers collected the data under the supervision of the researchers. The data were collected 15 days after the start of the semester to let the children become accustomed to the school. Data were collected simultaneously from the different age groups using a cross-sectional design. The testing environment was kept quiet, well-lit, and ventilated. The children were introduced to the tester a week before the test. On the day of the test, the children were shown a sample question and asked if they wanted to participate. The tests were voluntary and the children were made to feel comfortable. We scored the tests using the criteria established by Jordan et al., (2012a, 2012b) in the original scale. We gave a score of 1 for each correct answer in the subtests and 0 for questions answered incorrectly or not at all.

### Analysis of Data

The reason for choosing the latent profile analysis method in the study was to determine the latent profile of children's number sense using the observed variables of counting ability. The study employed a latent profile analysis method to identify the latent profiles of children's number sense using the observed variables of counting ability. The methodology followed the classification approach proposed by Lazarsfeld and Henry (1968) and was carried out in three stages.

The first stage involved establishing a latent profile model using the observed variables without determining which variables to include or the number of latent profiles. Assumptions were tested, including independence (observed variables of children's number sense are independent given latent class membership), homogeneity (observed variables have the same distribution within a latent class), and exogeneity (latent class membership is independent of observed variables).

In the second stage, we determined the latent profiles of children's number sense based on the estimated parameters for the model proposal and the observed variables, as well as latent profile probabilities. We chose the number of latent classes based on model fit indices and results interpretation. We assessed and reported the model fit and results, including the profile of each latent class in terms of observed number sense skills.

Finally, we validated the model using additional data or methods, such as cross-validation or structural equation modeling. We used logistic regression to investigate the relationship between the latent profiles of children's number sense and the co-variables (parental age and education

**Table 3** Descriptive Findings Regarding the Sub-dimensions of the NSS

Variable	# of items	Preschoolers		First graders	
		Mean	sd	Mean	sd
NSS Total	29	16.38	6.71	22.18	6.64
COUNT	3	2.81	0.53	2.89	0.55
NREC	4	1.25	1.23	2.70	1.15
NCOMB	7	4.71	1.99	5.52	1.62
NVCAL	4	2.84	1.24	3.15	1.12
SPROB	5	2.25	1.68	3.12	1.7
NCOMP	6	2.57	2.21	4.03	2.13

**Table 4** Model fit indices for the latent profiles analysis of the NSS

Model		AIC	BIC	BLRT_p	Entropy
Preschoolers	1 Profile	17,472.6	17,529.5		1.00
	2 Profiles	16,123.4	16,213.5	0.01	0.88
	3 Profiles	15,215.8	15,372.3	0.01	0.88
	4 Profiles	15,387.9	15,511.2	0.72	0.91
First Graders	1 Profile	16,715.8	16,772.6		1.000
	2 Profiles	14,607.8	14,697.8	0.01	0.932
	3 Profiles	13,645.0	13,768.1	0.01	0.929
	4 Profiles	13,400.4	13,556.6	0.92	0.710

LPA's did not converge when the number of profiles was greater than 4, so further estimates are not included here

level). We validated the model by using logistic regression, as recommended by Vermunt (2010).

### Assumptions of Model Fit

There are several fit indices for evaluating the fit of latent profile analysis, a part of the model fit assumption (Nylund et al., 2007). In this study, we analyzed the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Bootstrap Likelihood Ratio Test (BLRT) fit indices, selecting the model with the smallest AIC value, according to Akaike (1987), and the model with the smallest BIC value, according to Schwartz (1978). McLachlan and Peel (2000) explain that the BLRT value compares the K-profile model to the K-1-profile model and tests if adding a profile improves the K-1 model. After checking the model fits' homogeneity and profile separation, the entropy value is analyzed to determine profile classification uncertainty. A higher entropy value in latent profile analysis means higher uncertainty, while a value closer to 1 indicates lower uncertainty (Cheng, 2012; Collins & Lanza, 2010). Table 4 displays the fit index values (AIC, BIC, BLRT) used in this study and the entropy values used to identify the latent profile classes. The normality of the number sense scores for

both the preschoolers and the first graders was found to be between  $-1$  and  $+1$  and showed a normal distribution.

### Findings

The data analysis results are presented in the findings section, along with the research questions. The Number Sense Screener (NSS) subtest descriptive statistics (mean and standard deviation) and reliability statistics for the children are summarized in Table 3.

Table 3 shows that when the mean scores of number sense were analyzed for the preschoolers, the highest score was seen in Number Comparison ( $\bar{x} = 4.71$ ) and the lowest in Number Recognition ( $\bar{x} = 1.25$ ). For the first graders, the highest mean score was in Number Comparison ( $\bar{x} = 5.52$ ) and the lowest in Number Recognition ( $\bar{x} = 2.70$ ). Both the preschoolers and the first graders showed a similar pattern for the highest and lowest scores in their age group. The fit indices used to determine the latent profiles of children's number sense in latent profile analysis are presented in Table 4.

How many number sense profiles do preschoolers and first-graders have? The findings of this research question are described in Table 4.

Table 4 shows that the three-profile solution is the best fit for both preschoolers and first graders because when the number of profiles is three, they have the lowest BIC values and significant BLRT\_p values ( $p < 0.05$ ). Although the entropy values were high when the number of profiles set for the preschoolers was four, the BLRT\_p values were not significant. As the profile count among the first graders and the BIC value among the preschoolers were similar, we decided that the profile count should be three. We labeled the resultant three profiles Moderate Very Low, Moderate Low, and Moderate High.

How are number sense profiles distributed among preschoolers and first graders? The findings of this research question are described in Table 5.

Table 5 shows how the preschoolers and first graders were distributed among the three number sense profiles determined in Table 4. The largest profile size was Moderate Very Low with 51.9% of the students, followed by Moderate

High and Moderate Low with 35.1 and 13% of the students, respectively. While the Moderate Low profile for the first-grade students ranked second as it did for the preschoolers with 24.5%, we can see that the smallest and highest profiles have changed places with the largest profile being Moderate High at 66.3% and the smallest being Moderate Very Low at 9.2%

We determined the latent profiles by examining and interpreting the results from Table 5 and Fig. 2. Figure 2 illustrates the preschoolers' number sense profiles. We can see in Table 3 that 51.9% of preschoolers had moderate-very low number sense skills and scored below the mean in all sub-dimensions of NSS. The most striking difference was seen in the Counting Skill sub-dimension, which was lower than the mean of all other sub-dimensions. A moderate-low number sense profile was found in 13% of preschoolers, with scores close to the mean. The difference between the moderate very low and moderate low profiles was found to be in the mean of the Counting Skill scores. Meanwhile, 35% of preschoolers had a moderate high number sense profile with high Number Combinations scores. We concluded that Counting Skills differentiate the moderate very low and moderate low profiles and that Number Combinations scores differentiate the moderate low and moderate high profiles.

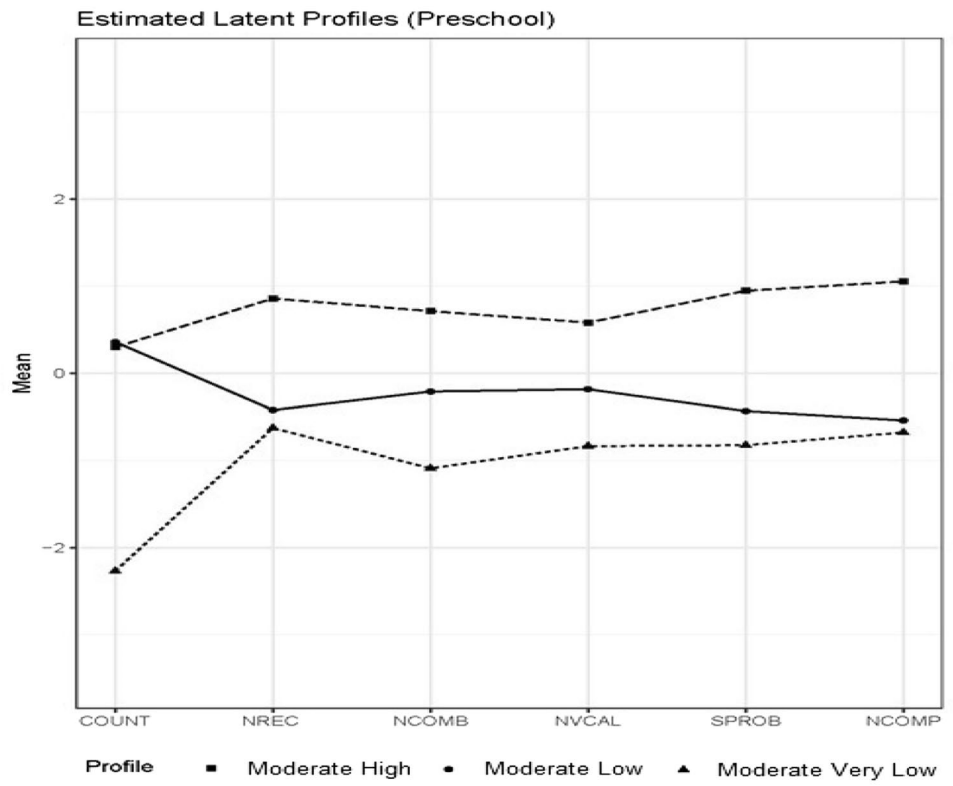
We determined the latent profiles of the first graders by examining the results from Table 5 and Fig. 3. Figure 3 illustrates the number sense profiles of the first graders. Only 9% of them had a moderate very low profile. All the sub-dimension values of the moderate very low-profile first graders were below the mean, with the lowest being the Counting mean. Meanwhile, 24.5% of the first graders had a moderate low profile, with mean scores below zero in Number Recognition, Number Comparison, Non-verbal Calculations, Story Problems, and Number Combinations. In the moderate low profile, only the counting score was above the mean. We found that 66.3% of the first graders had a moderate high profile and that the Number Combinations mean separated the moderate low profile from the moderate high profile. That 66% of the students fall into the high profile may indicate that number sense skills are acquired by the majority during elementary school education.

Do parental variables significantly predict the number sense profiles of preschoolers and first graders? The findings

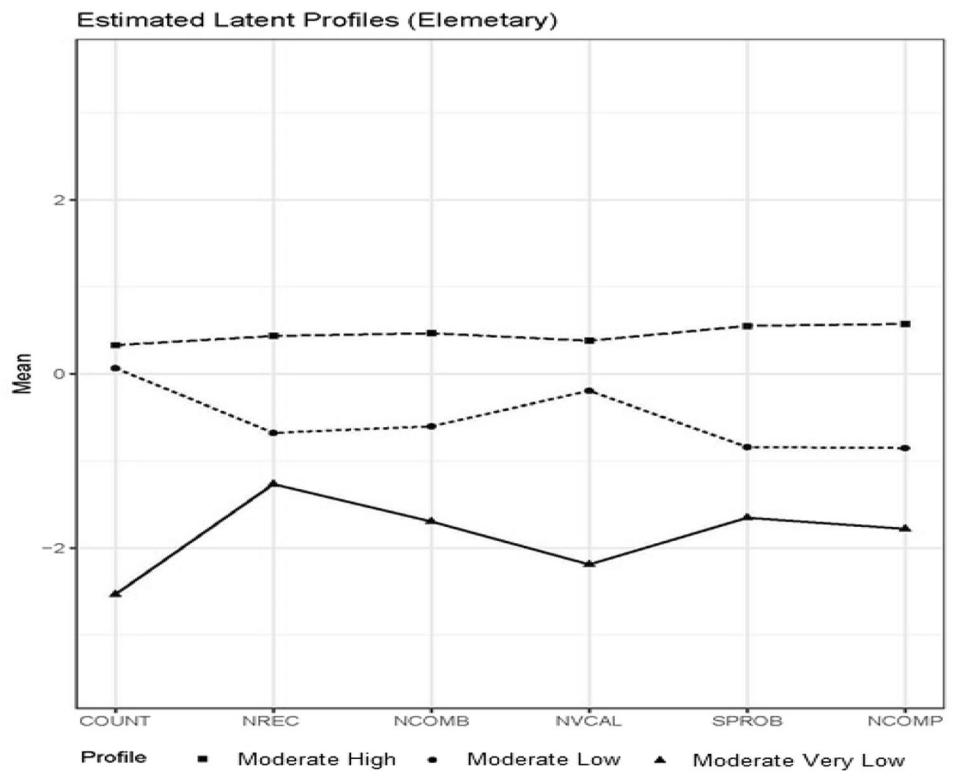
**Table 5** Profile sizes and assignment probabilities for the NSS

Profiles	Preschoolers			First graders		
	Profile size		Assignment probability	Profile size		Assignment probability
	(N)	(%)		(N)	(%)	
Moderate Very Low	440	51.9	.966	77	9.20	.980
Moderate Low	110	13.0	.975	206	24.5	.949
Moderate High	298	35.1	.950	557	66.3	.975

**Fig. 2** Estimated LPA profiles of the Preschoolers



**Fig. 3** Estimated LPA profiles of the First Graders



**Table 6** Multinomial logistic regression predicting NSS latent profile membership

Variable	Number sense profiles	Preschoolers			First graders		
		Beta (Odds Ratio)	95% CI	p	Beta (Odds Ratio)	95% CI	p
Mother's education	Moderate Very Low				−0.294	[−0.58, −0.04]	0.32
	Moderate Low	−0.321	[−0.50, −0.13]	.043*	−0.370	[−0.55, −0.19]	0.04*
	Moderate High	0.167	[0.04, 0.29]	.027*			
Father's education	Moderate Very Low				−0.507	[−0.75, −0.25]	0.04*
	Moderate Low	−0.111	[−0.28, 0.06]	0.52	−0.304	[−0.46, −0.14]	0.06
	Moderate High	0.278	[0.14, 0.41]	.036*			
Household income	Moderate Very Low				0.293	[0.16, 0.42]	0.02*
	Moderate Low	0.065	[−0.11, 0.18]	0.71	0.150	[−0.04, 0.34]	0.43
	Moderate High	−0.271	[−0.40, 0.13]	0.04*			

\* $p < .05$  \*\*reference category

of this research question are shown in Table 6. This table shows the demographic variables used to predict the profiles. We used multinomial logistic regression (MLR) to make the analysis. The MLR results for the profiles of the preschoolers and the first graders are shown in Table 6. The reference categories were set to “moderate very low” profile for the preschoolers and “moderate high” profile for the first graders because they are determined based on the highest number of students in each category (McNulty, 2021).

The results of the MLR analysis are presented in Table 6. We used the moderate very low profile as the reference category for preschoolers because it had the highest number of children. The results show that the mother's level of education is a significant predictor of the transition between number sense profiles. Analysis shows it to be a strong factor in determining whether the preschoolers move from the very low profile to low ( $p = 0.043$ ;  $p < 0.05$ ) or high profile ( $p = 0.027$ ;  $p < 0.05$ ). Similarly, the father's education level is a significant factor in determining the transition from the very low profile to the high profile ( $p = 0.036$ ;  $p < 0.05$ ). Household income is a significant predictor in the transition from the very low to the high profile ( $p = 0.004$ ;  $p < 0.05$ ).

We used the moderate high profile as the reference category for first graders because it had the highest number of students. The mother's education level is a significant predictor of the transition from the low profile to the high profile ( $p = 0.004$ ;  $p < 0.05$ ), while the father's education level is a significant predictor of the transition from the very low profile to the high profile ( $p = 0.004$ ;  $p < 0.05$ ). Household income is a significant predictor in the transition from the very low profile to the high profile based on the number sense profiles of the first graders ( $p = 0.02$ ;  $p < 0.05$ ). These results provide important information on factors that influence the latent profiles of the students.

What are the profile types for the number sense of preschoolers and first graders? In light of all the findings, we defined the profiles related to the models shown in Fig. 2 and

Fig. 3 to name and describe the number sense profiles of the preschoolers and first graders. The number sense profiles of the preschoolers are:

### Preschool Number Sense Profile I: Moderate Very Low

Since the preschoolers in Profile I had early number knowledge (counting, number recognition, number comparisons, and nonverbal calculation) and arithmetic skills (story problems and number combinations) far below the mean, we called this the *moderate very low* profile. This profile included 51.9% of the participant children who were just starting preschool (see Table 5). Analysis of their number sense components showed that counting was the lowest and number identification was the highest (see Fig. 2). We can say that the biologically developed “low order” dimension of number sense of these children was low and the “high order” dimension developed through education was also low. The MLR analyses showed that the mother's and father's education and household income variables were significant predictors in differentiating between the children in the moderate very low number sense profile and those in the preschool moderate high profile (see Table 6). Since the “high order” number sense components of the children in the moderate very low group were much lower than the mean, this suggests that not enough education to support number sense development was provided before preschool education began. In summary, we can say that the children in this profile require urgent intervention to improve their number sense development.

### Preschool Number Sense Profile II: Moderate Low

Since the preschoolers in Profile II had number knowledge and arithmetic skills slightly below the mean, we called this the *moderate low* profile. This profile included 13%

of the participant children who had just started preschool (see Table 5). Analysis of their number sense components showed that the lowest was number combinations and the highest was counting (see Fig. 2). We observed that the “low order” and “high order” dimensions of number sense of these children improved compared to the preschoolers in Profile I. The MLR analyses showed that the mother’s education level was a significant predictor in differentiating the children in the moderate very low number sense profile from the children in the moderate low profile. The father’s education and household income variables of the children in the moderate low number sense profile are different from those in the moderate very low profile (see Table 6). We can say, therefore, that the education level of the mothers of children in the moderate low number sense profile is critical for the development of number sense. In short, the children in this profile also require intervention to improve their number sense development.

### Preschool Number Sense Profile III: Moderate High

Since the preschoolers in Profile III had early number knowledge and arithmetic skills above the mean, we called this the *moderate high* profile. This profile includes 35.50% of the participant children who have just started preschool (see Table 5). Analysis of the number sense components of these children showed that the lowest was counting and the highest was number combinations (see Fig. 2). We observed that the “low order” and “high order” dimensions of number sense for these children were better than those in preschool number sense Profile I and Profile II. The MLR analyses showed that the mother’s education, father’s education, and household income variables were significant predictors in differentiating the children in the moderate high number sense profile from the children in the moderate very low profile (see Table 6). Children in the moderate high number sense profile did not differ significantly from children in the moderate low profile in terms of the father’s education level. This suggests that family and socio-economic factors in addition to primary biological abilities have an effect on the above-mean counting skills of children in the moderate low and moderate high number sense profiles. Considering that the number sense development of children in this profile was higher than the mean, the educational activities to be organized for these children should be suitable for the “high order” number dimension.

After examining the number sense profiles of the preschoolers, we then examined the number sense profiles of children who had just started the first grade. We named and described them as detailed below.

### First Grade Number Sense Profile I: Moderate Very Low

Since the first graders in Profile I had early number knowledge and arithmetic skills well below the mean, we called it the *moderate very low* profile. This profile included 9.20% of the participant children who started the first grade (see Table 5). Analysis of the number sense components of the children in this profile are analyzed showed that counting was the lowest and number identification was the highest (see Fig. 3). The development of number sense components was similar to the moderate very low profile for preschoolers. The fact that the children’s “high order” dimension of number sense is below the mean suggests that the preschool education they received did not support their number sense components. As such, the children’s “low order” dimension did not develop before they started preschool education. The “low order” dimension corresponds to the primary biological abilities in number sense, meaning that the children’s primary biological abilities were not developed before their preschool education began. The MLR analyses showed that the father’s education and household income variables were significant predictors in differentiating between the children in the moderate very low number sense profile from those in the moderate high profile (see Table 6). In short, we can say that first graders in this profile were the group that needs urgent intervention in terms of number sense development. Since children’s development (or underdevelopment) of the “low order” dimension of number sense involves biological competencies, they could be “at risk” children with respect to math learning skills. The development of the “low order” and “high order” dimensions of the number sense components should be monitored during the education that these children will receive in the first grade, and the authorized bodies should be informed for early intervention if no improvement is seen.

### First Grade Number Sense Profile II: Moderate Low

Since the first graders in Profile II had number knowledge and arithmetic skills slightly below the mean, we called it the *moderate low* profile. This profile included 24.50% of the participant children who had just started first grade (see Table 5). Analysis of their number sense components showed the lowest to be number combinations and the highest to be counting (see Fig. 3). The development of number sense components was similar to the moderate very low profile of preschoolers. We observed that the “low order” and “high order” dimensions of number sense of these children developed like the first-grade number sense Profile I. The MLR analyses showed that the mother’s education variable was a significant predictor in differentiating the children in the moderate low number sense profile from those in the

moderate high profile. The father's education and household income variables made no significant difference (see Table 6). We can say, therefore, that the mother's education level is critical in the development of number sense of first graders in the moderate low number sense profile. In summary, children in this profile are in the group that requires intervention in terms of number sense development. Interestingly, the counting skills of first graders in the moderate low group, in the "low order" dimension of number sense, were above the mean. This shows that the counting skill, one of the biologically developed number sense skills that children have, develops above the mean. The "high order" dimension of number sense can be expected to be above the mean due to the preschool education received by children in the first grade. We recommend monitoring the development of number sense components of children in the moderate low group. If their number sense development does not improve during the first grade, the moderate low group can be included in the risk group.

### First Grade Number Sense Profile III: Moderate High

Since the first-grade children in Profile III had early number knowledge and arithmetic skills above the mean, we called it the *moderate high* profile. This profile included 66.30% of the participant children who just started first grade (see Table 5). Analysis of the number sense components of these children showed that the lowest was counting and the highest was number combinations (see Fig. 3). We observed that these children's "low order" and "high order" dimensions of number sense were better than Profile I and Profile II. Considering that the children received preschool education, the development of "high order" components is normal. The MLR analyses showed that the father's education and household income variables were significant predictors in differentiating the children in the moderate high number sense profile from the children in the moderate very low I (see Table 6). This suggests that preschool education as well as family and socio-economic factors affect the above-mean counting skills of children in the moderate high number sense profile. Considering that the children in this profile had a higher level of number sense development, the educational activities to be organized for children should be planned around the "high order" number dimension components.

### Conclusion and Discussion

When we analyzed the number sense profiles of the preschoolers and the first graders, we observed three profiles: much lower than mean, lower than mean, and higher than mean. According to our findings, the profiles (based on the number sense profile descriptors) that require particular

attention are first graders who are below the mean and much below the mean. Studies (Okamoto & Case, 1996; Berch, 2005; Jordan et al., 2006, 2007, 2009, 2010; Östergren & Traff, 2013) have reported that while children develop "*early number knowledge*" biologically in the early years, their "*arithmetic skills*" develop as a result of mathematics education. The mean scores for both early number knowledge and arithmetic skills of first graders and preschoolers were below the mean in Profile I (moderate very low, 9.20%) and Profile II (moderate low, 24.50%). We determined that the arithmetic skills of the children (33.70%), which are expected to develop in line with the mathematics education they receive in the preschool period, were below the mean. Yet, 66.30% of the children who started the first grade showed a profile above the mean (Profile III). Since the development of number sense depends on both the child's maturity and the education received by the child, the process that needs to be discussed is a complex one. The proportional distribution of the profiles of preschoolers participating in this study sheds light on both maturation and education in the transition from preschool to elementary school. The proportional distribution of the Profile I (moderate very low, 51.90%), Profile II (moderate low, 13.00%), and Profile III (moderate high, 35.10%) preschoolers participating in this study increases in Profile I. Some 64.90% of preschoolers had number sense skills below the mean. This shows that children's number sense skills improve in elementary school rather than in the preschool period. The improvement in the latent profiles of number sense of elementary school children may be due to them maturing between preschool and elementary school, their readiness levels, and the learning outcomes of the math activities they did in preschool. Preschool education and growing older both play a critical role in the positive change in their number sense profiles. Polat Unutkan (2007) examined the literature on this result and found that the mathematics skills of six-year-old children were more developed than those of five-year-old children. Similarly, Avcı (2015) investigated the effect of age on mathematics achievement and found that the age variable significantly predicted the mathematics scores of preschoolers aged 48–66 months. In another study supporting our findings, Aunio and Niemivirta (2010) reported that age had a positive effect on children's mathematical performance and that the older age group was more successful than the younger age group. Hornburg et al. (2018) reported significant correlations between age and various arithmetic skills, including one-to-one counting, connecting numbers to quantities, set comparison, number comparison, and subitizing. Remarkably, the development of mathematical skills becomes more complex with age (Aunio & Niemivirta, 2010; Jordan et al., 2006). Aslan and Aktaş Arnas (2015) and Karademir and Akman (2019) also reported that children must receive preschool education if they are to develop early math skills.

However, the cross-sectional nature of the study limits the interpretation of these results.

According to Heuvel-Panhuizen (2008), children can perform pure counting by reaching level 3 during preschool education. If first graders are below the mean (Profile I–II) despite maturing and receiving preschool education, this constitutes a risk. Bird (2009) found that children with specific learning disabilities such as dyscalculia cannot subitize. In other words, counting skills are associated with dyscalculia. We must, therefore, consider number sense profiles when supporting the number sense skills of children with normal development and those at risk of dyscalculia. Providing early educational intervention for children with low number sense profiles is critical to preventing risky situations at later education levels. If we can identify the number sense components that cause differentiation in the number sense profiles of both preschool and elementary school children, we can increase the effectiveness of educational intervention at an early age. The preschoolers who participated in the study had similar early number knowledge and arithmetic skills, which are components of number sense, to the first-grade Profile I and Profile II. Equipping the learning environment with rich stimuli in support of the “low order” dimension of preschool children’s number sense can support the number sense skills that are expected to develop biologically. Children need preschool education to develop the “high order” dimension of number sense. And since improving the “low order” dimension of the preschoolers in the moderate very low profile will contribute to the development of the “high order” dimension, mathematics education can begin by supporting the early number-knowledge dimensions of the number sense components. In this respect, the Profile I (moderate very low) characteristics of preschoolers can correspond to Level 0 number sense development as defined by Van Den Heuvel-Panhuizen (2008). When we examined the Profile II characteristics of preschoolers, we found that the feature that distinguishes it from Profile I was children’s counting skills being above the mean. Profile II preschoolers were better prepared for preschool education and could be taught at Level I according to Heuvel-Panhuizen (2008), since their counting skills were better than those in Profile I. Educators can take this into consideration and use the “low order” component to support the “high order” component by associating it with counting when planning educational activities based on what the children do and do not know (Berch, 2005). While educators support children’s number sense skills in the “low order” dimension, Gelman and Gallistel’s (1978) counting principles can be used to support children’s development.

The findings of our study suggest that supporting the counting component of the early number knowledge dimension of the preschool and first-grade Profile I children is a key method of intervention because the counting component

was the critical feature that differentiated Profile I from Profile II and Profile III for both the preschoolers and first graders. Both the number sense development and mathematics achievement of children whose counting component does not develop from preschool to elementary school may be at risk in the following years. In their study to document the developmental trajectories of number knowledge in children from preschool to elementary school and determine the extent to which this predicts later mathematics achievement, According to Garon-Carrier et al. (2018) presented a 4-trajectory model characterized by low-increase (10% of children), medium-increase (39%), medium-fast-increase (32%) and high-increase (19%) groups. They found that children with a low numeracy profile always performed lower in mathematics at the age of 8–10 years than children with other profiles, suggesting the need for early intervention. Supporting children with low number sense in early childhood will positively affect their academic achievement in later years. Examining the literature on early education intervention for number sense skills, we see that Hawes et al. (2019) reported that children’s knowledge of symbolic numbers before they start elementary school coupled with a certain amount of support will increase their likelihood of being successful in mathematics when they start elementary school. Caferoğlu (2020) found a significant difference between the number sense of sixth- and seventh-grade students who had never received preschool education compared with those who had received preschool education for one year or more. Longitudinal studies on children’s mathematical skills have shown that early factual and conceptual skills such as number-link combinations, counting, pattern recognition, and calculation skills affect children’s acquisition of complex skills such as problem-solving in later years (Björn, et al., 2016; Fuchs et al., 2006; Rittle-Johnson et al., 2016). Moreover, children with retarded counting skills in early childhood continue to have difficulties improving their counting skills in the later stages of their education (Geary, 1993; Jordan et al., 2003). Supporting children’s counting skills is critical for helping children in the moderate very low profile to transition to the moderate low profile in preschool and elementary school and may help prevent the risk of developmental dyscalculia in children.

Another noteworthy finding of this study was that both the preschoolers and the first graders’ arithmetic skills (Profile III) were more developed than their early number knowledge. According to Heuvel-Panhuizen (2008), children who receive preschool education can reach Level 3. The fact that 66.30% of the first graders participating in this study had number sense development above the mean suggests that preschool education positively affects children’s number sense development. But the fact that 33.70% of the children who just started preschool education showed Profile III characteristics tells us that the activities to be organized should

be carried out individually according to the children's number sense development levels. Analyzing the findings of this study in this context, we observed that children with developing arithmetic skills were above the mean. Studies (Berch, 2005; Jordan et al., 2006, 2007, 2009, 2010a) show that early number sense development supports the development of arithmetic skills. If educators and researchers focus only on counting skills among the number sense components of the preschoolers and the first graders, they will not be able to distinguish between children in Profile II and Profile III. The counting skill means of both the preschoolers and first graders were very close to each other (Profile II and Profile III). The subcomponents of the arithmetic skills dimension, first number combinations and then story problems, are critical components in distinguishing between all three profiles of both the preschoolers and first graders. While Number combinations is an important component of arithmetic skills in differentiating between children whose number sense development is below the mean (Profile I–II) and those above the mean (Profile III), counting is an important component of early number knowledge in differentiating between children whose number sense development is below the mean (Profile I–II). Studies report individual differences in the level and development of children's number sense skills such as counting, number recognition, number comparison, non-verbal calculation, story problems, number combinations, and the relationship between these skills. Dowker (2008) found that some preschoolers were procedurally unable to count a sequence but understood how the number could be used to determine cardinality. Another group of children had basic counting skills but lacked basic knowledge, suggesting that some preschoolers may be successful in more complex tasks such as cardinality despite poor performance in basic numeracy skills such as counting. This result suggests that the acquisition of all numeracy skills does not follow the same developmental pathway for all individuals (Dowker, 2008). The number sense profiles of the children participating in this study were homogeneous and generalizable for their country of residence. But the number sense profiles of children living in different cultures and geographies may vary depending on both biological and environmental factors (Wu et al., 2015; Scalise et al., 2021; Gray & Reeve, 2016; Cahoon et al., 2021; Pitalis et al., 2018; Nanu et al., 2020). Comparing the results of this study with the number sense profiles of children from different cultures (Wu et al., 2015; Scalise et al., 2021; Gray & Reeve, 2016; Cahoon et al., 2021; Pitalis et al., 2018; Nanu et al., 2020), we found that what they have in common is that number sense develops with age. Longitudinal research has found that the age at which children understand the basics is central to later numeracy knowledge, better math achievement, and school readiness (Geary et al., 2018). In addition, Chu et al. (2016) found that the development of intelligence and executive

functions in four quantitative competencies (cardinality, discrete quantity discrimination, number recognition, non-verbal calculation) in preschool and the acquisition of these four quantitative competencies predicted later mathematics achievement. However, those studies used traditional variable-centered approaches to analysis that investigate the average level of development, assuming that the relationship between variables applies equally to all students. This poses a risk to the development of children's number sense.

The difference between children's number sense profiles may be due to their developmental characteristics and the education they receive but may also be influenced by the socio-cultural environment (Bronfenbrenner, 1979). Gould (2014), Ramani and Siegler (2011), and Starkey et al. (2004) found that children in the lower socio-economic level showed poor number sense performance. Jordan et al (2006), on the other hand, suggested that socioeconomic level is more effective in verbal number sense tasks than non-verbal tasks. The study by Tonizzi et al. (2021) reported that five-year-old Italian children from the middle-upper socioeconomic level who took part in their intervention program on number sense demonstrated higher numerical skills than children from lower socioeconomic levels and that all intervention groups scored higher when compared with the control group. This study showed that household income level was a significant predictor of differentiation between the number sense profiles of both the preschoolers and the first graders in the transition from low profile to high profile. We should note here that lack of social support, lack of material resources, housing instability, low parental education, and limited cognitive stimulation are among the indicators of low socioeconomic and cultural levels in children's lives (Evans, 2004; Janus & Duku, 2007). This study aimed to investigate whether parents' education levels significantly predicted the number sense profiles of preschoolers and first graders. The findings revealed that the mother's education level was a significant predictor of the transition from the moderate very low profile to the moderate low and moderate high profile for the preschoolers, as well as a significant predictor of the transition from the moderate profile to the moderate high profile for the first graders.

Another important finding was that the education level of the father was also a significant predictor of the number sense profiles of the children. PISA 2018 results and the related socio-cultural index also reveal the effect of the parents' education levels on mathematics and mathematical literacy. Aunio et al. (2006) stated that in their study with students aged between 4.5 and 7.5 years living in China and Finland to examine the intercultural change of number sense, children with highly educated mothers scored higher on number sense tests. They observed that the number sense scores of the students with highly educated fathers were also high, albeit not significant. Similarly, Şahin (2018) and Peker

(2019) concluded that higher mother's and father's education levels positively affected children's number sense skills. By contrast, Çekirdekçi (2015) stated that parents' education levels did not make a significant difference in children's number sense. Our study showed that the education level of the mother is a variable that causes a significant difference in the number sense profile of the children, consistent with the results of many other studies. Another important finding was that the education level of the father was a significant predictor of the transition from the moderate very low profile to the moderate high profile for both grade levels. The empirical findings in this study provide a new understanding of educating parents for further training.

Using technology to help develop number sense is good for children. According to Papadakis et al. (2016a), technological tools offer innovative opportunities to support mathematical achievement when used appropriately to children's development. Another study recommended that researchers conducting research into serious educational content for the application of technological tools should produce content for each profile, taking into account the dimensions and components of number sense that will cause differences between profiles (Papadakis et al., 2021). The results of that research presented the profile characteristics of children to experts who both prepare educational programs and produce content for digital applications. The experimental study conducted by Papadakis et al. (2018) found that mathematics activities conducted with technological tools supported children's understanding of numbers. Technological tools are thought to be critical for supporting the understanding of numbers in the Profile I and Profile II groups, which are the priority groups for intervention at the preschool and elementary levels. Children's profile characteristics can guide the development of content for digital applications. Furthermore, children's early arithmetic skills such as number combinations and story problems supported by technological tools in realistic mathematics activities (Papadakis et al., 2016b) can provide support for preschool and first-grade Profile III students, who already have strong number sense profiles. Papadakis et al. (2021) investigated the mathematics development of children in realistic mathematics education using tablets. They analyzed the content produced in terms of the Heuvel-Panhuizen (2008) number sense development levels. Since the study discussed the relationship between the Heuvel-Panhuizen (2008) number sense development levels and both preschool and first-grade profiles based on the analysis results, assessments elements that will distinguish children's profiles in the digital content to be developed can be added to the digital content. Considering how inexpensive access to technology is, it offers more advantages than physical objects when it comes to supporting the number sense skills of underprivileged children with low household incomes and low parent education levels (Sarama & Clements, 2009b).

Overall, the study suggests that developing technological content and mathematics education programs according to children's number sense profile characteristics can be an effective way of supporting their learning, especially for underprivileged children from poor socioeconomic backgrounds.

## Limitations and Suggestions for Future Research

The main limitation of this study is that it is cross-sectional. The extent to which children in different age groups have similar characteristics is a key limitation in cross-sectional studies. We, therefore, recommend that future studies be done longitudinally. The number of children included in the sample can also be increased in future studies. Our study showed that counting skills are a cause of differentiation in the children's number sense profiles, so we recommend planning experimental research to investigate counting skills. Future studies could conduct qualitative research to observe the sub-components of children's number sense and reveal the specific skills that differ.

## Recommendations

The results of the study showed that children have number sense skills in different profiles. This finding suggests that curriculum developers, developers of mobile apps, researchers, and educators should consider the individual differences of children. Focus should also be placed on the roles played by parents, not just educators, in supporting children's number sense skills, and parental education courses should be arranged on how to improve children's number sense. One of the most critical recommendations in this study is that early childhood education should be compulsory in Türkiye. The similarities between the number sense profiles of the preschoolers and the first graders and their distributions show that preschool education should begin at an earlier age.

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## Declarations

**Conflict of interest** The authors declare that they have no conflict of interest.

**Ethical Approval** All procedures performed in studies involving human participants were in accordance with the ethical standards of Afyon Kocatepe University—Türkiye research committee (document no: 91909) and the 1964 Helsinki Declaration and its later amendments or comparable ethical standards.

**Consent to Participate** All participants provided informed consent prior to participating in the study.

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